SMITHSONIAN CONTRIBUTIONS TO KNOWLEDGE.

DISCUSSION

OF THE

MAGNETIC AND METEOROLOGICAL OBSERVATIONS

MADE AT THE GIRARD COLLEGE OBSERVATORY, PHILADELPHIA,
IN 1840, 1841, 1842, 1843, 1844, AND 1845.

1840 – 1845

PART I.

INVESTIGATION OF THE ELEVEN YEAR PERIOD IN THE AMPLITUDE OF THE SOLAR-DIURNAL VARIATION AND OF THE DISTURBANCES OF THE MAGNETIC DECLINATION.

BY

A. D. BACHE, LL. D.

[ACCEPTED FOR PUBLICATION, JUNE, 1859.]

National Oceanic and Atmospheric Administration

Environmental Data Rescue Program

ERRATA NOTICE

One or more conditions of the original document may affect the quality of the image, such as:

Discolored pages
Faded or light ink
Binding intrudes into the text

This document has been imaged through the NOAA Environmental Data Rescue Program. To view the original document, please contact the NOAA Central Library in Silver Spring, MD at (301) 713-2607 x124 or www.reference@nodc.noaa.gov.

Information Manufacturing Corporation
Imaging Subcontractor
Rocket Center, West Virginia
September 14, 1999

COMMISSION

TO WHICH THIS MEMOIR HAS BEEN REFERRED.

Prof. BENJ. PEIRCE,
Prof. W. CHAUVENET.

JOSEPH HENRY,
Secretary S. I.

COLLINS, PRINTER,
PHILADELPHIA.

INTRODUCTION.

In co-operation with the scheme adopted at the British colonial observatories, a series of magnetic and meteorological observations were made at the Girard College magnetic observatory, in Philadelphia, with instruments purchased under the direction of the trustees of the college, the observations being made under the patronage of the American Philosophical Society, and finally completed for the use of the topographical bureau of the War Department.[1] These observations were made under my immediate direction, and were afterwards left under my general superintendence. The series commenced in May, 1840, and, with short interruptions, terminated in June, 1845, thus furnishing a five years' series of magnetic observations, taken bi-hourly up to October, 1843, and after that date hourly. The readings of each magnetic element were united into mean values, arranged according to hours of the day and days of the month and annual values, and presented graphically, under my direction, by Joseph S. Ruth, Esq., who had taken part in the observations, and who was at that time employed in the Coast Survey. As, owing to other laborious duties, the record could not then be submitted to a complete reduction and discussion, I have resumed the subject, with the aid of Charles A. Schott, Esq., assistant in the Coast Survey, by whom, under my immediate direction, the discussions contained in this paper have been made and prepared for publication. It is proper to state that this work has been performed out of office hours by Mr. Schott, as my assistant in this special matter, and at my own expense.

Although the magnetic observatories furnished by their judicious geographical location, a basis for the generalization of their results, it is, nevertheless, desirable to combine other results with them as confirmations, or as corrections. In the investigation of the disturbance law at Point Barrow, as compared with the same at Toronto, a very remarkable mutual relation was developed, and further examination may bring to light other dependencies of a mutual character.

According to the latest determination, the position of the Girard College observatory is in latitude 39° 58′ 23″ (north), and in longitude 75° 10′ 05″ = $5^h\ 00^m\ 40^s.3$

[1] See "Observations at the magnetic and meteorological observatory at the Girard College, Philadelphia, made under the direction of A. D. Bache, LL. D., and with funds supplied by the members of the American Philosophical Society and by the Topographical Bureau of the United States, 1840 to 1845. Printed by order of the Senate of the United States, and under the direction of the Topographical Bureau, second session of the twenty-ninth Congress. Washington, D. C., 1847." Three volumes record and one volume plates.

west of Greenwich.[1] From Philadelphia, Toronto bears 38° 45′ west of north (true), and is distant 4° 50′ in arc, or about 334 statute miles.

It is proposed in the present paper to investigate the law of the eleven year period, or as it is more frequently called, the decennial period, there being yet an uncertainty as to its precise length. It is supposed to have some direct or indirect connection with the solar spot period, which, according to late investigations by Prof. R. Wolf,[2] is said to exhibit corresponding disturbances.

The discussion is a contribution towards the determination of the epoch of the occurrence of a minimum (as to number and magnitude) in certain phases of the magnetic variations and disturbances, corresponding to a minimum in the solar spot period. The method of reduction is substantially the same as that adopted by General Sabine, and explained in his discussion of the Toronto and Hobarton[3] observations.

---

[1] This longitude depends on that of Cambridge observatory, for which $4^h\ 44^m\ 30^s.25$ has been adopted.

[2] Astronomische Nachrichten, No. 1091 (May, 1857).

[3] See three papers by General Sabine, on periodical laws discoverable in the mean effects of the larger magnetic disturbances. Philosophical Transactions of the Royal Society, 1851, 1852, and 1856.

INVESTIGATION OF THE ELEVEN YEAR PERIOD

IN THE

CHANGE OF THE AMPLITUDE OF THE SOLAR-DIURNAL VARIATION OF THE MAGNETIC DECLINATION, COMPRISING THE REGULAR AS WELL AS THE DISTURBED DIURNAL VARIATION.

WHILE the magnitude of the deflection is the only criterion for the recognition of a disturbance, the adoption of any limit of deviation from the normal value for the same hour, month, and year, must necessarily remain in some measure arbitrary, or, in other words, there must always remain, after the separation of the disturbances, a certain small amount of their effect in the remaining regular diurnal progression. General Sabine has shown that the results are not sensibly affected by a small variation in the line of separation of the disturbed from the undisturbed readings.[1]

To effect the separation, I made use of Peirce's criterion,[2] for the rejection of doubtful observations, applying it, however, to observations following a law different from the regular one.[3] From an examination of 465 hourly observations, distributed over different hours of the day and different months of the year, the following was the limit of separation:—

$9^d.3$	from six months	in	1840
8.1	"	"	1843
6.0	"	"	1845

The mean or 7.8 divisions, equal to $3'.6$ of arc, has been adopted provisionally. Accordingly, all numbers in the printed record of observations, differing 7.8 scale divisions (or 10.3 divisions for June, and, up to July 18, 1840), from the mean monthly value at each hour of observation, were marked in pencil. It was found that the ratio of the disturbed observations to the total number was 1 : 9.6, or for

---

[1] In the first discussion of the Toronto observations for the years 1843, 1844, 1845, the limit of $3'.6$ was adopted, corresponding to one disturbance in every 13.6 observations; in the second discussion $5'.0$ was substituted as preferable. Phil. Trans., 1856, art. xv.

[2] Gould's Astronomical Journal, vol. iv., No. 83, 1855.

[3] A similar application was made in the discussion of Dr. E. K. Kane's magnetic observations at Van Rensselaer Harbor, North Greenland, by Mr. Schott. Smithsonian Contributions to Knowledge, vol. x., 1858.

the years 1843, 1844, 1845, 1 : 13.3 nearly (the years 1843 and 1845 being incomplete, and omissions only approximately allowed for). For comparison with the Toronto observations we have the ratio 1 : 9.4 for the series 1841 to 1848, inclusive,[1] and 1 : 13.6 for the series 1843, 1844, 1845,[2] both for the limit 3′.6, which was afterwards raised to 5′.0.[3] It was thought desirable in comparing these results, and especially as the Girard College observations do not extend either way to years of maximum of disturbance, which would otherwise require the enlargement of the limit, to preserve the limit as pointed out by the criterion; hence a deviation from the normal of 8.0 scale divisions as a convenient number, 3′.64 of arc, has been adopted for the present discussion as constituting a disturbed observation. Previous to July 18, 1840, the declinometer had a different scale, one division being 20″.7, making the corresponding limit for the first month and a half, 10.6 divisions.

All observations therefore differing 8.0 scale divisions from the mean monthly value of their respective hour were marked by a pencil line; a new hourly mean was taken, omitting values so marked, and each observation was again examined with reference to its deviation from this new mean. The process was repeated, when necessary, so that in all cases values differing $8^d.0$ or more from the final mean, were excluded. The last mean thus obtained for each observing hour and each month has been called "the normal." The following tables of normals present the mean monthly declinometer readings for each observing hour, free from all disturbances, deviating either way 3′.64 or more, from the normal position of the magnet for the respective hour, month, and year. The observations having been made at the even Göttingen hours, the local times are 19½ minutes after the even hour.[4] The time given in the tables is mean local time, counting from midnight, or 0^h up to 24^h.

Increase in the scale readings, corresponds to a decrease of westerly declination. The value of one division of scale is 0′.453.

---

[1] Observations made at the Magnetical and Meteorological Observatory at Toronto, in Canada, under the superintendence of Colonel Edward Sabine, vol. ii., 1843, 1844, 1845, with abstracts of observations to 1852, inclusive. London, 1853.

[2] Phil. Trans. R. S., 1851, art. v.

[3] Observations made at the Magnetical and Meteorological Observatory at Toronto, in Canada, under the superintendence of Major-General Edward Sabine, vol. iii., 1846, 1847, 1848, with abstracts of observations to 1855, inclusive. London, 1857.

[4] The observations were made at the even Göttingen time, $6^h\ 00^m$, corresponding to $0^h\ 19\frac{1}{2}^m$ of Philadelphia time.

TABLE I.—NORMALS OF THE DECLINOMETER READINGS FOR EACH OBSERVING HOUR AND MONTH, IN THE YEAR 1840.

Observations taken $19\frac{1}{2}$ minutes after the hour indicated.

Philadelphia mean time.	0h.	2h.	4h.	6h.	8h.	10h.	Noon.	14h.	16h.	18h.	20h.	22h.
	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.
June[1]	494.4	495.0	497.5	504.0	502.7	493.8	485.5	483.4	487.9	492.8	492.5	493.6
July	497.3	497.2	498.9	504.7	505.5	495.4	484.5	484.0	488.7	493.3	495.5	496.3
August	495.3	495.7	498.8	506.4	509.1	489.4	480.5	481.9	488.2	493.2	494.9	496.1
September[2]	492.5	495.2	496.9	503.2	502.5	490.8	477.3	479.5	488.4	489.9	493.3	492.6
October	492.5	490.4	491.1	489.1	489.2	484.1	478.4	477.3	481.9	486.3	485.9	493.1
November[3]	481.1	480.6	482.9	483.7	486.4	481.7	474.2	472.5	477.5	480.8	483.6	482.7
December	477.9	475.2	479.8	479.5	480.5	480.6	470.7	471.6	472.7	478.5	479.0	481.2
Mean	490.14	489.90	492.27	495.80	496.56	487.86	478.73	478.60	483.61	487.83	489.24	490.80
Correction[4]	+5.21	5.10	5.33	4.68	5.17	5.85	5.05	4.65	4.46	4.36	4.75	5.25
Mean for '40	495.35	495.00	497.60	500.48	501.73	493.71	483.78	483.25	488.07	492.19	493.99	496.05
Correction for index[5]	+93.30											
Cor. mean for 1840.	588.65	588.30	590.90	593.78	595.03	587.01	577.08	576.55	581.37	585.49	587.29	589.35

TABLE II.—NORMALS OF THE DECLINOMETER READINGS FOR 1841.

Value of 1 div. = 0′.453. Time $19\frac{1}{2}$ minutes later than indicated.

Philadelphia mean time.	0h.	2h.	4h.	6h.	8h.	10h.	Noon.	14h.	16h.	18h.	20h.	22h.
	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.
January	579.3	577.0	578.6	576.9	580.7	581.9	570.0	568.8	570.3	574.2	578.0	580.1
February	575.0	573.2	575.6	577.8	582.1	479.5	569.5	566.0	569.5	572.4	574.4	575.8
March	577.1	577.6	580.9	582.9	586.8	578.9	569.4	567.7	571.8	576.4	577.4	577.7
April	580.0	581.9	582.9	585.6	587.6	579.4	568.8	566.1	571.7	576.9	578.0	579.1
May	579.1	579.8	581.9	587.4	589.1	578.6	569.4	567.9	573.6	577.4	578.5	580.1
June	571.7	572.2	574.7	583.3	582.6	571.1	561.6	560.3	565.0	570.1	570.9	570.8
July	569.9	568.5	571.6	578.4	581.2	571.8	558.9	557.3	562.3	567.2	568.8	568.6
August	568.4	570.3	571.6	580.1	583.9	568.9	558.3	556.9	564.0	566.8	568.6	568.9
September	565.1	564.5	565.5	569.4	571.1	564.1	553.6	554.5	559.5	562.9	563.8	564.0
October	566.8	566.3	565.5	567.6	569.4	568.2	564.0	562.3	564.7	573.5	568.6	569.3
November	557.2	558.5	558.5	557.6	561.7	557.1	551.8	549.9	553.4	554.9	558.0	558.6
December	560.1	559.3	560.5	559.6	560.1	558.1	552.9	551.7	555.8	559.6	563.3	561.6
Mean	570.81	570.76	572.32	575.55	578.03	571.47	562.35	560.78	565.13	569.36	570.70	571.22

[1] The readings from June 1st to July 18th, 15 hours, on the College building scale, were converted into observatory scale readings by subtracting $144^{d}.7$ at division 628.8 of the old scale, and converting the value of a division 0′.345 of the old into the corresponding reading for the value of a division 0′.453 of the new scale. The mean readings, thus corrected, of the first 18 days of July were then properly combined with the mean of the remaining days of the month.

[2] In the month of September, hour 8, the comparisons were made with the half monthly means, owing to the rapid change of the readings.

[3] On the 23d of November the index of the declinometer bar shifted 19.5 scale divisions; a correction of $+19^{d}.5$ has, therefore, been added to observations after this date, and, likewise, to all the readings of the following month.

[4] The corrections here given for referring the mean of the last seven months of the year to the mean for the whole year, are derived from the normals of the following year 1841 by comparing the mean of the same seven months with the annual mean of that year. Comparing the same months in the two years the character of the changes appears to be about the same.

[5] A further correction for change in the zero of the scale required to refer the readings of 1840 to the readings of subsequent years. Owing to a rearrangement of the instruments on January 7, 1841, the scale readings changed 112.8 divisions, and since 19.5 scale divisions had been added to the December readings, the resulting correction is the difference of the two, or $+93^{d}.30$.

In general during the year 1841 the readings are more changeable than during the following years.

The rearrangement of the instruments, and consequent shifting of the index of the scale, alluded to in the preceding notes, interrupted the observations between January 1 and January 12.

The normal for October, 14^h, was obtained by comparing with the half monthly means and taking the mean of the two results as in a similar case for the month of September of the previous year.

TABLE III.—NORMALS OF THE DECLINOMETER READINGS FOR 1842.

Value of 1 div. = 0′.453. Time $19\frac{1}{2}$ minutes later than indicated.

Philadelphia mean time.	0^h.	2^h.	4^h.	6^h.	8^h.	10^h.	Noon.	14^h.	16^h.	18^h.	20^h.	22^h.
	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.
January	564.3	563.8	565.3	565.9	570.9	566.4	556.7	556.0	562.9	563.2	566.1	567.8
February	564.5	564.3	563.8	565.2	567.8	565.5	558.2	559.9	558.0	561.9	565.3	565.5
March	564.8	564.1	565.4	566.1	571.8	565.9	555.6	553.9	556.4	560.3	564.5	564.9
April	563.3	565.4	566.1	568.5	569.7	563.6	554.0	552.5	555.1	560.6	561.3	563.0
May	563.3	564.3	566.0	571.2	569.5	560.0	552.6	552.3	557.7	560.8	561.8	562.3
June	564.6	563.7	567.2	573.7	573.0	565.2	555.1	552.5	558.3	561.8	563.7	564.1
July	566.0	566.0	568.4	576.6	576.4	565.8	556.3	553.8	558.5	562.4	564.2	567.1
August	564.8	566.0	568.5	573.7	575.0	560.0	552.3	553.7	561.5	562.2	564.1	564.5
September	567.4	567.8	570.0	576.8	574.9	561.2	556.0	555.4	562.0	565.7	566.7	566.6
October	563.1	563.1	564.4	566.0	568.8	564.0	556.0	555.0	558.2	564.3	565.0	565.3
November	564.2	563.8	565.6	566.9	569.2	563.3	556.6	557.3	561.2	564.0	565.5	565.0
December	561.7	5 0.7	562.1	562.7	565.5	564.2	556.6	556.2	560.1	562.0	563.5	563.8
Mean	564.33	564.42	566.07	569.44	571.04	563.76	555.50	554.54	559.16	562.42	564.31	564.99

TABLE IV.—NORMALS OF THE DECLINOMETER READINGS FOR 1843.

Value of 1 div. = 0′.453. Time $19\frac{1}{2}$ minutes later than indicated.

Philadelphia mean time.	0^h.	2^h.	4^h.	6^h.	8^h.	10^h.	Noon.	14^h.	16^h.	18^h.	20^h.	22^h.
	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.
January	...	...	...	...	...	...	...	555.4	...	...	...	...
February	...	...	...	...	...	...	...	555.9	...	...	...	...
March	...	...	...	...	...	....	...	5 7.2	...	...	...	...
April	569.7	570.0	571.0	574.7	576.2	566.2	557.8	555.7	562.6	564.8	568.5	568.7
May	567.0	567.3	569.6	574.6	575.6	565.7	556.0	556.2	562.2	566.4	566.9	567.3
June	566.0	565.6	568.4	574.1	573.9	564.8	556.4	556.0	561.1	564.3	564.0	565.6
July	566.9	565.9	568.2	574.2	574.6	564.5	555.1	554.1	559.5	563.6	563.8	565.6
August[1]	564.2	564.5	567.2	573.5	572.7	560.5	555.1	554.6	561.2	563.6	562.3	564.2
September	560.4	560.4	560.3	565.7	566.6	554.6	547.5	550.5	556.8	558.0	560.0	558.7
October	559.6	559.6	559.9	562.1	566.0	560.8	553.6	552.7	556.2	558.2	560.1	559.7
November	556.3	556.6	557.4	559.1	561.3	556.2	550.4	551.1	553.8	556.3	557.5	557.3
December	559.0	557.4	557.8	560.0	561.2	559.9	552.9	550.9	554.6	558.2	559.6	559.9
Mean	563.23	563.03	564.42	568.67	569.79	561.47	553.42	554.19	558.67	561.50	562.52	563.00
Correction[2]	+0.06	—0.11	—0.41	—1.24	—0.30	+0.63	+0.44	...	—0.02	—0.23	+0.33	+0.35
Cor'd mean	563.29	562.92	564.01	567.43	569.49	562.10	553.86	554.19	558.65	561.27	562.85	563.35

[1] The suspension threads of the declinometer gave way on the 9th of August, and again on the 10th of January, 1844, but after readjusting the instruments, the magnet returned almost exactly to its former reading—a mean of the two changes gave as a correction +18.7 divisions, which was accordingly added to all the readings of the year after August 9th, 21 hours.

[2] The correction to refer the mean of the nine last months to the mean of all the months is derived from the readings of the preceding year, as being more uniform in character than those for the year following.

The hourly readings commence on October 1, and are continued to the close of the series.

To make the readings of the odd hours of the months of October, November, and December comparable with those of the even hours during the whole year, the means of the even hours for the months of October, November, and December (1843) were compared with the corrected annual means respectively, which gave the corrections for the even hours; and the corrections for intermediate odd hours were obtained from those of the nearest even hours. The deductions from the series of observations at odd hours have but one-third of the weight of those obtained from the even series.

TABLE IV. (*b*).—Additional Normals for the odd hours of the Months of October, November, and December, 1843.

Value of 1 div. = 0′.453. Time 19½ minutes later than indicated.

Philadelphia mean time.	1h.	3h.	5h.	7h.	9h.	11h.	13h.	15h.	17h.	19h.	21h.	23h.
	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.
October	560.2	559.1	560.6	565.1	565.0	556.5	552.6	554.2	557.0	559.7	561.1	560.7
November	556.7	556.6	557.4	561.8	560.1	552.6	550.0	552.6	554.9	557.5	557.7	557.4
December	558.1	558.2	558.8	560.8	561.9	556.7	551.4	553.1	557.5	558.9	560.0	559.5
Mean	558.33	557.97	558.93	562.57	562.33	555.27	551.33	553.30	556.47	558.70	559.60	559.20
Correction	+5.01	+5.37	+6.36	+6.84	+4.92	+2.37	+2.09	+3.20	+3.74	+3.74	+4.08	+4.70
Cor'd mean	563.34	563.34	565.29	569.41	567.25	557.64	553.42	556.50	560.21	562.44	563.68	563.90

TABLE V.—Normals of the Declinometer Readings for 1844.

Value of 1 div. = 0′.453. Time 19½ minutes later than indicated.

Philadelphia mean time.	0h.	1h.	2h.	3h.	4h.	5h.	6h.	7h.	8h.	9h.	10h.	11h.
	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.
January	558.6	558.2	558.4	559.2	558.9	558.8	559.7	561.2	562.9	563.3	559.1	555.9
February	559.1	558.5	559.1	559.2	559.9	561.1	560.8	562.1	562.2	560.7	557.3	554.5
March	558.0	559.0	559.2	557.9	559.8	560.2	561.3	563.6	564.8	564.1	560.3	554.9
April	556.6	557.0	557.2	556.9	557.5	558.4	561.7	558.5	564.4	561.8	557.1	552.0
May	548.4	548.7	547.8	547.0	549.3	552.5	555.8	556.8	555.1	552.3	546.7	542.2
June	548.7	549.0	549.3	549.1	551.6	553.9	557.6	559.1	558.2	554.3	547.9	541.8
July	549.0	550.5	548.4	549.4	551.0	554.3	556.9	559.8	558.6	554.8	548.0	540.8
August	548.6	547.8	547.3	547.4	550.9	552.4	557.5	560.3	558.2	551.8	543.3	536.4
September	543.3	543.1	544.1	546.0	546.5	547.1	550.0	552.9	552.4	545.8	538.3	532.5
October	545.1	545.3	544.2	546.1	545.8	544.4	548.6	550.9	551.5	548.7	545.3	540.8
November	546.8	546.8	548.3	548.6	547.4	548.5	551.5	549.2	548.4	547.9	546.2	542.8
December	536.1	535.8	535.4	535.9	536.8	537.3	537.2	536.8	537.9	539.3	536.1	532.9
Mean	549.86	549.98	549.89	550.23	551.28	552.41	554.88	555.93	556.22	553.73	548.80	543.96

Philadelphia mean time.	Noon.	13h.	14h.	15h.	16h.	17h.	18h.	19h.	20h.	21h.	22h.	23h.
	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.
January	552.9	552.4	553.2	554.1	556.3	556.9	557.8	559.2	559.5	560.9	560.8	559.6
February	551.1	551.1	553.0	554.7	556.4	556.6	557.6	558.4	559.9	559.4	560.1	559.0
March	550.6	549.4	549.6	551.7	553.0	555.2	556.6	558.0	558.4	558.2	558.6	559.7
April	547.4	545.7	546.2	547.6	549.6	553.4	553.4	553.8	556.2	555.1	555.7	559.3
May	538.3	535.8	536.5	538.9	542.1	545.1	545.2	546.5	546.3	547.3	547.3	547.8
June	537.4	535.0	537.3	540.0	542.4	545.2	545.6	546.2	546.5	546.8	548.0	548.5
July	538.3	535.5	536.3	538.8	541.9	544.5	545.8	546.2	546.6	547.4	548.8	549.3
August	531.8	532.0	534.3	538.7	542.1	544.3	546.0	546.5	546.7	546.6	547.8	547.7
September	529.3	530.0	534.1	538.3	539.4	541.9	542.4	541.9	543.0	544.6	543.7	543.3
October	541.1	539.5	541.4	544.0	545.7	545.4	545.6	545.0	544.9	544.6	544.5	544.6
November	542.8	541.7	544.5	546.1	545.6	547.9	548.8	548.2	548.3	549.6	548.0	548.0
December	530.6	529.3	529.4	532.1	533.2	534.8	535 9	537.0	536.8	537.4	537.8	537.1
Mean	540.97	539.78	541.32	543.75	545.64	547.60	548.44	548.91	549.43	549.83	550.10	550.33

To the observations between January 1 and January 10 a correction of $+18^{d}.7$ was applied, as explained in the preceding note.

In the month of December the declination changed so rapidly as to require the use of half monthly means; the mean of the two results is inserted in the above table.

TABLE VI.—NORMALS OF THE DECLINOMETER READINGS FOR 1845.

Value of 1 div. $= 0'.453$. Time $19\frac{1}{2}$ minutes later than indicated.

Philadelphia mean time.	0h.	1h.	2h.	3h.	4h.	5h.	6h.	7h.	8h.	9h.	10h.	11h.
	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.
January	530.9	531.3	531.1	531.5	533.0	531.6	532.9	535.2	535.8	533.8	530.2	526.7
February	531.6	531.1	531.0	532.4	532.3	533.1	534.7	535.9	535.7	535.4	533.0	528.6
March	532.9	532.7	533.7	533.6	535.0	533.9	536.0	538.8	539.4	538.6	534.5	529.4
April	529.1	528.8	529.0	529.2	529.8	531.7	534.0	535.6	537.5	535.4	528.5	522.5
May	529.9	531.3	529.7	531.7	533.2	536.3	539.3	541.9	540.7	536.0	528.0	522.6
June	531.5	531.7	531.6	532.0	534.8	537.9	541.9	543.5	542.5	538.6	532.2	524.9
Mean	530.98	531.15	531.02	531.73	533.02	534.08	536.47	538.48	538.60	536.30	531.07	525.78
Correction	—2.42	—2.50	—2.58	—2.41	—2.26	—2.03	—1.81	—2.01	—2.21	—2.76	—3.30	—2.94
Cor'd mean	528.56	528.65	528.44	529.32	530.76	532.05	534.66	536.47	536.39	533.54	527.77	522.84
Philadelphia mean time.	Noon.	13h.	14h.	15h.	16h.	17h.	18h.	19h.	20h.	21h.	22h.	23h.
	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.
January	524.2	525.2	526.2	528.0	530.1	531.8	532.7	532.8	533.3	533.0	532.4	532.0
February	524.4	523.0	525.3	527.5	529.7	530.4	532.4	531.3	533.6	534.4	532.3	531.9
March	524.8	522.5	522.8	524.8	527.8	529.7	531.6	533.0	533.0	533.8	533.5	534.0
April	517.8	513.9	514.0	517.2	521.5	525.8	527.8	527.9	528.1	528.5	528.0	529.4
May	517.1	516.8	518.9	522.1	526.7	529.3	529.6	530.4	529.7	530.3	530.5	530.3
June	521.3	519.6	520.0	522.1	525.4	528.9	530.3	530.7	530.1	530.7	530.3	531.4
Mean	521.60	520.17	521.20	523.62	526.87	529.32	530.73	531.02	531.30	531.78	531.17	531.50
Correction[1]	—2.59	—2.28	—1.98	—1.80	—1.62	—1.64	—1.65	—1.99	—2.23	—2.36	—2.47	—2.44
Cor'd mean	519.01	517.89	519.22	521.82	525.25	527.68	529.08	529.03	529.07	529.42	528.70	529.06

For the purpose of comparing the annual means of the normals, or the mean march of the regular solar-diurnal variation for each year, the preceding results have been expressed analytically by means of Bessel's formula, and by the application of the method of least squares.

In these formulæ the angle θ is reckoned from midnight (Philadelphia), at the rate of 15° for each following hour. It was found unnecessary to carry the expressions beyond the third term, the fourth being generally smaller than the probable error of an hourly normal. We obtain accordingly—

$$\text{For 1840}\quad D = 586^{d}.73 + 6^{d}.214 \sin(\theta + 36^\circ\,35') + 4^{d}.588 \sin(2\theta + 217^\circ\,33') + 1^{d}.640 \sin(3\theta + 68^\circ\,50')$$
$$\text{" 1841}\quad D = 569.87 + 4.888 \sin(\theta + 30\;\;05) + 4.380 \sin(2\theta + 212\;\;38) + 1.581 \sin(3\theta + 50\;\;14)$$
$$\text{" 1842}\quad D = 563.33 + 4.944 \sin(\theta + 33\;\;49) + 4.211 \sin(2\theta + 217\;\;12) + 1.463 \sin(3\theta + 64\;\;42)$$
$$\text{" 1843}\quad D = 562.01 + 4.449 \sin(\theta + 36\;\;00) + 3.918 \sin(2\theta + 218\;\;05) + 1.811 \sin(3\theta + 68\;\;18)$$
$$\text{" 1844}\quad D = 548.89 + 4.486 \sin(\theta + 34\;\;35) + 3.872 \sin(2\theta + 222\;\;23) + 1.802 \sin(3\theta + 68\;\;53)$$
$$\text{" 1845}\quad D = 528.12 + 4.548 \sin(\theta + 35\;\;33) + 4.872 \sin(2\theta + 225\;\;35) + 1.987 \sin(3\theta + 61\;\;20)$$

[1] As indicated by the annual change in the readings, it was considered preferable to obtain the annual mean by deducing the correction to the mean of the first six months, from the readings of the preceding year and those of the year 1842.

Owing probably to the several accidental changes in the suspension of the bar, and consequent uncertainty in the precise amount of scale correction, the mean readings of each year, when compared with one another, exhibit differences not actually due to inequalities occasioned by declination changes. This question, however, does not directly bear upon the present investigation, which mainly depends on differences of readings, and it is proper to remark that the observed increase, giving the weight one-half to the mean of 1840 and of 1845, is under the supposition of a uniform annual change between these years, equal to 4′.50. From Mr. Schott's investigation[1] of the secular change of the declination at Philadelphia, supported by observations between the years 1701 and 1855, the annual increase between the years 1840 and 1845 is 4′.98, a result which accords tolerably well with actual observations. According to his formula, the declination on the first of January, 1843, the mean epoch of the present series was 3° 32′ west, with a probable error of ± 10′, which corresponds to the scale reading 560.31, deduced by taking into account the weights of the annual means.

We now proceed to the investigation of the inequality in the diurnal variation, changing the preceding formulæ for greater convenience into the following:—

For 1840 $\Delta = +\ 2'.815\ sin\ (15°n + 36°\ 35') + 2'.078\ sin\ (30°n + 217°\ 33') + 0'.743\ sin\ (45°n + 68°\ 50')$
" 1841 $\Delta = +\ 2.214\ sin\ (15\ n + 30\ \ 05) + 1.984\ sin\ (30\ n + 212\ \ 38) + 0.716\ sin\ (45\ n + 50\ \ 14)$
" 1842 $\Delta = +\ 2.240\ sin\ (15\ n + 33\ \ 49) + 1.908\ sin\ (30\ n + 217\ \ 12) + 0.663\ sin\ (45\ n + 64\ \ 42)$
" 1843 $\Delta = +\ 2.015\ sin\ (15\ n + 36\ \ 00) + 1.775\ sin\ (30\ n + 218\ \ 05) + 0.820\ sin\ (45\ n + 68\ \ 18)$
" 1844 $\Delta = +\ 2.032\ sin\ (15\ n + 34\ \ 35) + 1.754\ sin\ (30\ n + 222\ \ 23) + 0.816\ sin\ (45\ n + 68\ \ 53)$
" 1845 $\Delta = +\ 2.060\ sin\ (15\ n + 35\ \ 33) + 2.206\ sin\ (30\ n + 225\ \ 35) + 0.900\ sin\ (45\ n + 61\ \ 20)$

Where Δ = the regular solar-diurnal variation.
n = the number of hours after midnight.

To show the agreement between these expressions and the corresponding observed quantities, and to exhibit to the eye the character of the diurnal variation, the results have been thrown into curves. The observed bi-hourly means are represented in Fig. 2 (p. 11) by dots, and in no instance do they differ from the computed values by as much as $0^d.8$ or $0'.3$. As a specimen of the representation, I add the results for the year 1845:—

Hour.	Observed value.	Computed value.	$C - O$.	Hour.	Observed value.	Computed value.	$C - O$.
h. m.	d.	d.	d.	h. m.	d.	d.	d.
0 19½	528.56	528.99	+.43	12 19½	519.01	519.23	+.22
2 19½	528.44	528.48	+.04	14 19½	519.22	518.96	—.26
4 19½	530.76	530.26	—.50	16 19½	525.25	525.18	—.07
6 19½	534.66	535.11	+.45	18 19½	529.08	529.15	+.07
8 19½	536.39	535.97	—.42	20 19½	529.07	529.07	.00
10 19½	527.77	528.18	+.41	22 19½	528.70	528.86	+.16

The average probable error of any single representation by the formula is $\pm 0^d.22$ or $\pm 0'.10$.

By means of the preceding formulæ the following values were computed: 1. The time when the north end of the magnet reached its extreme eastern position, or, in other words, the epoch of the eastern elongation. 2. The corresponding maximum scale reading, or, more properly, the corresponding minimum of western declination. 3. The time of the occurrence of the western elongation; and 4. The corresponding

[1] Report on the progress of the U. S. Coast Survey for 1855, Appendix, No. 48, and an Appendix (p. 11) of the report for 1859.

maximum reading of western declination. In the last two columns the difference of the scale readings, or the amplitude of eastern and western elongation, is made out in scale divisions, and also in minutes of arc.

The inequality of this amplitude next requires our attention.

For	Epoch of eastern deflection.	Corresponding scale reading.	Epoch of western deflection.	Corresponding scale reading.	Amplitude.	
	h. m.	d.	h. m.	d.	d.	′
1840	7 26 A. M.	595.67	1 34 P. M.	575.71	19.96	9.08
1841	7 49	577.96	1 49	560.21	17.75	8.06
1842	7 36	571.24	1 37	553.96	17.28	7.83
1843	7 40	569.54	1 24	553.06	16.48	7.46
1844	7 32	556.50	1 18	539.99	16.51	7.51
1845	7 34	536.65	1 16	517.81	18.84	8.53
Mean	7 36 A. M. ±3		1 30 P. M. ±4			

The inequality constituting the ten or eleven year period is plainly exhibited in the last two columns of the above table, the progression in the numbers being quite regular. The year 1843 is clearly indicated as the year of the minimum range of the diurnal fluctuation, but whether the period is one nearer to ten or to eleven years cannot be decided from the Girard College observations, since they do not embrace a year of maximum amplitude. The epoch of the minimum, however, can be determined with more precision. For this purpose only, the values in the last column are represented by the formula,

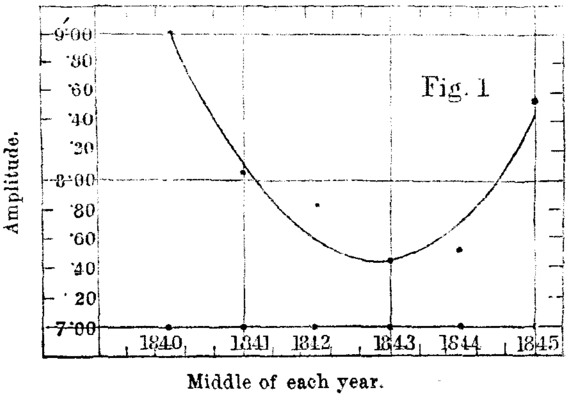
$$A = 9'.08 - 1'.14\ (t - 1840.5) + 0'.201\ (t - 1840.5)^2,$$

deduced by the method of least squares, and the quantities come out as follows:—

Year.	Observed amplitude.	Computed by formula.	Difference.	Year.	Observed amplitude.	Computed by formula.	Difference.
1840.5	9′.08	9′.08	0′.00	1843.5	7′.46	7′.47	—0′.01
1841.5	8.06	8.14	—0.08	1844.5	7.51	7.74	—0.23
1842.5	7.83	7.60	+0.23	1845.5	8.53	8.41	+0.12

Probable error of any single amplitude, ±0′.11.

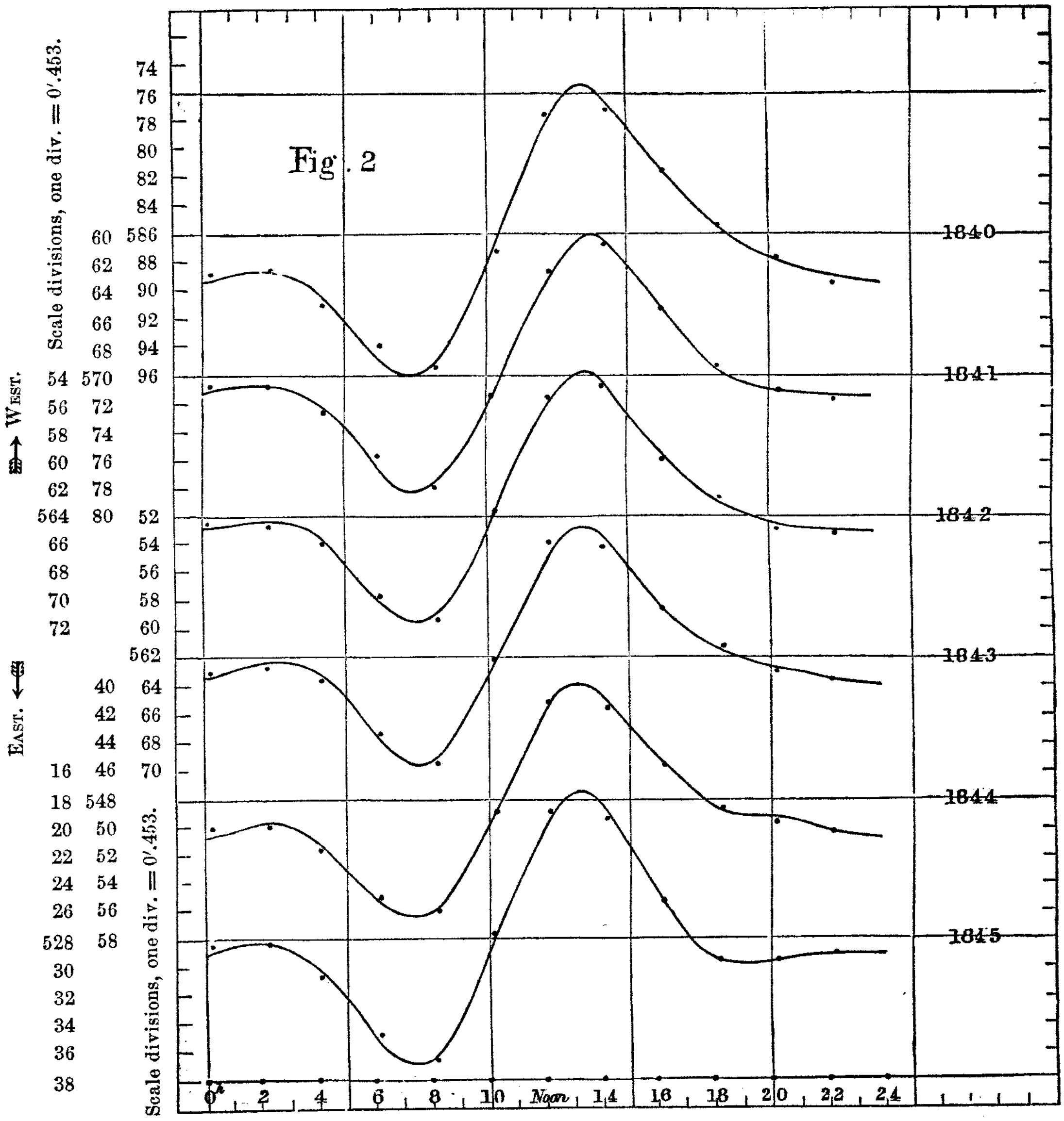
That portion of the ten or eleven year period which results from the preceding discussion of the differential observations of the magnetic declination, free from the effect of the disturbances as far as the latter can be eliminated, is shown graphically in Fig. 1.



Middle of each year.

The month of May, in the year 1843, is indicated by the formula as the epoch of the minimum amplitude.

Regular Solar-Diurnal Variation of the Declination.



Philadelphia mean time.

We now proceed to the discussion of the disturbances as far as they bear on the decennial inequality, taking in also some collateral results.

The total number of observations for changes of declination recorded and discussed amounts to 24,566; of these, 2357 were separated as disturbances differing eight scale divisions or more from their respective normals, leaving 22,209 observations, from which the preceding results were deduced. There is one disturbed observation in every 10.4 observations.

The discussion of the disturbances divides itself into two parts, that of the number and that of the amount of the larger deflections.

Owing to partial incompleteness in the number of observing months in some years, it became necessary to fill out the number for the annual inequality from the results of the complete years. Their number for each month in the complete years is given in the following table, the numbers for 1844 having first been divided by two, in order to make the hourly observations comparable with the bi-hourly in the years 1841 and 1842:—

Month.	1841.	1842.	1844.	Mean.	Ratio.
January	33	44	5	27	0.75
February	25	26	5	19	0.53
March	26	24	24	25	0.70
April	25	31	39	32	0.89
May	33	14	17	21	0.58
June	31	30	7	23	0.64
July	30	40	15	28	0.78
August	49	64	44	52	1.45
September	57	60	31	49	1.36
October	94	86	53	78	2.17
November	81	22	42	48	1.33
December	55	5	26	29	0.82
Sum	539	446	308	431	12.00
Mean				36	1.00

The last column contains the ratio of the mean monthly value to the mean annual value. By means of these ratios, and using the observed monthly values in each defective year, the numbers in the following table were filled up, all the deduced values being indicated by brackets. As in the preceding table, the values refer or were made to refer to bi-hourly observations:—

TABLE SHOWING THE NUMBER OF DISTURBANCES IN EACH MONTH OF THE YEARS 1840 TO 1845.

Month.	1840.	1841.	1842.	1843.	1844.	1845.	Mean.	Ratio.
January	(30)	33	44	(17)	5	19	25	0.77
February	(21)	25	26	(12)	5	13	17	0.52
March	(28)	26	24	(16)	24	14	22	0.68
April	(36)	25	31	21	39	22	29	0.91
May	(24)	33	14	15	17	11	19	0.58
June	8	31	30	12	7	12	17	0.53
July	44	30	40	20	15	(17)	28	0.86
August	40	49	64	80	44	(32)	51	1.59
September	56	57	60	27	31	(30)	44	1.36
October	94	94	86	16	53	(48)	68	2.12
November	19	81	22	8	42	(28)	35	1.08
December	83	55	5	4	26	(18)	32	1.00
Sum	344	539	446	230	308	91	387	12.00
Corrected sum and mean	483			275		264	32	1.00

The ratios in the last column show the annual inequality in the distribution of the disturbances. The principal maximum occurs in October,[1] the secondary in April; the two minima, nearly of equal amount, occur in the months of February and June. The progression of the numbers is regular.

[1] At Toronto this maximum occurred in September; the first minimum is likewise one month earlier at this station than at Philadelphia.

If we separate the numbers in accordance with westerly and easterly deflections we obtain the following table, deduced as in the former case. It may be remarked that on account of the separate ratios used for the interpolation of the western and eastern deflections, their sum in any one month does not give the corresponding number in the above table exactly, only the yearly sums having been preserved; and the same is true in regard to the table, showing the amount of the disturbances. Interpolated values as before are inclosed between brackets:—

Month.	1840.		1841.		1842.		1843.		1844.		1845.		Sums.		Ratios.	
	W.	E.	W.	E.	W.	E.	W.	E.	W.	E.	W.	E.	W.	E.	W.	E.
January	(36)	(2)	25	8	35	9	(22)	(7)	2	3	10	9	130	38	1.27	0.42
February	(17)	(3)	9	16	17	9	(13)	(9)	3	2	11	2	70	41	0.70	0.46
March	(23)	(5)	11	15	17	7	(15)	(6)	10	14	9	5	85	52	0.83	0.57
April	(27)	(5)	10	15	14	17	7	14	25	14	15	7	98	72	0.95	0.80
May	(17)	(4)	18	15	8	6	7	8	4	13	3	8	57	54	0.55	0.60
June	3	5	15	16	17	13	2	10	3	4	5	7	45	55	0.44	0.61
July	17	27	5	25	14	26	11	9	6	9	(7)	(11)	60	107	0.58	1.18
August	20	20	18	31	55	9	67	13	25	19	(20)	(11)	205	103	2.00	1.14
September	36	20	14	43	11	49	6	21	18	13	(11)	(21)	96	167	0.93	1.86
October	68	26	34	60	17	69	6	10	23	30	(15)	(30)	163	225	1.58	2.50
November	11	8	41	40	11	11	5	3	16	26	(15)	(14)	99	102	0.96	1.12
December	77	6	24	31	1	4	2	2	12	14	(8)	(10)	124	67	1.21	0.74
Sum	232	112	224	315	217	229	113	90	147	161	53	38	1232	1083	12.00	12.00
Corrected mean	352	131	...	...	...	...	163	112	...	...	129	135	...	...	...	...
Total	483		539		446		275		308		264		2315		...	

The ratios show a general correspondence in the numbers of westerly and easterly deflections; the westerly deflections seem to occur most frequently in August, while the easterly predominate in October; the secondary maximum of either series is in April. The minima remain nearly as before, one minimum of eastern deflection occurring in January.

With respect to the whole number of westerly and easterly deflections, we deduce the proportional sums from the following table:—

Year.	W.	E.	Sum.	
1840	352	131	483	Weight ½
1841	224	315	539	
1842	217	229	446	
1843	163	112	275	Weight ⅔
1844	147	161	308	
1845	129	135	264	Weight ½
Sum	1232	1083	2315	
Proportional sums by weight	937	912		

On account of the incompleteness of the record in the years 1840, 1843, and 1845, the number of eastern and western disturbances relative to the total number cannot be ascertained with accuracy. They are about equal in the record. At Toronto the eastern predominate over the western in the proportion of 1.17 to 1 (for the years 1841 to 1848), and nearly to the same extent for each year, taken separately.

The numbers in the column headed "sum" do not indicate the law of the eleven year period as plainly and systematically as did the investigation of the diurnal amplitude; yet giving half weight, on account of the want of record, to the sums for 1840 and 1845, the minimum number falls in the year 1843. More consistent results would, no doubt, have been obtained if the year 1845 had been complete.

If we distribute the disturbances (1942 in number for the even hours) according to their respective hours of occurrence, the following table results from observations between 1840 and 1845:—

Add 19½ m.	W	E.	Sum.	Ratios.		Add 19½ m.	W.	E.	Sum.	Ratios.	
				W.	E.					W.	E.
0h.	67	95	162	0.82	1.20	Noon	93	57	150	1.13	0.71
2	97	92	189*	1.18	1.16	14h.	79	54*	133*	0.95	0.67*
4	89	79	168	1.08	0.96	16	88	60	148	1.07	0.78
6	110*	63	173	1.35*	0.80	18	72	71	143	0.87	0.90
8	105	56	161	1.29	0.70	20	34*	133*	167	0.40*	1.66*
10	107	71	178	1.32	0.88	22	45	125	170	0.54	1.58

Maxima and minima values are distinguished by an asterisk.

The numbers in each vertical column show a regular progression; and the number of disturbances, irrespective of their direction, have a minimum at 2 P. M. and a maximum at 2 A. M.[1] The principal contrast is between the hours of the day and the hours of the night; in the former case the numbers being below, but in the latter above the mean value. This is in close correspondence with the Toronto results. The most striking result of the above table is—that the westerly disturbances have their minimum precisely at the hour (8 P. M.) when the easterly have their maximum value; and the exact coincidence of this result with that deduced by General Sabine for Toronto is not less remarkable. For the westerly disturbances, the hours 6 A. M. (maximum) and 8 P. M. (minimum), and for the easterly disturbances the hours 2 P. M. (minimum) and 8 P. M. (maximum), are specially contrasted. These results also agree with those found at Toronto; and the accordance with that station even goes so far as to exhibit the secondary minimum of eastern disturbances at 8 A. M. In connection with this subject it may be here stated, that the same distinguished magnetist found a singular mutual relation to subsist between the phenomena at Toronto and Point Barrow, on the shores of the Arctic Sea—the laws of the easterly deflection at one station being found to correspond for the same local hours, with those of the westerly deflections at the other station, and *vice versâ*. This contrast holds good for Philadelphia as well as for Toronto.

We now pass to the consideration of the amount of deflections caused by the disturbances, classifying the same according to years, months, and hours:—

[1] At Toronto the respective hours are 2 P. M. and 22 P. M.

Aggregate Values of the Disturbances, and Mean Values in the Different Years.						
Year.	Aggregate values.	Same corrected to 12 months.	Number.	Average value of a disturbance.	Same in minutes of arc.	Same at Toronto for comparison.
	d.	d.		d.	′	′
1840	5140.0 (7 months)	7155.5	483	14.8	6.70	...
1841	7844.4	7844.4	539	14.6	6.61	6.34
1842	6019.1	6019.1	446	13.5	6.11	5.90
1843	2465.7 (9 months)	2932.2	275	10.7	4.85	5.62
1844	4227.3	4227.3	308	13.7	6.21	6.49
1845	1138.6 (6 months)	3521.4	264	13.3	6.02	5.84

The table includes only the series of bi-hourly observations; the reduction of the numbers from incomplete years to the correct sum for the whole year being effected by means of ratios as in the discussion of the number of disturbances. For comparison the average value of a disturbance at Toronto is added. It must be remarked, that the amount of deviation from the normal, constituting a disturbance, was nearly but not quite the same at Toronto as at Philadelphia, so that the ratios of the corresponding numbers in the last two columns should be compared.

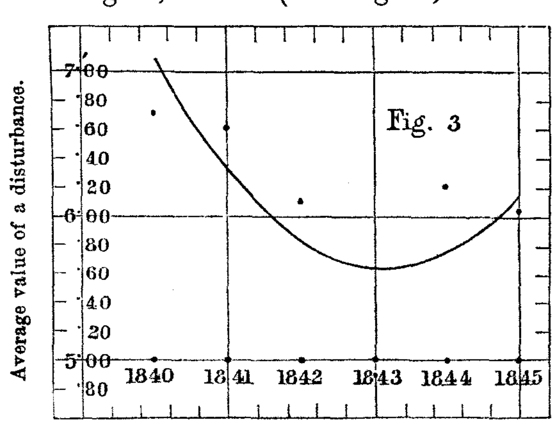
The eleven year period is well marked in the aggregate value of the disturbances, as well as in their average value in the different years; and the year 1843 is decidedly indicated as the minimum. To find a more precise value for the epoch of the minimum, the formula,

$$\delta = 7'.09 - 0'.930\,(t - 1840.5) + 0'.149\,(t - 1840.5)^2,$$

has been constructed, which represents the observed values as follows:—

Year.	Observed amount.	Computed amount.	Difference.	Year.	Observed amount.	Computed amount.	Difference.
1840.5	6′.70	7′.09	+0′.39	1843.5	4′.85	5′.64	+0′.79
1841.5	6.61	6.31	—0.30	1844.5	6.21	5.75	—0.46
1842.5	6.11	5.83	—0.28	1845.5	6.02	6.16	+0.14

The first and last value have only half weight. According to the formula, the minimum took place in August, 1843. (See Fig. 3.)



(Middle of each year.)

As the resulting epoch from the differential observations with the declinometer we find the month of June, 1843, by giving double weight to the result deduced from the inequality of the diurnal amplitude.

Separating into western and eastern disturbances we find—

Year	West Deflections.			East Deflections.		
	Aggregate value	n	Average value.	Aggregate value.	n	Average value.
	d.		′	d.		′
1840 . . .	5064.8	352	6.52	2090.7	131	7.20
1841 . . .	2935.5	224	5.93	4908.9	315	7.07
1842 . . .	2645.9	217	5.53	3373.2	229	6.70
1843 . . .	1741.6	163	4.85	1190.6	112	4.85
1844 . . .	2019.7	147	6.21	2207.6	161	6.21
1845 . . .	1489.2	129	5.25	2032.3	135	6.84

From which it appears that the easterly values preponderate over the westerly in the ratio of 1.14 to 1. The ratio from the Toronto observations between 1844 and 1848 is 1.28 to 1.

The following table shows the aggregate amount of disturbances in each month of the different years, or the annual inequality of the aggregate disturbances:—

Month.	1840.	1841.	1842.	1843.	1844.	1845.	Mean.	Ratio.
	d.	d.	d.	d.	d.	d.	d.	
January . . .	(418.4)	423.6	585.9	(171.0)	45.3	269.2	318.9	0.72
February . .	(323.0)	402.3	310.1	(131.9)	99.7	160.1	237.8	0.54
March . . .	(400.5)	327.9	264.4	(163.6)	430.0	167.4	292.3	0.66
April	(544.6)	294.7	481.1	281.7	601.5	289.7	415.6	0.94
May	(329.0)	442.8	184.4	206.8	205.5	111.0	246.6	0.56
June	83.1[1]	355.5	353.1	133.9	50.4	141.2	186.2	0.42
July	668.8[1]	416.8	546.8	271.5	168.3	(220.4)	382.1	0.87
August . . .	618.6	823.1	873.5	953.9	552.6	(434.2)	709.3	1.61
September . .	853.5	1242.7	779.9	301.5	448.6	(484.1)	685.0	1.56
October . . .	1319.1	1376.2	1253.2	195.0	668.1	(639.3)	908.5	2.06
November . .	314.6	1054.2	339.3	87.1	591.1	(387.4)	462.3	1.06
December . .	1282.3	684.6	47.4	34.3	366.2	(217.4)	438.7	1.00
Sum	7155.5	7844.4	6019.1	2932.2	4227.3	3521.4	5283.3	12.00

The last column of ratios of the aggregate value of the disturbances of each month to the mean of all, corresponds very closely to the analogous ratios deduced in a preceding table for the number of disturbances, giving the law in reference to the number and amount of disturbances in a year as the same, or nearly so. The maximum amount of disturbances occurs in October (at Toronto in September), the minimum amount in June (the same at Toronto); the secondary maximum occurs in April (as at Toronto), and the secondary minimum in February; but at Toronto in January, from comparison with the years 1843, 1844, 1845.

The next tables give the aggregate monthly values in the six years, separated into west and east deflections:—

[1] The differences of the disturbed readings from their respective normals during the month of June and part of July, 1840, were first converted from the old scale into equivalent new scale values.

West Deflections.								
Month.	1840.	1841.	1842.	1843.	1844.	1845.	Mean.	Ratio.
	d.	d.	d.	d.	d.	d.	d.	
January . . .	(495.5)	308.4	444.8	(170.4)	23.8	161.6	267.4	1.21
February . . .	(238.0)	147.2	217.1	(82.0)	28.0	69.9	130.4	0.59
March	(288.7)	127.2	168.5	(99.5)	172.8	117.5	162.4	0.73
April	(432.2)	97.9	216.9	98.9	370.1	171.0	229.5	1.04
May	(212.8)	229.5	84.4	109.7	43.5	8.3	114.7	0.52
June	30.9	170.4	194.2	21.7	12.6	65.9	82.6	0.37
July	186.7	51.1	140.5	153.3	28.9	(42.9)	100.6	0.46
August	275.9	228.4	721.3	809.7	304.5	(247.5)	431.2	1.95
September . .	495.3	257.8	116.7	65.2	249.3	(123.5)	217.9	0.99
October . . .	1019.9	422.5	172.5	74.4	340.3	(185.5)	369.2	1.67
November . . .	178.4	586.9	159.6	39.1	267.1	(196.9)	238.0	1.09
December . . .	1210.5	308.2	9.4	17.7	178.8	(98.7)	303.9	1.38
Sum	5064.8	2935.5	2645.9	1741.6	2019.7	1489.2	2647.8	12.00

East Deflections.								
Month.	1840.	1841.	1842.	1843.	1844.	1845.	Mean.	Ratio.
	d.	d.	d.	d.	d.	d.	d.	
January . . .	(27.9)	115.2	141.1	(22.7)	21.5	107.6	72.7	0.33
February . . .	(55.7)	255.1	93.0	(38.5)	71.7	90.2	100.7	0.46
March	(81.8)	200.7	95.9	(53.4)	257.2	49.9	123.2	0.56
April	(116.7)	196.8	264.2	182.8	231.4	118.7	185.1	0.84
May	(66.2)	213.3	100.0	97.1	162.0	102.7	123.6	0.56
June	52.2	185.1	158.9	112.2	37.8	75.3	103.6	0.47
July	482.1	365.7	406.3	118.2	139.4	(177.5)	281.5	1.29
August	342.7	594.7	152.2	144.2	248.1	(194.8)	279.4	1.28
September . .	358.2	984.9	663.2	236.3	199.3	(358.3)	466.7	2.12
October . . .	299.2	953.7	1080.7	120.6	327.8	(453.0)	539.2	2.46
November . . .	136.2	467.3	179.7	48.0	324.0	(187.6)	223.8	1.02
December . . .	71.8	376.4	38.0	16.6	187.4	(116.6)	134.4	0.61
Sum	2090.7	4908.9	3373.2	1190.6	2207.6	2032.2	2633.9	12.00

Maxima in September (mean of August and October) and April; minima in June and January as at Toronto.

The following table gives the aggregate values of the disturbances distributed into the different hours of the day, as deduced from bi-hourly observations made in 1840 to 1845:—

Philadelphia. Hour. (+ 19¼ m.)	Aggregate Values of Western Deflections, Eastern Deflections, and Sum.			Mean Aggregate Values for one Year.			Ratios.		
	W.	E.	Sum.	W.	E.	Sum.	W.	E.	Both combined.
	d.	d.	d.	d.	d.	d.			
0h.	897.4	1438.5	2335.9	149.6	239.8	389.4	0.83	1.24	1.04
2	1259.7	1278.2	2537.9	209.9	213.0	422.9	1.16	1.10	1.13
4	1255.5	1075.5	2331.0	209.2	179.3	388.5	1.16	0.92	1.04
6	1581.7	773.6	2355.3	263.6	128.9	392.5	1.46	0.67	1.06
8	1512.4	769.9	2282.3	252.1	128.3	380.4	1.39	0.67	1.02
10	1315.2	901.9	2217.1	219.2	150.3	369.5	1.22	0.77	0.99
Noon	1114.8	733.2	1848.0	185.8	122.2	308.0	1.03	0.63	0.83
14	1056.4	735.0	1791.4	176.1	122.5	298.6	0.98	0.63	0.80
16	1068.1	825.8	1893.9	178.0	137.6	315.6	0.99	0.72	0.85
18	902.1	965.2	1867.3	150.3	160.9	311.2	0.84	0.89	0.84
20	408.9	2175.4	2584.3	68.2	362.6	430.8	0.38	1.88	1.15
22	610.4	2180.3	2790.7	101.7	363.4	465.1	0.56	1.88	1.25
Sum	12982.6	13852.5	26835.1	2163.7	2308.8	4472.5	12.00	12.00	12.00
Mean	...	...	...	180.3	192.4	372.7	...	...	...

If we compare these ratios with the corresponding numbers in the preceding tables, showing the bi-hourly distribution in regard to the number of disturbances, we find, irrespective of the directions of the deflections, the 2 P. M. minimum preserved; the maximum occurs at 10 P. M. At Toronto, from a five years' hourly series, commencing with 1844, these hours are respectively 1 P. M. and 9 P. M. At Philadelphia, as at Toronto, the ratios are nearly invariable from 10 A. M. to 6 P. M., being then below unity; and again from 8 P. M. to 8 A. M., when they are above unity.

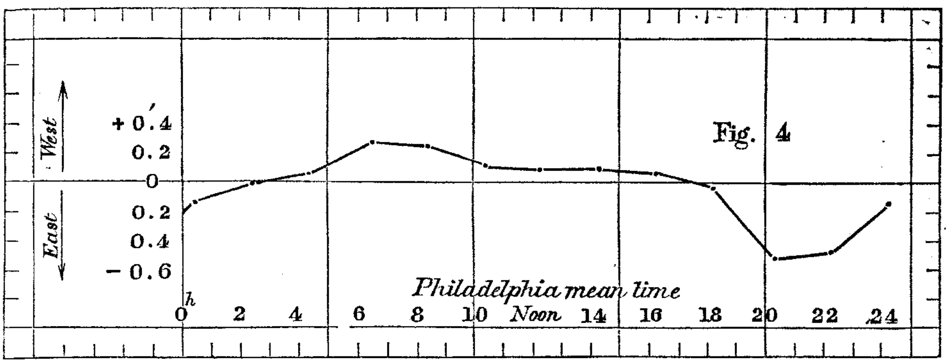
The easterly maximum and the westerly minimum at 8 P. M. appear again as a decided feature, and in general, the respective ratios exhibiting the diurnal distribution of the disturbances, both in an easterly and westerly direction, show almost a perfect correspondence in regard to both number and amount.

The next table exhibits the excess of westerly disturbance over easterly (the sign — indicating a defect, or excess of easterly over westerly) in the aggregate values of the five year series, and in the last column, the mean effect of the same at each even hour, is given as obtained by dividing the aggregate differential value of the preceding column by the actual number of days of observation during the whole period. The last column exhibits, therefore, the mean diurnal disturbance variation. The number of days is very nearly 1500.

Philadelphia mean time.	Excess of westerly over easterly values.	Diurnal Variation caused by the Larger Disturbances.		For comparison: Disturbance—variation at Toronto 1843–'44–'45 (at even hours).	Philadelphia mean time.	Excess of westerly over easterly values.	Diurnal Variation caused by the Larger Disturbances.		For comparison: Disturbance—variation at Toronto 1843–'44–'45 (at even hours).
		In scale divisions.	In minutes of arc.				In scale divisions.	In minutes of arc.	
	d.	d.	′	′		d.	d.	′	′
0h. 19½m.	—541.1	—0.36	—0.16	—0.36	Noon 19½m.	+ 381.6	+0.25	+0.11	+0.09
2 19½	— 18.5	—0.01	—0.01	—0.20	14 19½	+ 321.4	+0.21	+0.10	+0.04
4 19½	+180.0	+0.12	+0.05	—0.03	16 19½	+ 242.3	+0.16	+0.07	+0.03
6 19½	+808.1	+0.54	+0.24	+0.02	18 19½	— 63.1	—0.04	—0.02	—0.16
8 19½	+742.5	+0.50	+0.22	+0.10	20 19½	—1766.5	—1.18	—0.53	—0.56
10 19½	+413.3	+0.28	+0.13	+0.06	22 19½	—1569.9	—1.05	—0.47	—0.75

The law governing the disturbances during a solar day is clearly shown, and systematic in character. If we plot the disturbance curve on the same scale, or actually superpose it on the curves of the regular diurnal variation, the difference would hardly show to the eye. The diagram, showing the disturbance variation, has, therefore, been plotted on a larger scale. (See Fig. 4.)

DIURNAL DISTURBANCE VARIATION OF THE DECLINATION.



The curve has but one maximum and one minimum; its most prominent feature is the easterly deflection at 8 o'clock ($+19\frac{1}{2}^m$) P. M. (at Toronto it is at 9 P. M.). At that hour the maximum deflection amounts to 32″ of arc, and to 45″ at Toronto. The greatest westerly deflection occurs at 6^h ($+19\frac{1}{2}^m$) A. M., and amounts to but 14″; the Toronto hour is 8 A. M. with 6″, and from a five years' series of observation, with 31″ of deflection. The range of the disturbance variation equals 46″.[1] The disturbance amplitude, as well as the regular variation amplitude, is greater at Toronto than at Philadelphia, the occurrence of the maximum and minimum disturbance deflection seeming to be about one hour earlier at the latter station. From three in the morning till five in the afternoon the mean effect of the disturbances is to deflect the north end of the magnet to the west, and during the remaining hours (principally at night) to the east. The westerly and easterly disturbance deflections during a day balance within 0′.02.

The annual inequality in the amplitude of the diurnal disturbance variation might be satisfactorily shown by the proper combination of the results for consecutive years, comparing each two-year series successively; but owing to the small amount of the amplitude itself, and the incomplete or partly interrupted series of observations in the years 1840, 1843, and 1845, it was thought best to restrict the present discussion to the mean disturbance variation.

It is my intention to continue the discussion of the observations made at the Girard College Observatory.

After the above was written,[2] No. 1185 of the *Astronomische Nachrichten* came to hand, containing Prof. R. Wolf's interesting results on the close connection of the variation in frequency of the solar spots, and the corresponding inequality in the amplitude of the diurnal variation of the declination. He deduces for Munich the formula $\beta = 6'.273 + 0'.051\ a$ in which a is a relative number expressive of

---

[1] At Toronto 51″, and from a five years' series 83″.

[2] For former communications by Prof. R. Wolf, see Nos. 839, 1043, 1091, 1132, 1160, and 1181, *ibid.*

the frequency of the solar spots directly derived from observation and β the amplitude of the diurnal variation. He finds a very close correspondence between the computed and observed values of β, and gives in a table Dr. Lamont's and his own results between the years 1835 and 1850. He also reaffirms his former value for the average length of the solar spot period., viz., 11.11 years $\pm$ 0.04 years, the limits of variation being 8 and 16 years. This period is deduced from observations of maxima and minima since 1626.

For Philadelphia we have $\beta = 7'.080 + 0'.039\ \alpha$ representing the observed amplitudes as follows:—

Year.	α (from solar spot) observations.	β derived from α.	Observed amplitude, or β.	Difference obs'd and comp'd β.	Year.	α (from solar spot) observations.	β derived from α.	Observed amplitude, or β.	Difference obs'd and comp'd β.
1840	51.8	9'.10	9'.08	—0'.02	1843	8.4	7.'41	7'.46	+0'.05
1841	29.5	8.23	8.06	—0.17	1844	12.2	7.55	7.51	—0.04
1842	19.2	7.83	7.83	0.00	1845	32.4	8.34	8.53	+0.19

The correspondence between the observed diurnal amplitude and the same derived from observations of the solar spots is further exhibited by Fig. 5, the heavy line representing the magnetic, the other the solar amplitude curve. The dotted curve is from the Toronto magnetic observations, merely multiplied by $\frac{8}{9}$ to reduce (approximately) to the Philadelphia scale. The next maximum amplitude, according to the solar spot observations, would be in 1848, amounting to 11'.00; and the whole range of the inequality in the amplitude of the diurnal motion would, therefore, be 11'.00 — 7'.46 = 3'.54. The last quantity, it must be observed, is slightly variable with each period; thus, according to the solar spot observations, the year 1837 was a maximum, amplitude 11'.41; and the year 1856 a minimum, amplitude 7'.24, the difference being 4'.17.

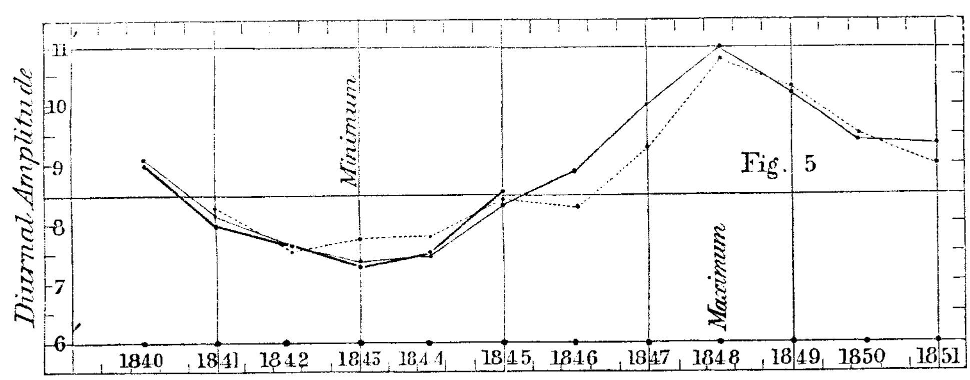


Fig. 5

It is much to be desired that this interesting branch of physical inquiry should be further studied, as it forms one of the links connecting terrestrial with cosmical phenomena.

SMITHSONIAN CONTRIBUTIONS TO KNOWLEDGE.

121

DISCUSSION

OF THE

MAGNETIC AND METEOROLOGICAL OBSERVATIONS

MADE AT THE GIRARD COLLEGE OBSERVATORY, PHILADELPHIA,
IN 1840, 1841, 1842, 1843, 1844, AND 1845.

PART II.

INVESTIGATION OF THE SOLAR DIURNAL VARIATION IN THE MAGNETIC DECLINATION AND ITS ANNUAL INEQUALITY.

BY

A. D. BACHE, LL. D.

[ACCEPTED FOR PUBLICATION, SEPTEMBER, 1860.]

INVESTIGATION

OF THE

SOLAR-DIURNAL VARIATION OF THE MAGNETIC DECLINATION, AND ITS ANNUAL INEQUALITY.

HAVING discussed, in Part I, the eleven-year period in the amplitude of the solar-diurnal variation, as well as in the disturbances of the magnetic declination, I now proceed to the analysis of the annual inequality of the solar-diurnal variation.

To obviate the difficulty which would occur in cases of months of unusual disturbance, if the crude observations were used, the normals or means freed from the disturbances have been employed in the discussion. This mode of proceeding not only obviates the necessity for rejecting the observations of particular months, but brings out the most consistent results which the observations can furnish, for both diurnal and annual variation. It is the course adopted by General Sabine in the third volume of his discussion of the Toronto observations.[1]

Returning, then, to the hourly normals, they are rearranged in the tables which follow, according to the different months of the year. The normals for 1840 are corrected for the index error by the addition of 93.3 scale divisions. All corrections for referring the partial monthly readings to the annual mean are, of course, omitted.

---

[1] Table LXVI, of this volume, exhibits the solar-diurnal variation of the declination after the separation and omission of the larger disturbances; whereas Table VII, of the preceding volume, similar in form, differs from the latter, being derived from all the observations including the disturbances.

HOURLY DECLINATION. NORMALS FOR JANUARY.[1]
Observations 19½ minutes later than indicated. Value of one scale division = 0′.453.
Increase of scale readings corresponds to a decrease of westerly declination.

YEAR.	0h.	1h.	2h.	3h.	4h.	5h.	6h.	7h.	8h.	9h.	10h.	11h.
	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.
1840	...	...	...	...	...	...	...	...	...	...	...	...
1841	579.3	...	577.0	...	578.6	...	576.9	...	580.7	...	581.9	...
1842	564.3	...	563.8	...	565.3	...	565.9	...	570.9	...	566.4	...
1843	...	...	...	...	...	...	...	...	...	...	...	...
1844	558.6	558.2	558.4	559.2	558.9	558.8	559.7	561.2	562.9	563.3	559.1	555.9
1845	530.9	531.3	531.1	531.5	533.0	531.6	532.9	535.2	535.8	533.8	530.2	526 7
Mean[2]	558.28	...	557.57	...	558.95	...	558.85	...	562.57	...	559.40	...
Same refer'd to its mean epoch[3]	565.25	564.80	564.35	565.62	565.70	564.66	565.47	567.74	569.27	569.51	566.65	561.88

YEAR.	Noon.	13h.	14h.	15h.	16h.	17h.	18h.	19h.	20h.	21h.	22h.	23h.
	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.
1840	...	...	...	...	...	...	...	...	...	...	...	...
1841	570.0	...	568.8	...	570.3	...	574.2	...	578.0	...	580.1	...
1842	556.7	...	556.0	...	562.9	...	563.2	...	566.1	...	567.8	...
1843	...	...	555.4	...	...	...	...	...	...	...	...	...
1844	552.9	552.4	553.2	554.1	556.3	556.9	557.8	559.2	559.5	560.9	560.8	559.6
1845	524.2	525.2	526.2	528.0	530.1	531.8	532.7	532.8	533.3	533.0	532.4	532.0
Mean[2]	550.95	...	551.92	...	554.90	...	556.97	...	559.22	...	560.27	...
Same refer'd to its mean epoch[3]	557.72	557.31	557.55	558.97	561.20	562.41	563.38	564.82	565.90	567.00	567.20	566.35

[1] The hours refer to mean local time, reckoned from midnight to 24 hours.

[2] The mean given is the simple mean of the four readings, and at 14^h of five readings, and is here inserted for comparison with the corrected mean in the line below, which would have been obtained if there had been no omissions in the observations.

[3] To obtain the normals referring to January of the mean year, the readings for the defective years 1840 and 1843 have been interpolated in the following manner: 1. *For the even hours.*—The normals for any two consecutive years differ simply by the annual effect of the secular change, which may be regarded as uniform when the same hours and months are compared, as in the present case. The values derived from the comparison of the several months of any two years differ, however, by the accidental errors of the observations; thus, taking the difference of the normals for 1840 and 1841, we obtain for the several months the values—

June . . .	$+15^d.7$	September . .	$+21^d.9$	December . .	$+20^d.0$
July . . .	20.5	October . . .	12.7		
August . . .	18.5	November . .	17.5	Mean .	16.86

Which mean corresponds exactly to the difference of the constant terms in Part I, for 1840 and 1841. By adding, therefore, 16.9 scale divisions to the normals for 1841, we obtain interpolated values for 1840. The values from January to May, 1840, were thus supplied. The normals for 1843 were supplied in a different manner, by making use of the readings at 2 P. M., which were taken for the purpose of keeping up the continuity of the series. Subtracting 0.6 scale division from the hourly readings of 1842, we obtain those for 1843—this being the difference at 14^h; in like manner, adding 2.2 scale divisions to the readings of 1844, we obtain a second value for the normals of 1843. The mean of these two independent determinations has been used in supplying the readings for 1843. The normals for 1840 and 1843, being thus supplied, the figures in the last line of the preceding table are obtained by simply taking the mean of the six readings at each even hour. 2. *For the odd hours.*—The difference in the mean readings for any given odd hour, in 1844 and 1845, from the two adjacent even hours, was applied to the normals of these hours, and the mean taken as the normal of the intermediate odd hour. Thus, the mean reading at noon of 1844 and 1845 is 538.55, at 13^h, 538.80, difference +0.25; which, added to the noon normal 557.72, gives 557.97; and, in like manner, by comparison with 14^h, the correction to its normal is —0.90, and the normal for 13^h becomes 556.65. The mean of the two results, 557.31, is the resulting normal for this hour as given in the table.

The same principle of interpolation was applied throughout the tables. Due attention must be paid, in the deductions, for the unequal weight of the normals for the even and odd hours; these weights being generally as 5 : 2, or proportional to the number of separate readings. The application of a nearly constant quantity to refer means from a defective number of years to the mean epoch of all the years, is not of much consequence in regard to the diurnal and annual inequalities, which depend mainly on differences of readings, but it is essential that no changes should have occurred in the zero of the scale during any interval under discussion.

Hourly Declination. Normals for February.

Observations 19½ minutes later than indicated. One division of scale = 0'.453.

Year.	0h.	1h.	2h.	3h.	4h.	5h.	6h.	7h.	8h.	9h.	10h.	11h.
	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.
1840	...	...	...	...	...	...	...	...	...	...	...	...
1841	575.0	...	573.2	...	575.6	...	577.8	...	582.1	...	579.5	...
1842	564.5	...	564.3	...	563.8	...	565.2	...	567.8	...	565.5	...
1843	...	...	...	...	...	...	...	...	...	...	...	...
1844	559.1	558.5	559.1	559.2	559.9	561.1	560.8	562.1	562.2	5 0.7	557.3	554.5
1845	531.6	531.1	531.0	532.4	532.3	533.1	534.7	535.9	535.7	535.4	533.0	528.6
Mean	557.55	...	556.90	...	557.90	...	559.62	...	561.95	...	558.82	...
Same refer'd to its mean epoch	563.88	563.10	563.13	563.90	564.23	565.25	565.93	567.88	568.53	567.97	565.42	561.47

Year.	Noon.	13h.	14h.	15h.	16h.	17h.	18h.	19h.	20h.	21h.	22h.	23h.
	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.
1840	...	...	...	...	...	...	...	...	...	...	...	...
1841	569.5	...	566.0	...	569.5	...	572.4	...	574.4	...	575.8	...
1842	558.2	...	559.9	...	558.0	...	561.9	...	565.3	...	565.5	...
1843	...	...	555.9	...	...	...	...	...	...	...	...	...
1844	551.1	551.1	553.0	554.7	556.4	556.6	557.6	558.4	559.9	559.4	560.1	559.0
1845	524.4	523.0	525.3	527.5	529.7	530.4	532.4	531.3	533.6	534.4	532.3	531.9
Mean	550.80	...	552.02	...	553.40	...	556.07	...	558.30	...	558.42	...
Same refer'd to its mean epoch	557.33	555.85	557.17	558.30	559.43	560.25	562.13	562.25	564.42	565.02	564.77	564.00

Hourly Declination. Normals for March.

Observations 19½ minutes later than indicated. One division of scale = 0'.453.

Year.	0h.	1h.	2h.	3h.	4h.	5h.	6h.	7h.	8h.	9h.	10h.	11h.
	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.
1840	...	...	...	...	...	...	...	...	...	...	...	...
1841	577.1	...	577.6	...	580.9	...	582.9	...	586.8	...	578.9	...
1842	564.8	...	564.1	...	565.4	...	566.1	...	571.8	...	565.9	...
1843	...	...	...	...	...	...	...	...	...	...	...	...
1844	558.0	559.0	559.2	557.9	559.8	560.2	561.3	563.6	564.8	564.1	560.3	554.9
1845	532.9	532.7	533.7	533.6	535.0	533.9	536.0	538.8	539.4	538.6	534.5	529.4
Mean	558.20	...	558.65	...	560.27	...	561.58	...	565.70	...	559.90	...
Same refer'd to its mean epoch	565.60	565.72	566.03	565.75	567.82	567.53	569.20	572.11	573.37	571.95	567.32	562.02

Year.	Noon.	13h.	14h.	15h.	16h.	17h.	18h.	19h.	20h.	21h.	22h.	23h.
	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.
1840	...	...	...	...	...	...	...	...	...	...	...	...
1841	569.4	...	567.7	...	571.8	...	576.4	...	577.4	...	577.7	...
1842	555.6	...	553.9	...	556.4	...	560.3	...	564.5	...	564.9	...
1843	...	...	557.2	...	...	...	...	...	...	...	...	...
1844	550.6	549.4	549.6	551.7	553.0	555.2	556.6	558.0	558.4	558.2	558.6	559.7
1845	524.8	522.5	522.8	524.8	527.8	529.7	531.6	533.0	533.0	533.8	533.5	534.0
Mean	550.10	...	550.24	...	552.25	...	556.22	...	558.32	...	558.67	...
Same refer'd to its mean epoch	557.52	555.75	555.97	557.75	559.63	561.85	563.68	565.31	565.75	566.04	566.08	566.94

HOURLY DECLINATION. NORMALS FOR APRIL.

Observations $19\frac{1}{2}$ minutes later than indicated. One division of scale = 0′.453.

YEAR.	0h.	1h.	2h.	3h.	4h.	5h.	6h.	7h.	8h.	9h.	10h.	11h.
	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.
1840	...	...	...	...	...	...	...	...	...	...	...	...
1841	580.0	...	581.9	...	582.9	...	585.6	...	587.6	...	579.4	...
1842	563.3	...	565.4	...	566.1	...	568.5	...	569.7	...	563.6	...
1843	569.7	...	570.0	...	571.0	...	574.7	...	576.2	...	566.2	...
1844	556.6	557.0	557.2	556.9	557.5	558.4	561.7	558.5	564.4	561.8	557.1	552.0
1845	529.1	528.8	529.0	529.2	529.8	531.7	534.0	535.6	537.5	535.4	528.5	522.5
Mean	559.74	...	560.70	...	561.46	...	564.90	...	567.08	...	558.96	...
Same refer'd to its mean epoch	565.93	566.42	567.05	567.12	567.85	568.31	571.17	569.90	373.32	570.98	565.18	559.76

YEAR.	Noon.	13h.	14h.	15h.	16h.	17h.	18h.	19h.	20h.	21h.	22h.	23h.
	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.
1840	...	...	...	...	...	...	...	...	...	...	...	...
1841	568.8	...	566.1	...	571.7	...	576.9	...	578.0	...	579.1	...
1842	554.0	...	552.5	...	555.1	...	560.6	...	561.3	...	563.0	...
1843	557.8	...	555.7	...	562.6	...	564.8	...	568.5	...	568.7	...
1844	547.4	545.7	546.2	547.6	549.6	553.4	553.4	553.8	556.2	555.1	555.7	559.3
1845	517.8	513.9	514.0	517.2	521.5	525.8	527.8	527.9	528.1	528 5	528.0	529.4
Mean	549.16	...	546.90	...	552.10	...	556.70	...	558.42	...	558.90	...
Same refer'd to its mean epoch	555.25	552.54	552.92	555.13	558.18	562.05	562.88	563.16	564.50	564.59	565.08	567.50

HOURLY DECLINATION. NORMALS FOR MAY.

Observations $19\frac{1}{2}$ minutes later than indicated. One division of scale = 0′.453.

YEAR.	0h.	1h.	2h.	3h.	4h.	5h.	6h.	7h.	8h.	9h.	10h.	11h.
	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.
1840	...	...	...	...	...	...	...	...	...	...	...	...
1841	579.1	...	579.8	...	581.9	...	587.4	...	589.1	...	578.6	...
1842	563.3	...	564.3	...	566.0	...	571.2	...	569.5	...	560.0	...
1843	567.0	...	567.3	...	569.6	...	574.6	...	575.6	...	565.7	...
1844	548.4	548.7	547.8	547.0	549.3	552.5	555.8	556 8	555.1	552.3	546.7	542.2
1845	529.9	531.3	529.7	531.7	533.2	536.3	539.3	541.9	540.7	536.0	528.0	522.6
Mean	557.54	...	557.78	...	560.00	...	565.66	...	566.00	...	555.80	...
Same refer'd to its mean epoch	563.95	565.16	564.27	564.72	566.47	569.28	572.10	574.01	572.67	569.07	562.42	557.72

YEAR.	Noon.	13h.	14h.	15h.	16h.	17h.	18h.	19h.	20h.	21h.	22h.	23h.
	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.
1840	...	...	...	...	...	...	...	...	...	...	...	...
1841	569.4	...	567.9	...	573.6	...	577.4	...	578.5	...	580.1	...
1842	552.6	...	552.3	...	557.7	...	560.8	...	561.8	...	562.3	...
1843	556.0	...	556.2	...	562.2	...	566.4	...	566.9	...	567.3	...
1844	538.3	535.8	536.5	538.9	542.1	545.1	545.2	546.5	546.3	547.3	547.3	547.8
1845	517.1	516.8	518.9	522.1	526.7	529.3	529.6	530.4	529.7	530.3	530.5	530.3
Mean	546.68	...	546.36	...	552.46	...	555.88	...	556.64	...	557.50	...
Same refer'd to its mean epoch	553.28	551.62	552.77	555.23	558.80	561.94	562.28	563.44	563.10	563.94	564.09	564.04

Hourly Declination. Normals for June.

Observations 19½ minutes later than indicated. One division of scale = 0′.453.

YEAR.	0h.	1h.	2h.	3h.	4h.	5h.	6h.	7h.	8h.	9h.	10h.	11h.
	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.
1840	587.7	...	588.3	...	590.8	...	597.3	...	596.0	...	587.1	...
1841	571.7	...	572.2	...	574.7	...	583.3	...	582.6	...	571.1	...
1842	564.6	...	563.7	...	567.2	...	573.7	...	573.0	...	565.2	...
1843	566.0	...	565.6	...	568.4	...	574.1	...	573.9	...	564.8	...
1844	548.7	549.0	549.3	549.1	551.6	553.9	557.6	559.1	558.2	554.3	547.9	541.8
1845	531.5	531.7	531.6	532.0	534.8	537.9	541.9	543.5	542.5	538.6	532.2	524.9
Mean	561.70	...	561.78	...	564.58	...	571.32	...	571.03	...	561.38	...
Same refer'd to its mean epoch	...	561.81	...	561.91	...	567.38	...	572.42	...	567.46	...	555.22

YEAR.	Noon.	13h.	14h.	15h.	16h.	17h.	18h.	19h.	20h.	21h.	22h.	23h.
	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.
1840	578.8	...	576.7	...	581.2	...	586.1	...	585.8	...	586.9	...
1841	561.6	...	560.3	...	565.0	...	570.1	...	570.9	...	570.8	..
1842	555.1	...	552.5	...	558.3	...	561.8	...	563.7	...	564.1	...
1843	556.4	...	556.0	...	561.1	...	564.3	...	564.0	...	565.6	...
1844	537.4	535.0	537.3	540.0	542.4	545.2	545.6	546.2	546.5	546.8	548.0	548.5
1845	521.3	519.6	520.0	522.1	525.4	528.9	530.3	530.7	530.1	530.7	530.3	531.4
Mean	551.77	...	550.47	...	555.57	...	559.70	...	560.17	...	560.95	...
Same refer'd to its mean epoch	...	549.42	...	552.80	...	558.76	...	560.26	...	560.58	...	561.65

Hourly Declination. Normals for July.

Observations 19½ minutes later than indicated. One division of scale = 0′.453.

YEAR.	0h.	1h.	2h.	3h.	4h.	5h.	6h.	7h.	8h.	9h.	10h.	11h.
	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.
1840	590.6	...	590.5	...	592.2	...	598.0	...	598.8	...	588.7	...
1841	569.9	...	568.5	...	571.6	...	578.4	...	581.2	...	571.8	...
1842	566.0	...	566.0	...	568.4	...	576.6	...	576.4	...	565.8	...
1843	566.9	...	565.9	...	568.2	...	574.2	...	574.6	...	564.5	...
1844	549.0	550.5	548.4	549.4	551.0	554.3	556.9	559.8	558.6	554.8	548.0	540.8
1845	...	...	...	...	...	...	...	...	...	...	...	...
Mean	568.48	...	567.86	...	570.28	...	576.82	...	577.92	...	567.76	...
Same refer'd to its mean epoch	561.77	563.26	561.15	562.07	563.60	567.16	570.02	572.67	571.23	567.61	561.00	535.47

YEAR.	Noon.	13h.	14h.	15h.	16h.	17h.	18h.	19h.	20h.	21h.	22h.	23h.
	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.
1840	577.8	...	577.3	...	582.0	...	586.6	...	588.8	...	589.6	...
1841	558.9	...	557.3	...	562.3	...	567.2	...	568.8	...	568.6	...
1842	556.3	...	553.8	...	558.5	...	562.4	...	564.2	...	567.1	...
1843	555.1	...	554.1	...	559.5	...	563.6	...	563.8	...	565.6	...
1844	538.3	535.5	536.3	538.8	541.9	544.5	545.8	546.2	546.6	547.4	548.8	549.3
1845	...	...	...	...	...	...	...	...	...	...	...	...
Mean	557.28	...	555.76	...	560.84	...	565.12	...	566.44	...	567.94	..
Same refer'd to its mean epoch	550.65	548.05	549.05	551.33	554.22	556.98	558.43	559.05	559.67	560.18	561.28	561.97

Hourly Declination. Normals for August.

Observations 19½ minutes later than indicated. One division of scale = 0′.453.

YEAR.	h.	1h.	2h.	3h.	4h.	5h.	6h.	7h.	8h.	9h.	10h.	11h.
	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.
1840	588.6	...	589.0	...	592.1	...	599.7	...	602.4	...	582.7	...
1841	568.4	...	570.3	...	571.6	...	580.1	...	583.9	...	568.9	...
1842	564.8	...	566.0	...	568.5	...	573.7	...	575.0	...	560.0	...
1843	564.2	...	564.5	...	267.2	...	573.5	...	572.7	...	560.5	...
1844	548.6	547.8	547.3	547.4	550.9	552.4	557.5	560.3	558.2	551.8	543.3	536.4
1845	...	...	...	...	...	...	...	...	...	...	...	...
Mean	566.92	..	567.42	...	570.06	...	576.90	...	578.44	...	563.08	...
Same refer'd to its mean epoch	560.40	559.85	560.60	560.80	563.40	565.00	570.20	573.35	571.60	565.01	556.32	549.14

YEAR.	Noon.	13h.	14h.	15h.	16h.	17h.	18h.	19h.	20h.	21h.	22h.	23h.
	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.
1840	573.8	...	575.2	...	581.5	...	586.5	...	588.2	...	589.4	...
1841	558.3	...	556.9	...	564.0	...	566.8	...	568.6	...	568.9	...
1842	552.3	...	553.7	...	561.5	...	562.2	...	564.1	...	564.5	...
1843	555.1	...	554.6	...	561.2	...	563.6	...	562.3	...	564.2	...
1844	531.8	532.0	534.3	538.7	542.1	544.3	546.0	546.5	546.7	546.6	547.8	547.7
1845	...	...	...	...	...	...	...	...	...	...	...	...
Mean	554.26	...	554.94	...	562.06	...	565.02	...	565.98	...	566.96	...
Same refer'd to its mean epoch	547.05	546.49	548.03	552.15	555.27	557.12	558.38	558.99	559.30	559.15	560.30	559.85

Hourly Declination. Normals for September.

Observations 19½ minutes later than indicated. One division of scale = 0′.453.

YEAR.	0h.	1h.	2h.	3h.	4h.	5h.	6h.	7h.	8h.	9h.	10h.	11h.
	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.
1840	585.8	...	588.5	...	590.2	...	596.5	...	595.8	...	584.1	...
1841	565.1	...	564.5	...	565.5	...	569.4	...	571.1	...	564.1	...
1842	567.4	...	567.8	...	570.0	...	576.8	...	574.9	...	561.2	...
1843	560.4	...	560.4	...	560.3	...	565.7	...	566.6	...	554.6	...
1844	543.3	543.1	544.1	546.0	546.5	547.1	550.0	552.9	552.4	545.8	538.3	532.5
1845	...	...	...	...	...	...	...	...	...	...	...	...
Mean	564.40	...	565.06	...	566.50	...	571.68	...	572.16	...	560.46	...
Same refer'd to its mean epoch	557.42	557.16	558.10	559.60	559.70	561.00	564.60	566.70	565.40	559.80	553.30	547.47

YEAR.	Noon.	13h.	14h.	15h.	16h.	17h.	18h.	19h.	20h.	21h.	22h.	23h.
	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.
1840	570.6	...	572.8	...	581.7	...	583.2	...	586.6	...	585.9	...
1841	553.6	...	554.5	...	559.5	...	562.9	...	563.8	...	564.0	...
1842	556.0	...	555.4	...	562.0	...	565.7	...	566.7	...	566.6	...
1843	547.5	...	550.5	...	556.8	...	558.0	...	560.0	...	558.7	...
1844	529.3	530.0	534.1	538.3	539.4	541.9	542.4	541.9	543.0	544.6	543.7	543.3
1845	...	...	...	...	...	...	...	...	...	...	...	...
Mean	551.40	...	553.46	...	559.88	...	562.44	...	564.02	...	563.78	...
Same refer'd to its mean epoch	544.25	543.81	546.77	551.44	553.00	555.31	555.63	556.04	557.05	558.26	556.97	557.00

Hourly Declination. Normals for October.

Observations 19½ minutes later than indicated. One division of scale = 0′.453.

YEAR.	0h.	1h.	2h.	3h.	4h.	5h.	6h.	7h.	8h.	9h.	10h.	11h.
	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.
1840	585.8	...	583.7	...	584.4	...	582.4	...	582.5	...	577.4	...
1841	566.8	...	566.3	...	565.5	...	567.6	...	569.4	...	568.2	...
1842	563.1	...	563.1	...	564.4	...	566.0	...	568.8	...	564.0	...
1843	559.6	560.2	559.6	559.1	559.9	560.6	562.1	565.1	566.0	565.0	560.8	556.5
1844	545.1	545.3	544.2	546.1	545.8	544.4	548.6	550.9	551.5	548.7	545.3	540.8
1845	...	...	...	...	...	...	...	...	...	...	...	...
Mean	564.08	...	563.38	...	564.00	...	565.34	...	567.64	...	563.14	...
Same refer'd to its mean epoch	557.45	557.71	556.72	557.33	557.50	556.67	559.08	561.23	561.48	560.04	556.70	552.36

YEAR.	Noon.	13h.	14h.	15h.	16h.	17h.	18h.	19h.	20h.	21h.	22h.	23h.
	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.
1840	571.7	...	570.6	...	575.2	...	579.6	...	579.0	...	586.4	...
1841	564.0	...	562.3	...	564.7	...	573.5	...	568.6	...	569.3	...
1842	556.0	...	555.0	...	558.2	...	564.3	...	565.0	...	565.3	...
1843	553.6	552.6	552.7	554.2	556.2	557.0	558.2	559.7	560.1	561.1	559.7	560.7
1844	541.1	539.5	541.4	544.0	545.7	545.4	545.6	545.0	544.9	544.6	544.5	544.6
1845	...	...	...	...	...	...	...	...	...	...	...	..
Mean	557.28	...	556.40	...	560.00	...	564.24	...	563.56	...	565.04	...
Same refer'd to its mean epoch	551.12	549.62	550.43	552.39	554.15	555.68	557.67	557.47	556.98	558.12	558.15	558.22

Hourly Declination. Normals for November.

Observations 19½ minutes later than indicated. One division of scale = 0′.453.

YEAR.	0h.	1h.	2h.	3h.	4h.	5h.	6h.	7h.	8h.	9h.	10h.	11h.
	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.
1840	574.4	...	573.9	...	576.2	...	577.0	...	579.7	...	575.0	...
1841	557.2	...	558.5	...	558.5	...	557 6	...	561.7	...	557.1	...
1842	564.2	...	563.8	...	565.6	...	566.9	...	569.2	...	563.3	...
1843	556.3	556.7	556.6	556.6	557.4	557.4	559.1	561.8	561.3	560.1	556.2	552.6
1844	546.8	546.8	548.3	548.6	547.4	548.5	551.5	549.2	548.4	547.9	546.2	542.8
1845	...	...	...	...	...	...	...	...	...	...	...	...
Mean	559.78	...	560.22	...	561.02	...	562.42	...	564.06	...	559.56	...
Same refer'd to its mean epoch	554.15	554.21	554.77	555.20	555.28	555.30	557.13	557.98	557.98	556.90	553.87	550.00

YEAR.	Noon.	13h.	14h.	15h.	16h.	17h.	18h.	19h.	20h.	21h.	22h.	23h.
	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.
1840	567.5	...	565.8	...	570.8	...	574.1	...	576.9	...	576.0	...
1841	551.8	...	549.9	...	553.4	...	554.9	...	558.0	...	558.6	...
1842	556.6	...	557.3	...	561.2	...	564.0	...	565.5	...	565.0	...
1843	550.4	550.0	551.1	552.6	553.8	554.9	556.3	557.5	557.5	557.7	557.3	557.4
1844	542.8	541.7	544.5	546.1	545.6	547.9	548.8	548.2	548.3	549.6	548.0	548.0
1845	...	...	...	...	...	...	...	...	...	...	...	...
Mean	553.82	...	553.72	...	556.96	...	559.62	...	561.24	...	560.98	...
Same refer'd to its mean epoch	548.52	547.32	548.72	550.76	551.60	553.25	554.35	555.26	555.62	556.36	555.35	555.35

HOURLY DECLINATION. NORMALS FOR DECEMBER.

Observations 19½ minutes later than indicated. One division of scale = 0'.453.

YEAR.	0h.	1h.	2h.	3h.	4h.	5h.	6h.	7h.	8h.	9h.	10h.	11h.
	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.
1840	571.2	...	568.5	...	573.1	...	572.8	...	573.8	...	573.9	...
1841	560.1	...	559.3	...	560.5	...	559.6	...	560.1	...	558.1	...
1842	561.7	...	560.7	...	562.1	...	562.7	...	565.5	...	564.2	...
1843	559.0	558.1	557.4	558.2	557.8	558.8	560.0	560.8	561.2	561.9	559.9	556.7
1844	536.1	535.8	535.4	535.9	536.8	537.3	537.2	536.8	537.9	539.3	536.1	532.9
1845	...	...	...	...	...	...	...	...	...	...	...	...
Mean	557.62	...	556.26	...	558.06	...	558.46	...	559.70	...	558.44	...
Same refer'd to its mean epoch	550.57	549.92	549.32	550.38	551.05	551.35	551.45	551.75	552.60	553.75	551.25	547.78

YEAR.	Noon.	13h.	14h.	15h.	16h.	17h.	18h.	19h.	20h.	21h.	22h.	23h.
	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.
1840	564.0	...	564.9	...	566.0	...	571.8	...	572.3	...	574.5	...
1841	552.9	...	551.7	...	555.8	...	559.6	...	563.3	...	561.6	...
1842	556.6	...	556.2	...	560.1	...	562.0	...	563.5	...	563.8	...
1843	552.9	551.4	550.9	553.1	554.6	557.5	558.2	558.9	559.6	560.0	559.9	559.5
1844	530.6	529.3	529.4	532.1	533.2	534.8	535.9	537.0	536.8	537.4	537.8	537.1
1845	...	...	...	...	...	...	...	...	...	...	...	...
Mean	551.40	...	550.62	...	553.94	...	557.50	...	559.10	...	559.52	...
Same refer'd to its mean epoch	544.47	543.45	543.62	546.35	547.02	549.40	550.43	551.50	551.92	552.35	552.43	551.60

The following table contains the recapitulation of the monthly normals for each hour of the day, and for the mean epoch 1842 to 1843, and forms the basis for the discussion of the diurnal variation and its annual inequality. The table exhibits at one view the mean hourly readings for each month, unaffected by the larger disturbances.

RECAPITULATION.—MONTHLY DECLINATION-NORMALS FOR EACH HOUR OF THE DAY, AND FOR THE MEAN EPOCH 1842–43.

Increasing scale divisions denote an easterly movement of the north end of the magnet. The readings belong to an hour 19½ minutes later than indicated by the figures at the head of the columns. Value of a scale division = 0′.453. Readings derived from five years of observations between 1840 and 1845.

PHILADELPHIA MEAN TIME.

MEAN EPOCH 1842–43.	0h.	1h.	2h.	3h.	4h.	5h.	6h.	7h.	8h.	9h.	10h.	11h.
	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.
January	565.25	564.80	564.35	565.62	565.70	564.66	565.47	567.74	569.27	569.51	566.65	561.88
February	563.88	563.10	563.13	563.90	564.23	565.25	565.93	567.88	568.53	567.97	565.42	561.47
March	565.60	565.72	566.03	565.75	567.82	567.53	569.20	572.11	573.37	571.95	567.32	562.02
April	565.93	566.42	567.05	567.12	567.85	568.31	571.17	569.90	573.32	570.98	565.18	559.76
May	563.95	565.16	564.27	564.72	566.47	569.28	572.10	574.01	572.67	569.07	562.42	557.72
June	561.70	561.81	561.78	561.91	564.58	567.38	571.32	572.42	571.03	567.46	561.38	555.22
July	561.77	563.26	561.15	562.07	563.60	567.16	570.02	572.67	571.23	567.61	561.00	553.47
August	560.40	559.85	560.60	560.80	563.40	565.00	570.20	573.35	571.60	565.01	556.32	549.14
September	557.42	557.16	558.10	559.60	559.70	561.00	564.60	566.70	565.40	559.80	553.30	547.47
October	557.45	557.71	556.72	557.33	557.50	556.67	559.08	561.23	561.48	560.04	556.70	552.36
November	554.15	554.21	554.77	555.20	555.28	555.30	557.13	557.98	557.98	556.90	553.87	550.00
December	550.57	549.92	549.32	550.38	551.05	551.35	551.45	551.75	552.60	553.75	551.25	547.78

MEAN EPOCH 1842–43.	Noon.	13h.	14h.	15h.	16h.	17h.	18h.	19h.	20h.	21h.	22h.	23h.	Mean.
	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.
January	557.72	557.31	557.55	558.97	561.20	562.41	563.38	564.82	565.90	567.00	567.20	566.35	564.20
February	557.33	555.85	557.17	558.30	559.43	560.25	562.13	562.25	564.42	565.02	564.77	564.00	562.98
March	557.52	555.75	555.97	557.75	559.63	561.85	563.68	565.31	565.75	566.04	566.08	566.94	564.86
April	555.25	552.54	552.92	555.13	558.18	562.05	562.88	563.16	564.50	564.59	565.08	567.50	564.03
May	553.28	551.62	552.77	555.23	558.80	561.94	562.28	563.44	563.10	563.94	564.09	564.04	563.18
June	551.77	549.42	550.47	552.80	555.57	558.76	559.70	560.26	560.17	560.58	560 95	561.65	560.84
July	550.65	548.05	549.05	551.33	554.22	556.98	558.43	559.05	559.67	560.18	561.28	561.97	560.24
August	547.05	546.49	548.03	552.15	555.27	557.12	558.38	558.99	559.30	559.15	560.30	559.85	559.07
September	544.25	543.81	546.77	551.44	553.00	555.31	555.63	556.04	557.05	558.26	556.97	557.00	656.07
October	551.12	549.62	550.43	552.39	554.15	555.68	557.67	557.47	556.98	558.12	558.15	558.22	556.43
November	548.52	547.32	548.72	550.76	551.60	553.25	554.35	555.26	555.62	556.36	555.35	555.35	553.97
December	544.47	543:45	543.62	546.35	547.02	549.40	550.43	551.50	551.92	552.35	552.43	551.60	549.82
Mean	...	...	...	...	...	...	...	...	...	...	...	...	559.64

This table shows plainly the relation of the mean hourly position of the magnet of each month to its general mean position, after the separation of the larger disturbances, and also, by running the eye along any horizontal line, the solar-diurnal variation for each month. It does not, however, show distinctly the annual inequality, on account of the changes in the numbers by the secular change. To eliminate the effect of this change, each hourly normal has been compared, in the following table, with the corresponding mean monthly value, as given in the last right-hand column; the sign + indicating a westerly direction, and — an easterly direction,[1] of the north end of the magnet from the mean monthly position. The scale divisions have been converted into minutes of arc.

[1] The sign + being generally taken to signify west declination, it has been retained to indicate a movement of the north end of the magnet to the west.

TABLE OF THE SOLAR DIURNAL VARIATION OF THE MAGNETIC DECLINATION FOR EACH MONTH OF THE YEAR, SHOWING THE ANNUAL INEQUALITY.

Observations 19½ minutes later than indicated in the headings.

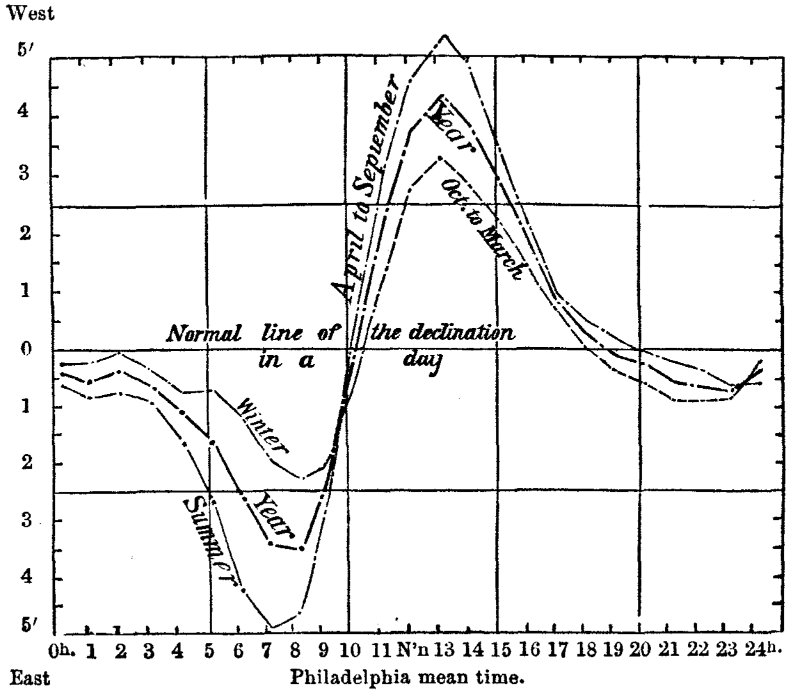
PHILADELPHIA MEAN TIME.

MEAN EPOCH 1842–43.	0h.	1h.	2h.	3h.	4h.	5h.	6h.	7h.	8h.	9h.	10h.	11h.
January	—0′.47	—0′.27	—0′.07	—0′.64	—0′.68	—0′.21	—0′.57	—1′.61	—2′.29	—2′.40	—1′.11	+1′.06
February	—0.41	—0.06	—0 07	—0.42	—0.56	—1.03	—1.34	—2.22	—2.51	—2.26	—1.11	+0.68
March	—0.34	—0.39	—0.53	—0.40	—1.35	—1.21	—1.97	—3.28	—3.85	—3.21	—1.12	+1.29
April	—0.86	—1.09	—1.37	—1.40	—1.73	—1.94	—3.24	—2.65	—4.21	—3.15	—0.50	+1.93
May	—0.35	—0.90	—0.49	—0.70	—1.49	—2.77	—4.04	—4.90	—4.30	—2.66	+0.35	+2.47
June	—0.39	—0.44	—0.43	—0.48	—1.70	—2.97	—4.75	—5.25	—4.62	—3.00	—0.25	+2.54
July	—0.68	—1.37	—0.41	—0.82	—1.53	—3.18	—4.44	—5.63	—4.98	—3.34	—0.35	+3.07
August	—0.60	—0.36	—0.69	—0.78	—1.96	—2.68	—5.03	—6.47	—5.68	—2.69	+1.25	+4.50
September	—0.61	—0.49	—0.92	—1.60	—1.64	—2.23	—3.86	—4.81	—4.23	—1.69	+1.26	+3.89
October	—0.46	—0.58	—0.13	—0.41	—0.48	—0.10	—1.20	—2.17	—2.28	—1.63	—0.12	+1.84
November	—0.09	—0.11	—0.36	—0.55	—0.59	—0.60	—1.44	—1.81	—1.81	—1.33	+0.05	+1.80
December	—0.34	—0.05	+0.23	—0.26	—0.55	—0.69	—0.73	—0.87	—1.27	—1.78	—0.64	+0.93
Summer	—0.58	—0.78	—0.72	—0.96	—1.68	—2.63	—4.23	—4.95	—4.67	—2.76	+0.29	+3.07
Winter	—0.35	—0.24	—0.16	—0.45	—0.70	—0.64	—1.22	—1.99	—2.33	—2.10	—0.67	+1.27
Year	—0.47	—0.51	—0.44	—0.71	—1.19	—1.64	—2.72	—3.47	—3.50	—2.43	—0.19	+2.17

MEAN EPOCH 1842–43.	Noon.	13h.	14h.	15h.	16h.	17h.	18h.	19h.	20h.	21h.	22h.	23h.
January	+2′.94	+3′.12	+3′.01	+2′.36	+1′.36	+0′.81	+0′.37	—0′.28	—0′.77	—1′.27	—1′.36	—0′.98
February	+2.55	+3.23	+2.62	+2.12	+1.61	+1.24	+0.38	+0.33	—0.65	—0.93	—0.81	—0.46
March	+3.33	+4.13	+3.02	+3.22	+2.36	+1.36	+0.53	—0.20	—0.40	—0.54	—0.55	—0.95
April	+3.98	+5.20	+5.02	+4.03	+2.64	+0.90	+0.52	+0.39	—0.21	—0.26	—0.47	—1.57
May	+4.49	+5.24	+4.71	+3.60	+1.99	+0.56	+0.41	—0.12	+0.04	—0.35	—0.41	—0.39
June	+4.11	+5.16	+4.70	+3.64	+2.38	+0.95	+0.51	+0.27	+0.30	+0.12	—0.05	—0.36
July	+4.35	+5.53	+5.07	+4.03	+2.73	+1.47	+0.81	+0.53	+0.26	+0.03	—0.47	—0.78
August	+5.45	+5.71	+5.00	+3.14	+1.72	+0.88	+0.32	+0.04	—0.10	—0.04	—0.56	—0.36
September	+5.35	+5.56	+4.17	+2.09	+1.39	+0.35	+0.20	+0.01	—0.45	—1.00	—0.41	—0.42
October	+2.40	+3.08	+2 72	+1.83	+1.04	+0.35	—0.56	—0.47	—0.25	—0.76	—0.78	—0.81
November	+2.46	+3.01	+2.37	+1.45	+1.08	+0.33	—0.18	—0.59	—0.74	—1.09	—0.63	—0.63
December	+2.42	+2.89	+2.81	+1.57	+1.27	+0.19	—0.27	—0.76	—0.96	—1.15	—1.18	—0.81
Summer	+4.62	+5.40	+4.78	+3.42	+2.14	+0.85	+0.46	+0.19	—0.03	—0.25	—0.40	—0.65
Winter	+2.68	+3.24	+2.76	+2.09	+1.46	+0.71	+0.05	—0.33	—0 63	—0.95	—0.88	—0.77
Year	+3.65	+4.32	+3.77	+2.76	+1.80	+0.78	+0.25	—0.07	—0.33	—0.60	—0.64	—0.71

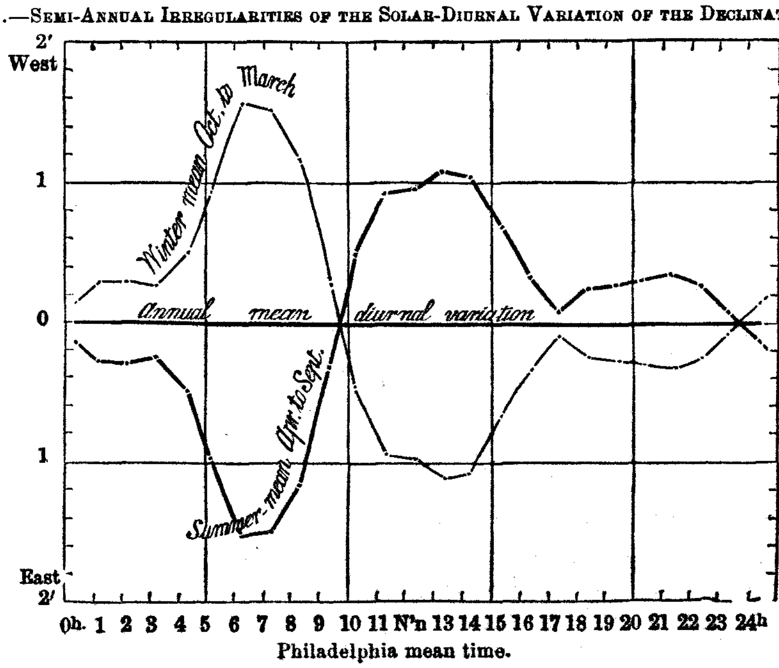
The distinctive features of the above table are next to be considered analytically as well as graphically. The inequality in the diurnal variation is most conspicuous when the tabular numbers in the horizontal lines for the months of February and August are compared. The annual variation appears plainest by carrying the eye over the vertical column at the hours 6 or 7 A. M. The annual variation depends on the earth's position in its orbit; the diurnal variation being subject to an inequality depending on the sun's declination. The diurnal range is greater when the sun has north declination, and smaller when south declination; the phenomenon passing from one state to the other about the time of the equinoxes. To show the diurnal variation at these periods, the summer and winter means, as well as the annual means, were tabulated. The months from April to September (inclusive) comprise the summer period, and from October to March (inclusive) the winter period. The first diagram (A) shows this variation, and contains the type curves for these half yearly periods. We find for the summer months a diurnal range of nearly 10½ minutes, and for the winter months of but 5½ minutes. These and other curves will be further analyzed hereafter.

(A).—MEAN SOLAR-DIURNAL VARIATION OF THE DECLINATION FOR SUMMER, WINTER, AND THE WHOLE YEAR.



The second diagram (B) exhibits the same phenomenon in a different way; the yearly curve of the first diagram being straightened out and forming the axis of the second diagram, which thus shows the deviations from the annual mean value for the two seasons when the sun has north and south declination. The ordinates are obtained by subtracting the annual mean from either the summer or winter mean in the preceding table. This diagram exhibits, in quite a characteristic manner, the course of the annual variation at the different hours of the day, at

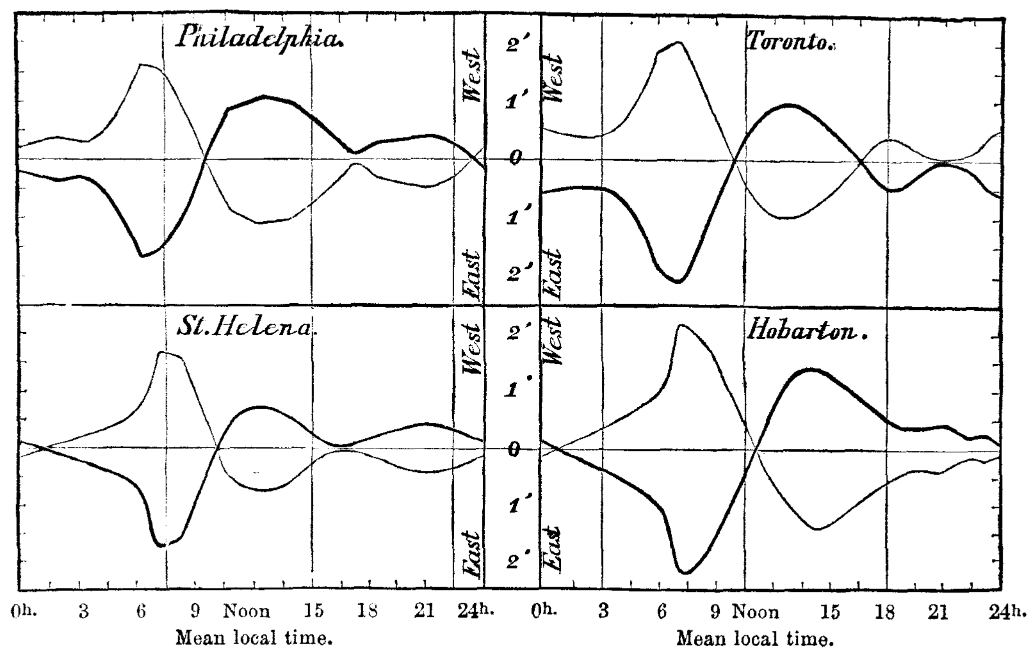
(B).—SEMI-ANNUAL IRREGULARITIES OF THE SOLAR-DIURNAL VARIATION OF THE DECLINATION.



the season for which the diagram is constructed. Thus, at the hour of 6 or 7 in the morning, the annual variation is a maximum, disappearing at a quarter before 10 A. M., and reaching a second (secondary) maximum value at 1 P. M. It almost disappears soon after 5 P. M., and a third still smaller maximum is reached after 9 P. M. Half an hour before midnight, the annual variation again disappears. At (and before and after) the principal maximum, between 6 and 7 in the morning, the annual variation causes the north end of the magnet to be deflected to the east in summer and to the west in winter; at 1 P. M., the deflections are to the west in summer and to the east in winter. The range of the diurnal motion is thus increased in summer and diminished in winter; the magnet being deflected in summer more to the east in the morning hours, and more to the west in the afternoon hours, or having greater elongations than it would have if the sun moved in the equator. In winter, the converse is the case. The range of the annual variation from summer to winter is about 3′.0, and its daily range about 2′.6 at Philadelphia.

(C).—COMPARATIVE DIAGRAM OF THE SEMI-ANNUAL DEFLECTION OF THE SOLAR-DIURNAL VARIATION.

Sun {north / south} of the equator {━━ / ——}



The next diagram (C) has been projected in order to illustrate the semi-annual inequality of the diurnal variation at four principal magnetic stations.[1] The general features of the Philadelphia curve most nearly resemble those exhibited in the St. Helena curve; and relatively, the Toronto and Hobarton curves appear to represent rather extreme than normal shapes. The Philadelphia and St. Helena

[1] The annual variation of the diurnal motion has been made the subject of a particular discussion by General Sabine, in papers presented to the British Association and the Royal Society. See Reports of the British Association, 1854, pp. 355–368, and Transactions of Royal Society, May 18, 1854, pp. 67–82; also, article XXVIII, Philosophical Transactions, 1851.

curves have another feature in common: the amplitude at its maximum value, shortly after 6 A. M., is less than the amplitude at Toronto and Hobarton; and, upon the whole, the Philadelphia type confirms the idea that all forms partake of the same general character, more or less affected by incidental irregularities.

In reference to the annual variation, General Sabine, in the "rectifications and additions" to the last volume of Humboldt's *Cosmos*, expresses himself as follows: "Thus, in each hemisphere, the semi-annual deflections concur with those of the mean annual variation for half the year, and consequently augment them, and oppose and diminish them in the other half. At the magnetic equator, there is no mean diurnal variation, but in each half year the alternate phases of the sun's annual inequality constitutes a diurnal variation, of which the range in each day is about 3′ or 4′, taking place every day in the year except about the equinoxes; the march of this diurnal variation being from east in the forenoon to west in the afternoon, when the sun has north declination, and the reverse when south declination." According to the same authority, the *annual* variation is the *same* in both hemispheres, the north end of the magnet being deflected to the east in the forenoon, the sun having north declination; when in the *diurnal* variation, the north end of the magnet at that time of the day is deflected to the east in the northern hemisphere and to the west in the southern hemisphere. In other words, in regard to direction, the law of the annual variation is the same, and that of the diurnal variation the opposite, in passing from the northern to the southern magnetic hemisphere.

I next proceed to consider more in detail the annual variation at the hours of 6 and 7 in the morning and of 1 and 2 in the afternoon, these being the hours of the principal and secondary maxima respectively. By subtracting the annual mean from each monthly value at the respective hours, we obtain from the preceding general table the following columns:—

ANNUAL VARIATION AT THE HOURS OF THE PRINCIPAL AND SECONDARY MAXIMA OF RANGE.

$\left.\begin{matrix}+\\-\end{matrix}\right\}$ indicates $\left\{\begin{matrix}\text{west}\\\text{east}\end{matrix}\right\}$ deflection from the mean annual position.

	6h. A. M.	7h. A. M.	Mean.	1h. P. M.	2h. P. M.	Mean.
January	+2′.15	+1′.86	+2′.01	—1′.20	—0′.76	—0′.98
February	+1.38	+1.25	+1.31	—1.09	—1.15	—1.12
March	+0.75	+0.19	+0.47	—0.19	—0.75	—0.47
April	—0.52	+0.82	+0.15	+0.88	+1.25	+1.06
May	—1.32	—1.43	—1.38	+0.92	+0.94	+0.93
June	—2.03	—1.78	—1.90	+0.84	+0.93	+0.89
July	—1.72	—2.16	—1.94	+1.21	+1.30	+1.25
August	—2.31	—3.00	—2.66	+1.39	+1.23	+1.31
September	—1.14	—1.34	—1.24	+1.24	+0.40	+0.82
October	+1.52	+1.30	+1.41	—1.24	—1.05	—1.14
November	+1.28	+1.66	+1.47	—1.31	—1.40	—1.35
December	+1.99	+2.60	+2.30	—1.43	—0.96	—1.20
	Maximum range at the above hours, 5′.0; the easterly deflection being greater by 0′.4 than the westerly.			Range at the hours 1 and 2 P.M. 2′.7; the eastern and western deflections being equal.		

A general inspection of the above columns containing the mean values shows that, approximately, the solstices are the turning epochs of this annual variation, the signs changing at the time of the equinoxes. To ascertain how nearly this is true, and in order to obtain a more precise expression, the means of the two columns (after changing the signs in the second) for each month respectively, were put into an analytical form, using Bessel's well-known formula for periodic functions—

$$\Delta a = +\ 1'.78\ sin\ (\theta + 90^\circ) + 0'.32\ sin\ (2\theta + 180^\circ)\ ;$$

$$\text{or, } \Delta a = +\ 1'.78\ cos\ \theta - 0'.32\ sin\ 2\theta\ ;$$

the angle θ counting from January 1st.

The maximum values will occur on the first of January and the first of July; and the transition from a positive to a negative value, and the reverse, will take place on the first of April and the first of October, the equation $1.78\ cos\ \theta = 0.32\ sin\ 2\ \theta$, being only satisfied for $\theta = 90^\circ$ and 270°. That the angles C_1 and C_2 should be exactly 90° and 180° is remarkable. The monthly values are satisfied as follows:—

Middle of	By observation.	By calculation.
January	+1'.50	+1'.56
February	+1.22	+0.94
March	+0.47	+0.30
April	—0.46	—0.30
May	—1.16	—0.94
June	—1.40	—1.56
July	—1.59	—1.56
August	—2.00	—0.94
September	—1.03	—0.30
October	+1.28	+0.30
November	+1.41	+0.94
December	+1.76	+1.56

The regular progression of the monthly values is a feature of the annual variation deserving particular notice. There is no sudden transition from the positive to the negative side, or *vice versâ*, at or near the time of the equinoxes (certainly not at the vernal equinox); on the contrary, the annual variation seems to be regular in its progressive changes. The method here pursued is entirely different from that employed by General Sabine for the same end, but the results are, nevertheless, in close accordance. He remarks (in the British Association report above cited): "When a mean is taken corresponding to the 10th or 11th day after the equinox, the transition from the character of the preceding six months has already commenced and advanced very far towards its completion, and, by the middle of October, is quite complete; apparently, the progress of the change is somewhat more tardy in the March than in the September equinox." From the above analysis, we have found that the transition took place *ten* days after either equinox, and also that the turning points occur ten days after the solstices.

For the more precise determination of the law of the phenomenon, and in order to render the results of similar investigations comparable with one another, the *regular* solar-diurnal variation is now to be expressed as a function of the time. The preceding tabular values, given in minutes of arc, when treated as required by Bessel's[1] periodic function, furnish the following expressions for each month of the year:—

[1] For another development of the formula, see Rev. Dr. H. Lloyd, "On the Mean Results of Observations," Transactions Royal Irish Academy, 1848, Vol. XXII, Part I. Dublin, 1849.

For January, $\Delta_d = +1'.423 \sin (15\,n + 225^\circ\ 09') + 1'.491 \sin (30\,n + 16^\circ\ 38')$
$+0'.579 \sin (45\,n + 220^\circ\ 23') + 0'\ 548 \sin (60\,n + 53^\circ\ ..\ )$

For February, $\Delta_d = +1'.469 \sin (15\,n + 211^\circ\ 09') + 1'.456 \sin (30\,n + 20^\circ\ 50')$
$+0'.472 \sin (45\,n + 231^\circ\ 59') + 0'.352 \sin (60\,n + 60^\circ\ ..\ )$

For March, $\Delta_d = +2'.098 \sin (15\,n + 206^\circ\ 46') + 1'.827 \sin (30\,n + 26^\circ\ 34')$
$+0'.693 \sin (45\,n + 230^\circ\ 10') + 0'.413 \sin (60\,n + 84^\circ\ ..\ )$

For April, $\Delta_d = +2'.906 \sin (15\,n + 213^\circ\ 21') + 2'.001 \sin (30\,n + 34^\circ\ 01')$
$+0'.926 \sin (45\,n + 223^\circ\ 29') + 0'.245 \sin (60\,n + 80^\circ\ ..\ )$

For May $\Delta_d = +2'.746 \sin (15\,n + 210^\circ\ 38') + 2'.377 \sin (30\,n + 45^\circ\ 50')$
$+0'.970 \sin (45\,n + 251^\circ\ 57') + 0'.100 \sin (60\,n + 161^\circ\ ..\ )$

For June, $\Delta_d = +2'.883 \sin (15\,u + 204^\circ\ 09') + 2'.438 \sin (30\,n + 44^\circ\ 15')$
$+0'.941 \sin (45\,n + 254^\circ\ 03') + 0'.216 \sin (60\,n + 114^\circ\ ..\ )$

For July, $\Delta_d = +3'.310 \sin (15\,n + 204^\circ\ 19') + 2'.465 \sin (30\,n + 38^\circ\ 48')$
$+1'.047 \sin (45\,n + 251^\circ\ 38') + 0'.092 \sin (60\,n + 176^\circ\ ..\ )$

For August, $\Delta_d = +3'.161 \sin (15\,n + 211^\circ\ 37') + 2'.849 \sin (30\,n + 52^\circ\ 16')$
$+1'.375 \sin (45\,n + 265^\circ\ 49') + 0'.201 \sin (60\,n + 51^\circ\ ..\ )$

For September, $\Delta_d = +2'.706 \sin (15\,n + 220^\circ\ 05') + 2'.372 \sin (30\,n + 55^\circ\ 54')$
$+1'.126 \sin (45\,n + 261^\circ\ 14') + 0'.414 \sin (60\,n + 115^\circ\ ..\ )$

For October, $\Delta_d = +1'.271 \sin (15\,n + 226^\circ\ 29') + 1'.325 \sin (30\,n + 33^\circ\ 12')$
$+0'.727 \sin (45\,n + 230^\circ\ 52') + 0'.150 \sin (60\,n + 47^\circ\ ..\ )$

For November, $\Delta_d = +1'.259 \sin (15\,n + 229^\circ\ 06') + 1'.257 \sin (30\,n + 39^\circ\ 15')$
$+0'.390 \sin (45\,n + 236^\circ\ 30') + 0'.242 \sin (60\,n + 87^\circ\ ..\ )$

For December, $\Delta_d = +1'.212 \sin (15\,n + 231^\circ\ 46') + 1'.321 \sin (30\,n + 23^\circ\ 34')$
$+0'.367 \sin (45\,n + 205^\circ\ 46') + 0'.418 \sin (60\,n + 32^\circ\ ..\ )$

In like manner, we obtain for the summer half-year (from April to September inclusive), for the winter half-year (from October to March inclusive, and for the whole year, the following expressions for the diurnal variation:—

For summer half-year, $\Delta_d = +2'.936 \sin (15\,n + 210^\circ\ 36') + 2'.404 \sin (30\,n + 46^\circ\ 07')$
$+1'.031 \sin (45\,n + 253^\circ\ 37') + 0'.178 \sin (60\,n + 132^\circ\ 20')$

For winter half-year, $\Delta_d = +1'.420 \sin (15\,n + 220^\circ\ 41') + 1'.399 \sin (30\,n + 26^\circ\ 39')$
$+0'.520 \sin (45\,n + 227^\circ\ 26') + 0'.310 \sin (60\,n + 61^\circ\ 17')$

For the whole year,[1] $\Delta_d = +2'.167 \sin (15\,n + 213^\circ\ 55') + 1'.875 \sin (30\,n + 38^\circ\ 52')$
$+0'.759 \sin (45\,n + 244^\circ\ 40') + 0'.198 \sin (60\,n + 83^\circ\ 05')$

---

[1] For the purpose of showing the correspondence when the above equation is deduced *independently*, from the observations at the even and odd hours, I add here the values for the two cases:—

From even hours, $\Delta d = +2'.170 \sin (15\,n + 213^\circ\ 27') + 1'.888 \sin (30\,n + 38^\circ\ 59')$
$+0'.729 \sin (45\,n + 244^\circ\ 57') + 0'.183 \sin (60\,n + 83^\circ\ 26')$

From odd hours, $\Delta d = +2'.159 \sin (15\,n + 215^\circ\ 19') + 1'.835 \sin (30\,n + 38^\circ\ 31')$
$+0'.848 \sin (45\,n + 243^\circ\ 49') + 0'.242 \sin (60\,n + 82^\circ\ 01')$

The relative weights of the results by the even hours and the odd hours are as 3 : 1.

If, for the purpose of comparison with the previous results in Part I of this discussion, and with other similar expressions, we change the angles C_1, C_2, C_3, C_4, by 180°, which is equivalent to an easterly deviation from the mean for positive results and to a westerly deviation for negative results, we find—

For Philadelphia, $\Delta d = +2'.167 \sin (\ \theta + 33^\circ\ 55') + 1'.875 \sin (2\theta + 218^\circ\ 52')$
$+0'.759 \sin (3\theta + 64^\circ\ 40') + 0'.198 \sin (4\theta + 263^\circ\ 05')$

For Dublin, $\Delta d = +3'.519 \sin (\ \theta + 64^\circ\ 18') + 2'.127 \sin (2\theta + 225^\circ\ 22')$
$+0'.688 \sin (3\theta + 70^\circ\ 40') + 0'.322 \sin (4\theta + 242^\circ\ 27')$

This latter expression is copied from the Rev. H. Lloyd's discussion of the Dublin observations in 1840–'43.

For a comparison of the monthly equations, the reader may also consult similar expressions ob-

In determining the least square coefficients in these equations, allowance has been made for the different weights due to the readings at the even and odd hours. θ is reckoned from midnight at the rate of 15° an hour. To compare the numerical quantities of the angles C_1, C_2, C_3, C_4, in the general expression—

$$\Delta_d = B_1 \sin(\theta + C_1) + B_2 \sin(2\theta + C_2) + B_3 \sin(3\theta + C_3) + B_4 \sin(4\theta + C_4),$$

with the same quantities in the formula of the diurnal variation (pp. 8 and 9 of Part I), 180° must first be added or subtracted from each angle given there; since, in the discussion of Part I, *increasing* numbers correspond to a *decrease* of western declination, the scale being thus graduated, whereas, in the *present* case, *increasing* positive numbers correspond to an *increase* of western declination, as stated above.

The following table exhibits the close correspondence of the computed and observed mean annual value of the regular solar-diurnal variation:—

Philadelphia mean time.	Diurnal Variation.		C—O.	Philadelphia mean time.	Diurnal Variation.		C—O.
	Computed.	Observed.			Computed.	Observed.	
0h. 19½m.	—0′.49	—0′.47	—0′.02	Noon 19½m.	+3′.69	+3′.65	+0′.04
1 19½	—0.48	—0.51	+0.03	13h. 19½	+4.28	+4.32	—0.04
2 19½	—0.51	—0.44	—0.07	14 19½	+3.81	+3.77	+0.04
3 19½	—0.67	—0.71	+0.04	15 19½	+2.77	+2.76	+0.01
4 19½	—1.09	—1.19	+0.10	16 19½	+1.71	+1.80	—0.09
5 19½	—1.82	—1.64	—0.18	17 19½	+0.88	+0.78	+0.10
6 19½	—2.77	—2.72	—0.05	18 19½	+0.33	+0.25	+0.08
7 19½	—3.49	—3.47	—0.02	19 19½	—0.07	—0.07	0.00
8 19½	—3.44	—3.50	+0.06	20 19½	—0.38	—0.33	—0.05
9 19½	—2.29	—2.43	+0.14	21 19½	—0.57	—0.60	+0.03
10 19½	—0.24	—0.19	—0.05	22 19½	—0.62	—0.64	+0.02
11 19½	+2.03	+2.17	—0.14	23 19½	—0.57	—0.71	+0.14

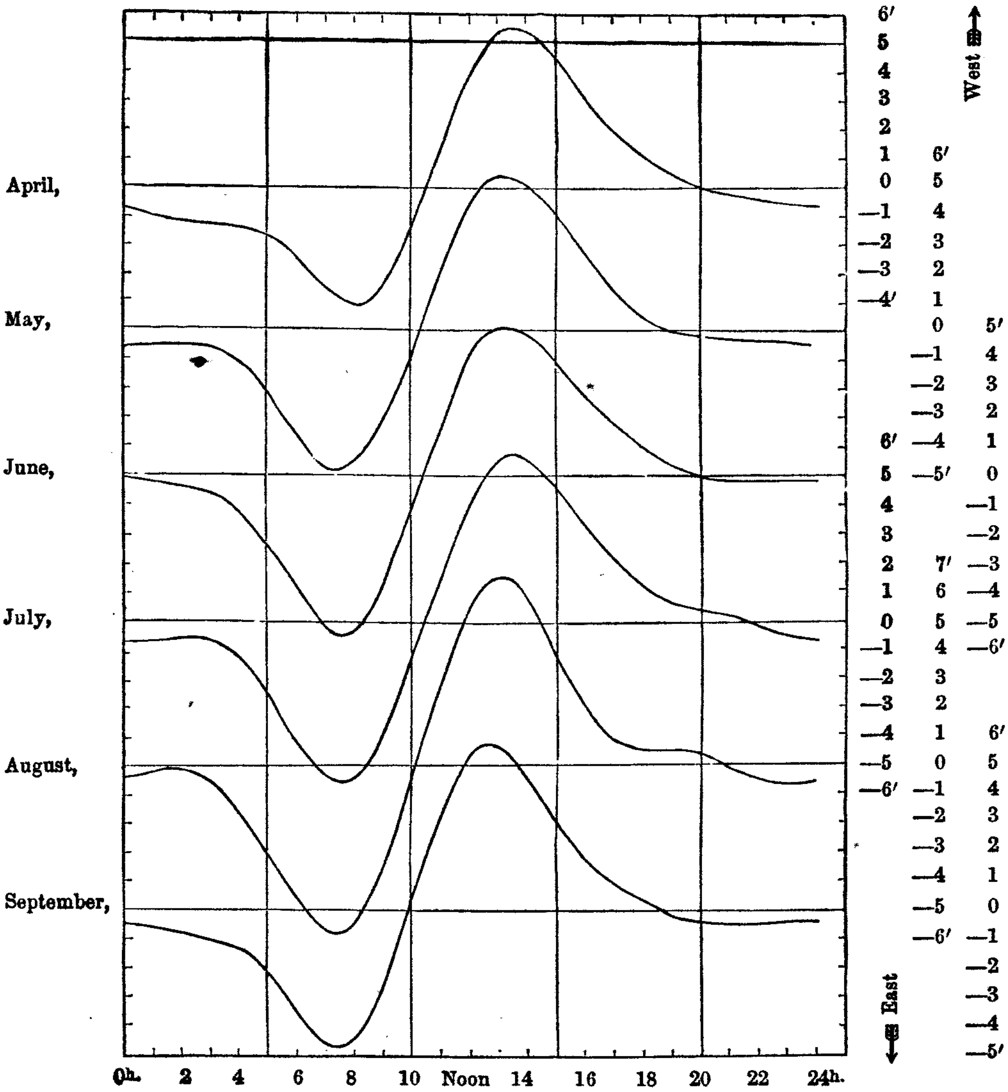
The maximum difference at any one hour is less than 11″, and the probable error of any single hourly result is $\pm 0'.05$. The probable error of any single computed value from a monthly expression is $\pm 0'.19$.

By means of the preceding equations, the hourly values of the diurnal variation for each month of the year have been computed; and the results, projected in curves, are given in Diagrams D and E. The first contains the curves for the six months of the summer half-year, and the second those of the winter half-year. Positive ordinates correspond to a westerly movement, and negative ordinates to an easterly movement, of the north end of the magnet. The diagram following (F) contains the type curves for summer, winter, and the whole year, all being upon the same scale.

---

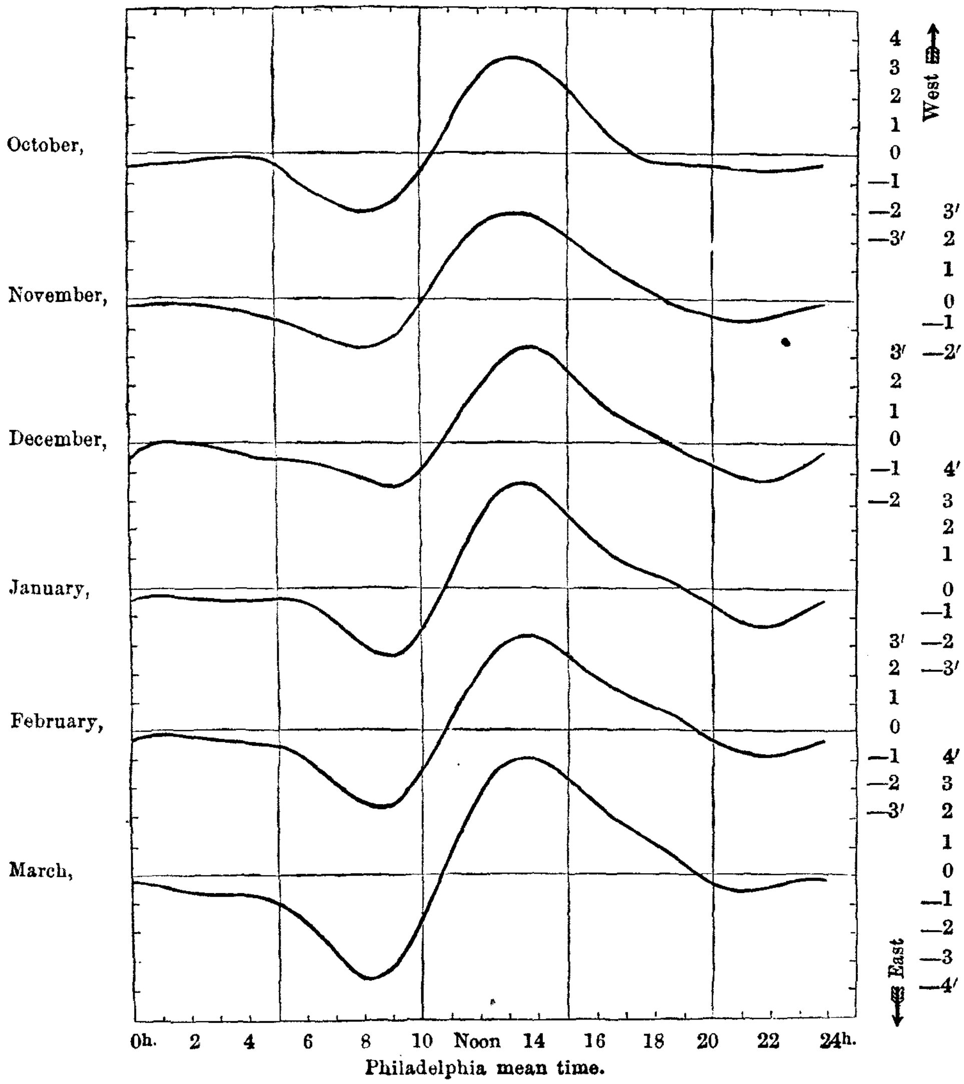
tained by Mr. Karl Kreil from his discussion of declinometer observations at Prague, extending over ten consecutive years (1840–'49), and selected from a thirteen years' series, in order to obtain mean results *unaffected* by the smaller inequality of the ten or eleven year period with which our results are still affected. Part I of the present discussion, however, affords ready means of changing slightly the numerical values of the coefficients B_1, B_2, B_3, B_4, in our equations, in order to obtain the values we would have obtained, had we discussed a consecutive eleven year series of observations or one extending over a series of years corresponding to the actual length of the solar period then observed. Mr. Kreil's discussion will be found in Vol. VIII of the proceedings of the mathematical and physical section of the Imperial Academy of Sciences at Vienna (1854.)

REGULAR SOLAR-DIURNAL VARIATION OF THE MAGNETIC DECLINATION, SUMMER HALF YEAR.

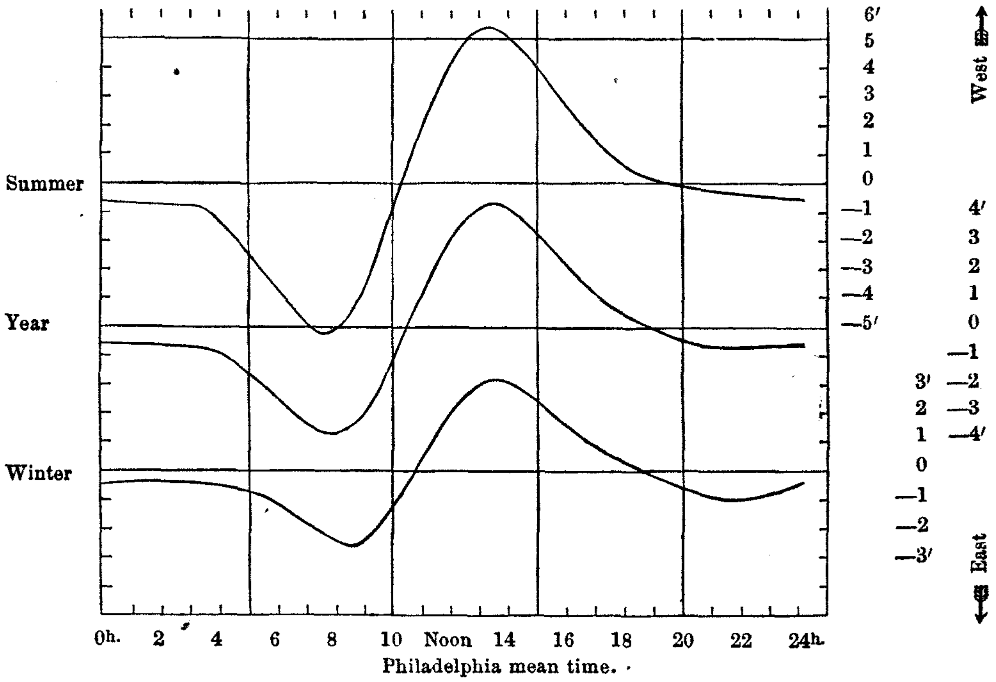


Philadelphia mean time.

REGULAR SOLAR-DIURNAL VARIATION OF THE MAGNETIC DECLINATION, WINTER HALF YEAR.



TYPE-CURVES OF THE REGULAR SOLAR-DIURNAL VARIATION OF THE DECLINATION.



REGULAR SOLAR DIURNAL VARIATION OF THE MAGNETIC DECLINATION COMPUTED FOR EVERY MONTH OF THE YEAR, AND FOR THE PRINCIPAL SEASONS.

MEAN EPOCH 1842-43.	0h.	1h.	2h.	3h.	4h.	5h.	6h.	7h.	8h.	9h.	10h.	11h.
January	—0′.52	—0′.24	—0′.30	—0′.48	—0′.48	—0′.38	—0′.55	—1′.24	—2′.12	—2′.45	—1′.59	+0′.26
February	—0.30	—0.09	—0 14	—0.32	—0.49	—0.71	—1.18	—1.90	—2.52	—2.49	—1.49	+0.24
March	—0.25	—0.32	—0.55	—0.71	—0.81	—1.09	—1.84	—2.93	—3.67	—3.40	—1.81	+0.55
April	—0.88	—1.14	—1.34	—1.44	—1.54	—1.88	—2.64	—3.55	—4.02	—3.42	—1.54	+1.11
May	—0.58	—0.59	—0.50	—0.59	—1.18	—2.32	—3.75	—4.74	—4.66	—3.24	—0.81	+1.93
June	—0.19	—0.28	—0.41	—0.69	—1.32	—2.45	—3.87	—5.02	—5.13	—3 80	—1.23	+1.69
July	—0.80	—0.82	—0.67	—0.72	—1.34	—2.62	—4.22	—5.42	—5.45	—4 04	—1.47	+1.47
August	—0.62	—0.33	—0.21	—0.59	—1.60	—3.09	—4.68	—5.71	—5.48	—3.67	—0.58	+2.87
September	—0.52	—0.72	—0.90	—1.06	—1.42	—2.23	—3.51	—4.55	—4.50	—2.84	+0.11	+3.18
October	—0.44	—0.52	—0.30	—0.20	—0.16	—0.45	—1.33	—1.72	—2.19	—1.92	—0.79	+0.77
November	—0.23	—0.21	—0.32	—0.44	—0.56	—0.75	—1.18	—1.68	—1.91	—1.50	—0.37	+1.11
December	—0.36	+0.01	—0.02	—0.34	—0.60	—0.67	—0.73	—1.00	—1.44	—1.64	—1.09	+0.26
Summer	—0.63	—0.71	—0.71	—0.81	—1.33	—2.43	—3.84	—4.92	—4.91	—3.43	—0.83	+2.12
Winter	—0.41	—0.26	—0.29	—0.42	—0.50	—0.68	—1.09	—1.73	—2.25	—2.15	—1.12	+0.56
Year	—0.49	—0.48	—0.55	—0.62	—0.94	—1.54	—2.41	—3.31	—3.54	—2.80	—1.02	+1.32

MEAN EPOCH 1842-43.	Noon.	13h.	14h.	15h.	16h.	17h.	18h.	19h.	20h.	21h.	22h.	23h.
January	+2′.26	+3′.40	+3′.34	+2′.46	+1′.52	+0′.92	+0′.57	+0′.08	—0′.64	—1′.29	—1′.45	—1′.08
February	+1.96	+2.97	+3.02	+2.42	+1.71	+1.17	+0.76	+0.26	—0.36	—0.85	—0.97	—0.70
March	+2.71	+3.86	+3.85	+3.17	+2.33	+1.65	+1.02	+0.35	—0.31	—0.70	—0.67	—0.43
April	+3.60	+5.06	+5.18	+4.28	+2.98	+1.76	+0.88	+0.27	—0.14	—0.38	—0.54	—0.67
May	+4.06	+5.07	+4.88	+3.85	+2.48	+1.22	+0.39	—0.02	—0.12	—0.14	—0.21	—0.43
June	+3.99	+5.00	+4.79	+3.79	+2.60	+1.59	+0.87	+0.38	+0.07	—0.10	—0.13	—0.15
July	+3.90	+5.26	+5.37	+4.54	+3.28	+2.04	+1.16	+0.66	+0.39	+0.18	—0.15	—0.53
August	+5.44	+6.35	+5.55	+3.75	+1.98	+0.87	+0.50	+0.45	+0.26	—0.13	—0.56	—0.77
September	+5.18	+5.54	+4.48	+2.99	+1.68	+0.85	+0.33	—0.11	—0.44	—0.56	—0.55	—0.44
October	+2.60	+3.17	+3.00	+2.20	+1.08	+0.25	—0.39	—0.36	—0.39	—0.52	—0.69	—0.69
November	+2.31	+2.81	+2.58	+1.90	+1.18	+0.57	+0.06	—0.40	—0.59	—0.92	—0.77	—0.49
December	+1.86	+2.95	+3.04	+2.32	+1.34	+0.57	+0.11	—0.28	—0.78	—1.22	—1.33	—0.96
Summer	+4.35	+5.29	+4.99	+3.89	+2.57	+1.43	+0.64	+0.18	—0.05	—0.17	—0.33	—0.44
Winter	+2.21	+3.12	+3.09	+2.38	+1.52	+0.84	+0.37	—0.09	—0 55	—0.89	—0.94	—0.72
Year	+3.25	+4.22	+4.09	+3.14	+2.06	+1.12	+0.45	+0.05	—0.32	—0.52	—0.58	—0.58

In the above table + signifies westerly and — easterly deflection; it may be compared with similar tables constructed for Toronto,[1] Dublin,[2] and Prague.[3] It will be observed that the preceding table, which gives the observed variation, refers to an epoch $19\frac{1}{2}$ minutes later than the exact local hour (that is, to an exact Göttingen hour), whereas the computed table refers to the exact Philadelphia hours.

From the computed tabular values, aided by the diagrams, we can now deduce some of the general features of the diurnal variation and its annual inequality.

The general character of the diurnal motion (see type-curves) is nearly the same throughout the year; the most eastern deflection is reached a quarter before 8 o'clock in the morning (about a quarter of an hour earlier in summer, and half an hour later in winter); near this hour the declination is a minimum; the north end of the magnet then begins to move westward, and reaches its western elongation about a quarter after one o'clock in the afternoon (a few minutes earlier in summer). At this time the declination attains its maximum value. The diurnal curve presents but a single wave, slightly interrupted by a deviation occurring during the hours near midnight (from about 10 P. M. to 1 A. M.), when the magnet has a direct or westerly motion; shortly after 1 A. M. the magnet again assumes a retrograde motion, and completes the cycle by arriving at its eastern elongation shortly before 8 o'clock in the morning. This nocturnal deflection is well-marked in winter, vanishes in the summer months, and is hardly perceptible in the annual curve. According to the investigations of General Sabine, it is probable that, if we had the means of entirely obliterating the effect of disturbances, this small oscillation would almost disappear. In summer, when it has no existence, the magnet remains nearly stationary between the hours of 8 P. M. and 3 A. M., a feature which is also shown by the annual type-curve.

The two preceding plates show a close general resemblance in the diurnal curves for the six months when the sun has north declination, and a similar resemblance in the other six months when it has south declination.

The analytical expressions give the epoch and amount of variation with greater precision. The hours of minimum and maximum deflection are obtained from the equation $\frac{d\,\Delta_d}{d\,n} = o$; and the hours of the mean declination, when the curves cross the axis of abscissæ, from the condition $\Delta_d = o$. The following table contains these results for each month and the two principal seasons of the year, also the critical interval between the two adjacent hours of the mean position.

---

[1] Vol. III., Table LXVI.; compare also with Table VII. of Vol. II.
[2] Trans. Royal Irish Academy, Vol. XXII., Part I., Table III.
[3] Academy of Sciences at Vienna, Vol. VIII. of Math. Section, Table II.

Month.	Eastern elongation A. M.	Western elongation P. M.	Critical interval from minimum to maximum.	Epoch of Mean Declination.		Critical interval.
				A. M.	P. M.	
January	8h. 58m.	1h. 27m.	4h. 29m.	10h. 52m.	7h. 08m.	8h. 16m.
February	8 34	1 32	4 58	10 52	7 26	8 34
March	8 07	1 34	5 27	10 46	7 32	8 46
April	8 12	1 27	5 15	10 34	7 40	8 56
May	7 29	1 21	5 52	10 19	6 57	8 38
June	7 33	1 20	5 47	10 25	8 26	10 01
July	7 36	1 28	5 52	10 30	9 32	11 02
August	7 18	1 05	5 47	10 10	8 40	10 30
September	7 30	0 45	5 15	9 58	6 45	8 47
October	8 00	1 17	5 17	10 30	5 23	6 53
November	7 54	1 08	5 14	10 16	6 08	7 52
December	8 54	1 40	4 46	10 50	6 17	7 27
Summer	7h. 33m.	1h. 8m.	5h. 35m.	10h. 17m.	7h. 43m.	9h. 26m.
Winter	8 24	1 25	5 01	10 40	6 49	8 09
Year	7 48	1 16	5 28	10 26	7 08	8 42

We likewise obtain:

Secondary minimum of eastern deflection in winter	9h. 42m. P. M.	Amount —0′.97
" maximum of western " " "	1 15 A. M.	" —0 .26
Differences:	3h. 33m.	0′.71

and

Secondary minimum of eastern deflection for the year	10h. 11m. P. M.	Amount —0′.62
" maximum of western " " "	1 13 A. M.	" —0 .47
Differences:	3h. 02m.	0′.15

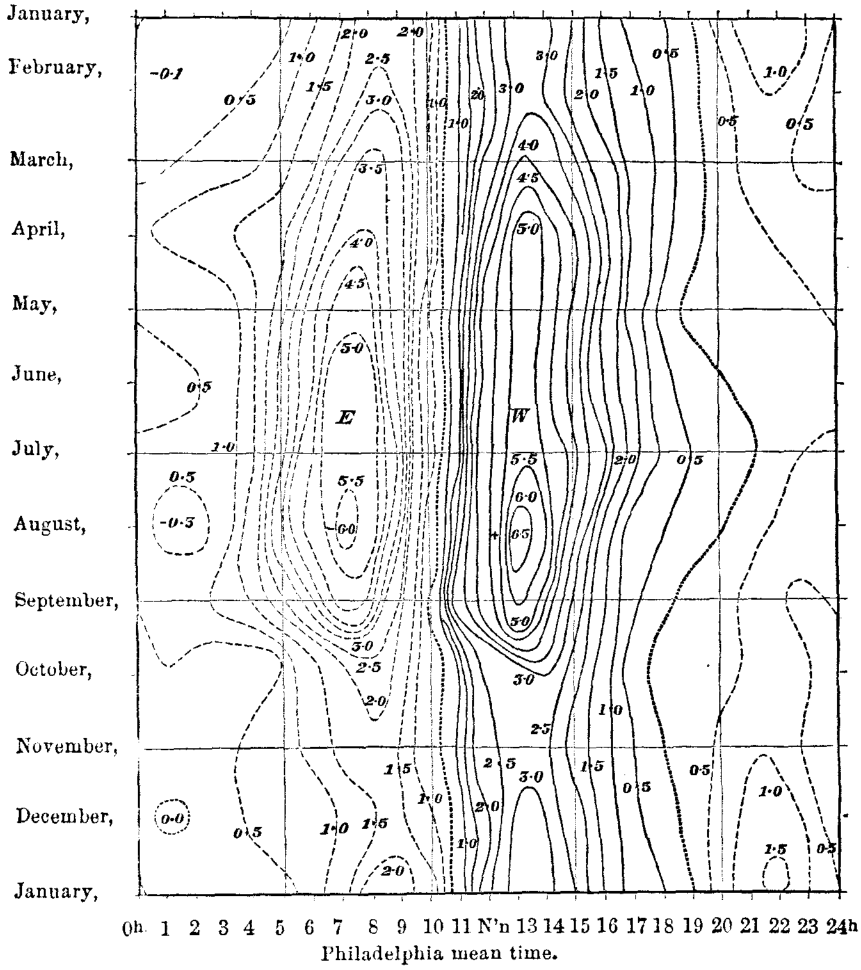
The effect of the seasons on the critical hours is well marked in the above table. The eastern elongation occurs earliest between the summer solstice and the autumnal equinox, and latest about the winter solstice. The western elongation occurs earliest about the autumnal equinox, and latest about the winter solstice; and the same holds good for the morning epoch of the mean declination. The afternoon epoch, however, occurs earliest, shortly after the autumnal equinox, and latest, shortly after the summer solstice.

The critical hours which vary least during the year are those of the western elongation and those of the morning mean declination. The extreme difference between the value for any month and the mean annual value is 31 minutes in the former and 28 minutes in the latter.

The following graphical representation of three variables (Diagram G) will serve to show at a glance the various features of the diurnal variation and its annual inequality: The magnetic surface is formed by contour lines, 0′.5 apart; the dotted curves are lines of mean position, the curves represented by dashes correspond to eastern, and the full curves to western deflection from the normal position. This diagram, as well as the computed tabular values from which it has been constructed, serve equally to furnish the correction necessary to reduce any single observation taken at any hour of the day and month to its mean value. It also enables us in a measure to dispense with developing the annual variability of the coefficients B_1 B_2 B_3 . . . and C_1 C_2 C_3 . . . . (or rather the equivalents a_1 b_1 a_2 b_2 a_3 b_3 . . . . from which they are derived) in the general expression $A + B_1 \sin(\theta + C_1) +$ etc. In most cases either a tabular or graphical interpolation between the two adjacent monthly values will fully answer the purpose. The diagram also distinctly exhibits the diurnal minima and maxima, the former represented by a valley, the latter by a ridge in the magnetic surface.

The magnitude of the diurnal range is next to be considered.

DIAGRAM SHOWING THE DEFLECTION (IN MINUTES OF ARC) OF THE NORTH END OF THE MAGNET FROM ITS MONTHLY NORMAL POSITION FOR EVERY HOUR OF THE DAY AND MONTH OF THE YEAR, DERIVED FROM THE DECLINOMETER OBSERVATIONS AT PHILADELPHIA BETWEEN 1840 AND 1845.



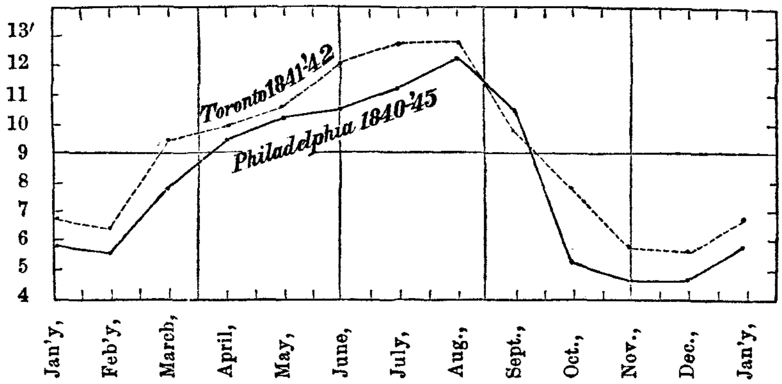
The following table contains the amount of the deflection at the eastern and western elongations and the diurnal amplitude of the declination for each month of the year, derived from the preceding equations:—

	Deflection at		Diurnal range.		Deflection at		Diurnal range.
	E. Elong.	W. Elong.			E. Elong.	W. Elong.	
January	—2′.46	+3′.52	5′.98	July	—5′.58	+5′.46	11′.04
February	—2.64	+3.11	5.75	August	—5.79	+6.36	12.15
March	—3.73	+4.03	7.76	September . . .	—4.71	+5.60	10.31
April	—4.02	+5.28	9.30	October	—2.18	+3.23	5.41
May	—4.89	+5.16	10.05	November . . .	—1.92	+2.85	4.77
June	—5.26	+5.06	10.32	December . . .	—1.65	+3.14	4.79

The diurnal range for the summer months is 10′.45, for the winter months 5′.56, and for the whole year 7'.89; all corresponding to an epoch removed about one year and a half from the epoch of a minimum of the solar period.

The numbers expressing the diurnal range exhibit three remarkable features, viz., the maximum value in the month of August, the sudden falling off in the months of September and October (see the graphical representation), and the

DIURNAL RANGE OF THE DECLINATION.



minimum value in November or December. Otherwise the progression is regular; the curve is *single-crested*, a feature equally true for the *eastern* as well as for the *western* deflection when viewed separately. This latter circumstance is of special importance, since it is probable that it is mostly by the interference of these two separate curves that we observe at other stations the curve of the diurnal range at some stations apparently to be a double-crested one. The curves for Milan, Munich, Göttingen, Brussels, Greenwich, Dublin, etc., for instance, exhibit two maxima, one after the vernal equinox, and a second, generally the smaller one, about the summer solstice, with more or less regularity. The system to which Philadelphia belongs is exemplified by the annual curve of the diurnal range at Prague and at some Russian stations, especially at Nertschinsk, but principally at Toronto, for which last station the curve is shown in the diagram. Neither station appears to have a tendency to a secondary maximum about the month of April, leaving the maximum about a month and a half after the summer solstice, a well-marked North American feature.

Annual Variation of the Declination.—In connection with the preceding discussion the annual inequality in the magnetic declination next claims attention.

This subject presents greater difficulty, inherent in the observations, than the diurnal inequality; not so much on account of the length of the period as on account of the difficulty of keeping the instrument in precisely the same condition of adjustment throughout the year. In the first part of this discussion I have already had occasion to refer to this circumstance while investigating the annual effect of the secular change, and it was there shown that the Philadelphia observations share in this respect the difficulties of those of other stations,[1] in consequence of which the results must be received with caution.

---

[1] It may be proper to give here, in full, Dr. Lloyd's instructive note on this subject, in his discussion of the Dublin observations: "The determination of the annual variation is much more difficult

Returning to the last vertical column in the table, headed "mean," we have there the monthly values of the declinometer readings (in scale divisions), and in their differences when compared month for month, the joint effect of the secular change, and of the annual inequality. To eliminate the effect of the secular change, we determine its annual amount as follows: Subtracting the mean annual reading 559.64, corresponding to July 1, from each monthly mean, and putting x = monthly effect of the secular change (considered as uniform), each monthly mean reading furnishes an equation for the determination of x, thus: for

January	$4.56 = 5.5\,x$
February	$3.34 = 4.5\,x$
March	$5.22 = 3.5\,x$
April.	$4.39 = 2.5\,x$, etc.,

which, when combined by least squares, give $x = 1^d.227$, hence the annual change $14^d.7$ or $6'.7$ of *increasing westerly* declination.[1]

Deducting the effect of the secular change, and comparing the monthly remainders with their mean values, we obtain the annual inequality of the declination as follows:—

MONTH.	Mean reading.	Reduct'n for sec. change.	Reduced reading.	Annual in-equality.	MONTH.	Mean reading.	Reduct'n for sec. change.	Reduced reading.	Annual in-equality.
	d.	d.	d.	d.		d.	d.	d.	d.
January . . .	564.20	—6.75	557.45	+2.2	July	560.24	+0.61	560.85	—1.2
February . . .	562.98	—5.52	557.46	+2.2	August . . .	559.07	+1.84	560.91	—1.3
March	564.86	—4.29	560.57	—0.9	September . .	556.07	+3.07	559.14	+0.5
April	564.03	—3.07	560.96	—1.3	October . . .	556.43	+4.29	560.72	—1.1
May	563.18	—1.84	561.34	—1.7	November . .	553.97	+5.52	559.49	+0.2
June	560.84	—0.61	560.23	—0.6	December . .	549.82	+6.75	556.57	+3.1

The sign $\begin{cases} - \\ + \end{cases}$ in the annual inequality indicates an $\begin{cases} \textit{easterly} \\ \textit{westerly} \end{cases}$ deflection.

According to these results the magnet (north end) is deflected to the east of its mean annual position in summer, and to the west in winter. It is, however, desirable to test the result by submitting the first and the second $2\frac{1}{2}$ years of observations separately to the same process of investigation. The first 31 months in the years 1840, '41, and '42, give a result almost identical with that just deduced;

---

than that of the diurnal, both on account of the much smaller frequency of the period, and the difficulty of preserving the instrument in the same unchanged condition during the much longer time, or of determining and allowing for its changes when they do occur. Accordingly, although the annual period may be traced in the observations of Gilpin and is decidedly displayed in those of Bowditch, it has evaded the researches of recent observers. There is but a faint indication of its existence in the Göttingen observations, which were made at the hours of 8 A. M. and 1 P. M., and Professor Gauss and Dr. Goldschmidt find, in their analysis of these observations no important fluctuation dependent on season. A similar negative result is deduced by Dr. Lamont from the Munich observations, which were made twelve times in the day."

[1] This value ($+6'.7$), as resulting from a different combination of observed and partly interpolated values, may not be preferable to that ($+4'.5$) deduced in Part I. of this discussion, but must necessarily be employed in the present investigation. The most reliable value, $+5'.0$, was deduced from independent observations, as already remarked, and lies between the two.

the remaining 27 months in the years 1843, '44, and '45, when discussed in the same manner, give a rather different result.

Some improvements, however, can be made in the preceding investigation by omitting the December mean of 1844, which is obviously about 12 scale divisions too small; the observed value is $535^{d.}.2$, and the interpolated value $547^{d.}.0$. An examination of the first series shows a defect in the monthly means of 1841, between May and June, requiring a constant correction of + 8.0 scale divisions for the remaining months after May, as may be seen by the following table:—

Year.	May.	June.	Diff. May-June.	
1841	578.5	571.2?	. . . .	Computed value for June, 579.2
1842	561.8	563.6	—1.8	
1843	566.2	565.0	+1.2	
1844	546.5	547.5	—1.0	
1845	529.7	531.0	—1.3	
Mean			—0.7	

The following values then remain for the discussion, and they should be considered as forming the basis from which the legitimate results are to be deduced. The numbers marked with an asterisk have been increased by $8^{d.}.0$. Interpolated values are between brackets, and were obtained by comparing the means of the remaining months of the year with the corresponding means of every other year; by this process several values are obtained for each interpolated number; the resulting mean is given in the table. The high value of 1841, and the low value of 1844, for the month of May, in some measure compensate.

YEAR.	Jan.	Feb.	March.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.
1840	(586.9)	(585.9)	(586.3)	(586.4)	(584.1)	586.9	588.4	587.4	585.2	579.9	573.9	570.5
1841	576.3	574.3	577.0	578.1	578.5	*579.2	*576.7	*576.9	*571.1	*575.2	*564.4	*567.4
1842	564.1	563.3	562.0	561.9	561.8	563.6	565.2	563.8	566.7	562.7	563.5	561.6
1843	(566.5)	(565.6)	(565.9)	567.2	566.2	565.0	564.7	563.6	558.3	559.0	556.1	557.6
1844	558.0	558.0	557.6	555.2	546.5	547.5	547.5	546.2	542.2	545.3	547.2	(547.0)
1845	531.0	531.3	532.0	527.1	529.7	531.0	(529.9)	(529.0)	(527.0)	(526.2)	(522.7)	(522.1)
Means	563.8	563.1	563.5	562.6	561.1	562.2	562.1	561.2	558.4	558.1	554.6	554.4
	560.4											
Correction for sec. changes	—4.4	—3.6	—2.8	—2.0	—1.2	—0.4	+0.4	+1.2	+2.0	+2.8	+3.6	+4.4
Corrected means.	559.4	559.5	560.7	560.6	559.9	561.8	562.5	562.4	560.4	560.9	558.2	558.8
Annual variation	+1.0	+0.9	—0.3	—0.2	+0.5	—1.4	—2.1	—2.0	—0.0	—0.5	+2.2	+1.6
(in arc)	+0'.5	+0'.4	—0'.1	—0'.1	+0'.2	—0'.6	—1'.0	—0'.9	—0'.0	—0'.2	+0'.9	+0'.7

This last result accords in general with that before deduced, but is much to be preferred.

From June to October the north end of the magnet is accordingly to the eastward of the mean annual position (after the elimination of the secular change), and in the remaining months of the year it is to the westward of this position. From the vernal equinox till after the summer solstice the motion is to the eastward or retrograde in regard to the advance of the secular change (to the westward); this is in conformity with the law as given by Dr. Lloyd in the Dublin discussion, where the motion of the magnet is to the westward at this period of the year, or the reverse of the Philadelphia deflection, but the secular change is likewise reversed, the west declination diminishing at Dublin (at the same time or more accurately between 1840 and '43).

For further comparison I give here the results deduced from seven years' observation at Toronto between the years 1845 and '51, a previous working up of a three years' series (middle year 1846) not being deemed sufficiently distinctive in its results. The secular change is here 2'.0 per annum, increasing westerly declination, whereas it was 4'.4 per annum at Philadelphia in 1843; as in the above result + indicates west, — east deflection.

ANNUAL VARIATION AT TORONTO BETWEEN 1845 AND 1851.

Jan.	Feb.	March.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
+0'.1	—0'.5	—0'.2	0'.0	—0'.1	—0'.5	—0'.8	—0'.2	+0'.7	+1'.0	+0'.3	+0'.3

In regard to the amount of the inequality, the two stations agree remarkably well, the range remaining slightly below 2' of arc. It has been supposed that this range at the same station is increasing or diminishing as the secular change increases or diminishes.

It may further be remarked that the general mean resulting from the above discussion at Philadelphia, viz., 560.4, is identical with the value given in Part I. of the discussion, there deduced by an entirely different combination. The annual effect of the secular change, + 4'.4, is likewise in very close conformity with the value given in Part I., as found by a very different process.

The monthly values of the annual variation may serve to give the corrections to observed declinations in any month of the year needed to refer the same to the mean declination of the year, and may also be used in the more refined discussion of the secular change, in both cases, only, when the greatest accuracy is required.

SMITHSONIAN CONTRIBUTIONS TO KNOWLEDGE.

132

DISCUSSION

OF THE

MAGNETIC AND METEOROLOGICAL OBSERVATIONS

MADE AT THE GIRARD COLLEGE OBSERVATORY, PHILADELPHIA,
IN 1840, 1841, 1842, 1843, 1844, AND 1845.

PART III.

INVESTIGATION OF THE INFLUENCE OF THE MOON ON THE MAGNETIC DECLINATION.

BY

A. D. BACHE, LL. D.

[ACCEPTED FOR PUBLICATION, SEPTEMBER, 1860.]

INVESTIGATION

OF THE

INFLUENCE OF THE MOON ON THE MAGNETIC DECLINATION.

THE existence of a sensible lunar effect on the magnetic declination has already been established by the labors of Broun, Kreil, Sabine, and others. It is nevertheless important to add the weight of new numerical results to those already obtained.

In the discussions of the Philadelphia observations of magnetic declination, already presented to the Association, I have shown how the influence of magnetic disturbances, of the eleven year period of the solar diurnal variation and its annual inequality, of the secular change, and of the annual variation may be severally eliminated, leaving residuals from which the lunar influence is to be studied. Each observation was marked with its corresponding lunar hour and the hourly normals used for comparison.

This method of treatment of the subject is that followed by General Sabine in his discussion of the results of the British observations.[1]

The details of the method will be better understood by an example.

The time of the moon's passage over the meridian of Philadelphia (upper transit) was obtained from the American Almanac, the small correction for the difference

---

[1] In reference to methods and results, in general, on this subject, the following papers may be consulted: Observations in Magnetism and Meteorology made at Makerstown, in Scotland, in the observatory of General Sir Thomas M. Brisbane, Bart., in 1845 and 1846, forming vol. xix., part i. of the Trans. Royal Society of Edinburgh. By John Allan Broun. Edinburgh, 1849; also vol. xix. part ii., containing the general results (1850).

Einfluss des Mondes auf die magnetische Declination by Carl Kreil. Vol. iii. of the Proceedings of the Mathematical and Physical Section of the Imperial Academy of Sciences of Vienna, 1852; also, vol. v., ibid., 1853.

Philosophical Trans. Royal Society, art. xix., 1853: On the Influence of the Moon on the Magnetic Declination at Toronto, St. Helena, and Hobarton. By Col. E. Sabine.

Phil. Trans. Royal Society, art. xxii., 1856: On the Lunar-diurnal Magnetic Variation at Toronto. By Major General E. Sabine. And—

Phil. Trans. Royal Society, art. i., 1857: On the evidence of the Existence of the Decennial Inequality in the Solar-diurnal Magnetic Variations and its Non-existence in the Lunar-diurnal Variation, of the Declination at Hobarton. By Major General E. Sabine.

of longitude being neglected. The observation nearest to the local mean solar time of the moon's transit was marked with a zero, signifying $0^{h.}$ of lunar time. The time of the inferior transit was next obtained; and the observation nearest to it in time was marked 12^{h}. The greatest difference in interval between the moon's transit and the time of observation could in no instance exceed half an hour. In the bi-hourly series, the observations nearest the moon's transit, or to either hour angle, one hour before or one hour after the transit was marked. The mean of a number of differences for the same hours thus gave a result corresponding nearly enough with the hour. The number of observations intermediate between those marked $0^{h.}$ and $12^{h.}$ were marked with the corresponding hour angle by interpolation, care being taken to note the nearest full hour against each observation in the bi-hourly series. The hourly series begins with October, 1843. In the case of thirteen observations within twelve lunar hours, the one nearest midway between the two consecutive lunar hours was omitted.

In the month of March, 1842, which is selected as an example of the details of working the bi-hourly series, the number of observations available is 298, of which 148 correspond to western and 150 to eastern hour angles. In the abstract which follows + indicates a deviation of the north end of the magnet to the west, and — a deviation to the east of the respective normal position for the hour. The hourly normals are given in the first part of the discussion. No difference exceeds eight divisions, this being the limit in number indicated by the criterion.

LUNAR-DIURNAL VARIATION FROM OBSERVATIONS AT PHILADELPHIA IN MARCH, 1842.
Differences from the hourly normals.

☽'s Upper transit. Western hour-angles.

	0h.	1h.	2h.	3h.	4h.	5h.	6h.	7h.	8h.	9h.	10h.	11h.
	+4.1	+6.1	−0.1	−5.0	+1.3	−0.9	−0.1	+1.0	+2.5	+3.1	+0.9	−4.1
	−2.5	−4.2	+2.0	+1.0	+7.2	+0.2	+7.8	0.0	+1.2	−2.0	+2.1	−1.8
	+3.1	−1.4	+4.6	+6.1	+1.4	+0.9	−0.6	−0.5	+4.2	−2.7	+1.6	−6.8
	−4.3	+4.3	−0.7	+1.8	+0.3	+1.2	+2.3	−3.0	+1.0	+5.2	−0.4	−2.0
	−1.9	−0.5	+3.9	−1.3	−0.6	−2.9	−0.5	+1.4	+0.7	−1.8	−1.1	−5.2
	+6.4	+1.0	+3.6	+1.9	−4.8	+1.3	−0.1	+2.1	−3.3	+7.0	+1.7	+5.6
	+1.7	+3.3	+1.6	−0.9	+3.1	−2.3	−1.6	−6.2	+2.3	−1.1	+6.5	+6.6
	−0.7	+0.2	−3.2	−0.9	+2.9	−3.8	−3.8	−3.1	−4.5	−5.5	−1.9	−1.8
	+0.8	−5.1	+3.0	0.0		−7.2	−6.3	−5.9	−2.9	−3.0	+2.6	+1.1
	+2.9	−2.7	+3.1	+1.8		−3.7	−2.4	−4.6	−0.8	+4.4	−1.7	+0.7
	−1.7	+2.8		−3.0		+0.5	+0.5	−5.1	−0.9	−2.2	−0.9	−1.6
		−1.3		−1.9		+2.9	−0.2	−4.6		−1.6	−0.5	
		+2.2		−1.4			−5.9	−4.2		−1.9	−2.9	
				+2.9			+3.5	−1.6		−1.9		
				+2.1				+2.0				
				+1.6								
Means	+0d..72	+0.38	+1.78	+0.30	+1.35	−1.15	−0.53	−2.15	−0.05	−0.29	+0.46	−0d..84

☽'s Lower transit. Eastern hour-angles.

	12h.	13h.	14h.	15h.	16h.	17h.	18h.	19h.	20h.	21h.	22h.	23h.
	−2.7	−0.5	−2.9	+0.5	+5.6	+0.5	+0.6	−4.1	+1.6	−4.9	+4.4	−4.2
	−0.2	−2.4	−1.9	−0.4	−3.4	−1.7	−2.8	+1.3	+6.5	−5.4	−0.3	+0.6
	−1.7	+3.0	+2.9	−0.6	−2.0	−0.6	−1.6	+2.5	−5.1	−0.4	+6.9	−3.1
	−1.0	−0.6	+0.5	−7.2	−1.4	−3.2	−1.0	−0.6	−0.6	+0.4	−0.9	−0.1
	+3.2	−0.2	−5.2	−0.5	−2.3	−0.4	−6.0	+0.3	+0.6	+0.7	+1.6	+1.2
	+0.9	+2.7	+3.4	−1.1	+1.4	+2.2	−3.3	−4.2	+0.2	+0.1	−1.1	+1.7
	+2.4	+2.6	+4.5	+3.4	+1.0	+1.4	−3.8	−3.3	−0.7	−0.4	−2.8	+1.6
	+1.9	+4.9	+5.4	+4.7	−1.0	+2.3	+2.9	−0.3	−3.6	−2.2	−1.5	−3.5
	+7.4	−0.7	−1.1	+7.6	+5.8	−0.1	−0.7	−2.0	+5.9	+1.5	+1.4	−4.0
	+0.3	−4.6	+6.4	+3.4	+3.0	−0.5	−3.0	+3.6	−1.0	+1.2	+3.0	−3.3
	+5.3	−3.5	+1.6	−0.1	−0.3	−3.0	+2.5	−6.6	−1.6	−0.8	−0.4	+0.6
	−1.2	−0.3	+1.5		−1.1	−1.0	−0.7	−3.7	+3.4	+5.6	−3.4	+5.1
					+3.7		+4.9				−6.5	+2.8
					−1.5							
					+0.6							
					−1.1							
Means	+1d..22	+0.20	+1.25	+0.88	+0.44	−0.34	−0.92	−1.43	+0.47	−0.39	+0.03	−0d..35

Number of observations or differences at western hour-angles . . 148
" " " " eastern " . . 150
Total 298

The following table contains the number of observations used in the discussion of the lunar-diurnal variation:—

	1840.	1841.	1842.	1843.	1844.	1845.
January	...	168	265	...	577	591
February	...	263	257	...	571	535
March	...	293	298	...	551	575
April	...	283	278	276	522	561
May	...	276	285	309	596	603
June	300	276	280	300	566	542
July	272	292	267	290	593	...
August	269	262	254	244	541	...
September	253	250	247	283	522	...
October	223	214	221	571	517	...
November	271	230	289	590	517	...
December	237	268	316	595	549	...
Sum	1825	3075	3257	3458	6622	3407
Total sum 21644						

If divided into western and eastern hour-angles, the annual numbers stand as follows:—

	Western hour-angles.	Eastern hour-angles.
1840	916	909
1841	1523	1552
1842	1618	1639
1843	1724	1734
1844	3288	3334
1845	1700	1707
Sum	10769	10875

The preceding mean results will be found inserted in their proper place in the following abstract of the mean monthly values for each observing month between 1840 and 1845.

Proceeding in this way the following results are obtained for the different months discussed.

☽'s Upper transit. Moon's hour-angle.

1846	0h.	1h.	2h.	3h.	4h.	5h.	6h.	7h.	8h.	9h.	10h.	11h.
	d.	d.	d.	d.	d.	d	d.	d.	d.	d.	d.	d.
June[1]	—0.23	—0.25	—1.28	+0.95	—1.09	+0.11	—0.21	+0.30	—1.12	+1.60	—0.02	+0.55
July[2]	+0.52	+1.87	—0.56	+2.04	—1.98	+1.60	—1.34	+0.40	—0.21	+0.47	+0.11	+0.75
August	—0.71	—0.10	+1.41	+0.73	+1.05	+1.20	—0.50	—0.44	+0.[illegible]0	+0.86	+0.20	+0.75
September[3]	+1.74	—0.52	+1.05	—0.87	—0.40	—2.05	—0.67	—1.18	+0.49	+0.28	+0.52	+1.53
October	+0.77	—1.13	+0.37	+0.98	+0.25	+1.23	—0.01	+0.71	—0.78	—0.63	—0.68	—3.61
November[4]	+1.11	+1.04	+1.21	+0.77	+1.07	+1.44	—0.39	—0.53	—1.44	—2.03	—0.08	—1.61
December	—1.43	+1.14	+0.37	+0.37	+0.16	—0.90	—0.73	—1.44	—1.03	+1.01	—0.81	+1.24

☽'s Lower transit.

1846	12h.	13h.	14h.	15h.	16h.	17h.	18h.	19h.	20h.	21h.	22h.	23h.
	d.	d	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.
June[1]	+0.50	+0.75	+0.38	+0.86	+0.19	+1.65	—0.72	+0.68	—1.35	+0.69	—2.30	+0.98
July[2]	+1.15	—1.08	—0.41	+0.32	—1.71	+1.03	+0.15	—0.18	—0.37	+1.00	—1.38	—0.03
August	+0.18	—1.56	—0.91	—0.65	—1.15	—0.03	+0.06	—2.61	+1.50	—1.30	—1.27	—0.50
September[3]	+0.64	+0.38	+0.63	+2.25	+0.84	+1.26	—0.61	—0.01	—1.05	—0.61	—0.23	+0.20
October	+0.53	—0.59	+0.30	+1.18	—1.19	+0.63	—0.31	—0.99	—0.40	—0.40	+1.51	+1.05
November[4]	+0.75	—0.62	+0.02	—0.82	—0.49	+0.01	—0.02	+1.09	+0.88	+0.57	+0.14	+0.18
December	+0.91	—0.78	—0.67	—1.82	—0.06	—0.70	—2.57	+1.21	+0.63	+0.86	+0.64	+1.48

[1] The tabular values for this month are expressed in parts of the new or observatory scale, the quantities having been converted from parts of the old or college scale into parts of the new scale.

[2] The tabular numbers refer to the new scale, the values for the first eighteen days of the month having been converted as above.

[3] Attention was paid to the half-monthly normals for the hour $8^{h.}\ 19\frac{1}{2}^{m.}$ (mean observatory time).

[4] The index correction, on and after the twenty-third day of the month, was applied before the differences were taken.

☽'s Upper transit. Moon's hour-angle.

1841.	0h.	1h.	2h.	3h.	4h.	5h.	6h.	7h.	8h.	9h.	10h.	11h.
	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.
January	+0.86	−1.07	+0.54	+1.39	+0.50	−2.01	+0.89	−0.11	−1.52	+0.48	−0.12	−1.10
February	+1.48	−2.17	+1.12	+0.49	+0.49	+0.10	−0.10	−0.57	−0.38	+0.32	+0.92	+1.40
March	+1.67	+0.82	+0.64	+1.00	+0.61	+0.40	−0.39	−1.07	−1.21	+0.69	−0.65	−0.91
April	+1.57	+1.01	+0.45	+0.97	+0.20	+0.12	+0.39	+1.40	−0.27	−1.52	+0.48	−1.43
May	+0.19	+2.11	+0.69	+1.94	−0.05	+0.92	−0.39	−0.60	−0.73	−0.20	−0.94	+1.21
June	−0.56	+1.77	+0.07	+0.45	+2.18	+1.25	−1.15	−0.59	−2.40	−1.13	−0.42	−1.24
July	+0.84	+1.86	+0.46	−1.06	−0.62	−1.52	−0.80	−0.55	−0.88	−1.71	−0.24	+1.63
August	+1.95	+1.31	+1.73	+1.42	−1.17	−1.46	−1.48	−1.39	−2.06	−2.24	−1.72	+0.60
September	+1.05	+0.10	−0.45	−0.17	−3.50	−0.54	−0.55	−0.83	−1.47	+0.86	+1.29	+0.03
October[1]	−1.15	+0.26	−0.77	−0.06	−1.31	−0.82	−0.66	−0.61	−1.73	+1.73	+0.22	+1.09
November	+0.01	−0.08	+0.02	+0.54	+0.23	−1.08	+1.54	+0.52	+1.39	+0.02	−0.24	−0.06
December	−0.41	+0.10	+0.45	−0.71	−0.94	+0.55	−0.51	+1.09	+0.62	−0.47	+0.48	+0.08

☽'s Lower transit.

1841.	12h.	13h.	14h.	15h.	16h.	17h.	18h.	19h.	20h.	21h.	22h.	23h.
	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.
January	+1.33	+0.57	−0.04	−0.51	−0.50	+0.21	+0.25	−2.10	−0.21	−1.32	−0.07	0.00
February	−0.03	−1.30	−0.78	−0.30	−1.23	−2.01	−1.12	−1.08	+0.60	+1.30	+0.56	+1.07
March	+0.15	+0.18	+1.05	+0.23	−0.15	−0.59	−0.23	−0.93	−0.47	−0.98	+1.89	+0.35
April	+1.35	−1.05	−0.09	+0.90	−0.02	−1.13	−0.32	−1.67	−0.89	−0.13	−0.63	+0.02
May	+0.42	+1.44	+0.56	+0.24	−1.21	−0.89	−2.64	−0.85	−2.20	−1.09	+0.96	+0.90
June	+0.11	−1.42	−0.13	+0.67	+1.18	−0.53	+0.62	−1.14	+1.79	+0.01	−0.22	+0.80
July	+1.26	+1.50	+1.09	+1.76	+0.32	+0.45	−0.80	+0.01	−0.95	+0.27	−0.87	+0.44
August	+2.28	+0.51	+1.97	+1.19	+0.62	−1.81	−0.50	−1.07	−0.59	+1.66	+0.06	+1.20
September	+0.37	+0.41	+1.21	+0.95	−1.66	−0.44	−0.25	−0.45	+0.45	+0.19	+0.85	+0.44
October[1]	−1.73	+1.04	+0.76	+0.34	+0.18	+1.60	+0.97	+3.14	+1.30	+3.10	−0.61	−1.54
November	+1.01	+0.03	−1.20	−0.30	−1.89	−1.33	−0.72	−0.49	+0.50	−1.89	+0.79	−0.27
December	+0.73	−0.59	+0.80	−0.49	+0.71	−0.92	−0.67	−1.27	+0.12	+1.21	+1.76	+0.83

☽'s Upper transit. Moon's hour-angle.

1842.	0h.	1h.	2h.	3h.	4h.	5h.	6h.	7h.	8h.	9h.	10h.	11h.
	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.
January	−0.30	+0.64	−0.53	+0.02	+0.66	−0.61	+0.14	−1.48	+0.44	−1.20	+0.26	−1.84
February	−0.73	+0.88	+0.36	−0.13	−0.83	+0.67	+0.18	−1.80	−0.92	−0.73	−0.27	+0.04
March	+0.72	+0.38	+1.78	+0.30	+1.35	−1.15	−0.53	−2.15	−0.05	−0.29	+0.46	−0.84
April	−0.77	+0.92	+0.53	+0.37	−0.07	−0.39	−0.20	−1.65	+0.27	−0.42	+1.21	+0.10
May	−0.57	+1.78	+0.01	−0.16	+0.18	−1.01	−1.41	−0.97	−0.92	+0.08	−0.43	+0.42
June	+0.38	+0.69	−0.95	+1.64	−0.18	+0.77	−0.25	−0.32	+0.76	+1.18	+0.38	−0.74
July	+0.78	+0.16	+0.69	−0.07	+0.60	−0.76	−2.08	+0.08	−1.65	+0.87	−1.04	+3.03
August	+0.88	+0.82	−0.08	−1.03	+1.17	−0.91	−0.95	+0.67	+0.72	−1.24	−0.17	+1.65
September	+0.71	−0.52	−0.13	−0.95	+0.67	+0.96	−0.82	+0.34	+0.82	+0.35	+0.62	+1.36
October	+3.46	+0.38	+0.77	−0.29	+0.06	+0.02	−0.25	−2.21	−0.98	−1.39	+0.52	−1.09
November	−0.05	+0.38	−1.07	−0.48	−0.36	−1.10	−0.53	+0.43	−0.95	+0.54	+0.14	+0.29
December	−0.59	−0.36	−0.34	−1.15	−0.75	+0.26	−0.57	+0.24	+0.39	+0.64	+0.87	+0.16

☽'s Lower transit.

1842.	12h.	13h.	14h.	15h.	16h.	17h.	18h.	19h.	20h.	21h.	22h.	23h.
	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.
January	−0.18	−1.19	−0.11	−2.13	+0.17	−0.61	+0.45	+1.05	+0.72	+0.66	−0.12	+0.62
February	−0.91	+0.71	+0.52	+0.40	−0.97	−0.86	−1.11	+0.44	−0.12	+0.14	+0.08	+0.84
March	+1.22	+0.20	+1.25	+0.88	+0.44	−0.34	−0.92	−1.43	+0.47	−0.39	+0.03	−0.35
April	+3.28	−0.86	+0.13	−0.12	−1.05	−1.34	−1.36	+0.15	−1.22	+0.19	−0.94	+1.11
May	+1.13	+1.78	+1.59	+1.10	−0.59	−0.52	−0.68	−1.47	−1.05	+0.15	−0.70	+1.01
June	+0.20	−0.82	+1.45	+0.33	+1.73	−1.19	+0.05	−1.36	−1.04	−1.43	−1.35	−1.37
July	−0.32	+1.84	−0.86	−0.72	+0.59	−0.95	−0.27	+0.03	−1.22	+0.09	−0.58	+0.68
August	−0.68	+2.50	−1.34	+0.59	−1.41	−0.67	−0.79	−0.58	−0.96	−0.26	+1.68	+0.81
September	+0.46	+1.11	−1.94	+0.25	+0.99	−0.45	−1.64	+0.10	−1.70	+2.14	+1.50	+0.96
October	+1.31	+1.68	−0.62	+0.74	−1.87	−0.14	+1.08	+0.43	−0.16	−0.25	+0.71	−0.56
November	+0.47	+0.40	+0.91	−1.13	+0.02	+0.11	−0.22	−1.46	+0.05	+0.68	+0.94	+1.58
December	+0.53	+0.35	+0.12	−0.45	−1.12	+0.15	+0.35	+0.54	+0.40	+0.57	+0.21	+0.07

[1] At $14^{h.}\ 19\frac{1}{2}^{m.}$ (observatory time) the difference from the half-monthly normals was used.

☽'s Upper transit. Moon's hour-angle.

1843.[1]	0h.	1h.	2h.	3h.	4h.	5h.	6h.	7h.	8h.	9h.	10h.	11h.
	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.
April	+0.87	+1.47	+1.66	+0.39	−0.42	−1.30	−2.64	−1.72	−1.99	−0.12	−1.63	−0.48
May	+0.94	+0.89	+1.54	+0.45	+0.27	+0.38	+0.23	−1.02	−0.79	−1.01	+0.47	+1.08
June	−0.13	−1.58	+0.18	−0.81	+0.67	+1.21	−0.31	+0.83	+0.16	+0.61	−0.10	+1.30
July	+2.10	+0.91	−0.71	+0.65	+0.69	+0.54	−0.62	+0.56	−0.39	−2.29	+1.05	−0.16
August[2]	−1.56	−0.81	−2.28	+1.17	−0.05	−1.12	+0.32	−1.24	+0.26	−0.22	−0.69	+0.46
September	−0.71	+0.26	−0.58	−0.85	−1.08	−0.23	−0.30	+1.74	−0.74	+0.37	−0.42	+0.58
October[3]	+1.05	+0.14	+0.28	+0.17	−0.03	−0.93	+0.19	−0.52	−1.16	+0.27	+0.33	+0.33
November	+0.52	+0.16	−0.72	−0.47	−0.80	−0.84	−0.57	−0.72	−0.02	+0.23	−0.17	+0.72
December	−0.41	−0.24	−0.64	−1.15	−0.88	−0.41	+0.07	+0.08	+0.39	+0.99	+1.09	+1.28

☽'s Lower transit.

1843.	12h.	13h.	14h.	15h.	16h.	17h.	18h.	19h.	20h.	21h.	22h.	23h.
	d.	d.	d	d.	d.	d.	d.	d.	d.	d.	d.	d.
April	+0.79	+1.92	+0.72	−0.06	+0.53	+0.05	−1.10	−1.05	−0.22	−1.06	−0.56	+1.58
May	+0.67	+0.74	+1.01	−0.58	−1.01	−1.03	−1.43	−0.27	−0.52	−0.49	+0.70	+0.08
June	+0.94	+1.46	−0.55	+0.29	−0.99	−0.05	−0.63	+0.07	−0.38	−0.22	+0.74	−0.20
July	−0.25	+0.61	+0.66	+0.66	−0.43	−1.10	−2.00	−1.05	−0.20	−0.06	−0.54	+1.73
August[2]	+0.91	−0.59	−0.77	+0.59	−1.85	+0.01	−1.00	+1.37	−0.92	+0.74	+0.49	+0.06
September	+1.63	+1.85	+0.78	+2.32	+1.15	−0.29	−0.86	+1.08	+0.65	−0.37	−0.90	−0.78
October[3]	+0.76	+1.50	+1.30	+0.53	−0.71	−0.92	−1.76	−0.70	−0.08	+0.50	−0.37	+0.78
November	+0.67	+0.45	−0.33	−0.25	−0.54	+0.04	−0.24	+0.17	+1.06	+1.00	+0.27	+0.50
December	+0.83	+0.51	+0.60	+0.62	+0.28	−1.14	−0.59	−0.74	−0.46	+0.46	+0.24	−0.42

☽'s Upper transit. Moon's hour-angle.

1844.	0h.	1h.	2h.	3h.	4h.	5h.	6h.	7h.	8h.	9h.	10h.	11h.
	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.
January[4]	−0.79	−0.18	−0.26	+0.07	+0.20	+0.94	+0.58	+0.19	+0.22	+0.37	−0.46	+0.43
February	+1.43	+0.87	+0.67	−0.52	−0.69	−0.82	−0.56	−0.74	−0.29	+0.77	+1.03	+0.96
March	+1.10	+1.06	+0.42	+0.04	−0.72	−0.55	−0.69	−0.16	+1.18	+0.05	+0.93	−0.02
April	−0.52	+0.08	+0.23	+0.54	+0.09	+0.35	−0.49	−0.12	−0.55	−0.41	+0.16	−0.04
May	+0.76	+1.17	+0.88	+0.27	+0.02	−0.49	−0.18	−0.60	−0.35	−0.10	+0.14	+0.27
June	+1.11	+0.68	+1.07	+0.44	+0.09	−0.64	−0.24	−1.33	−1.58	−1.47	−0.40	+0.22
July	+1.09	+1.27	+0.78	+0.97	+0.18	−0.73	−1.05	−1.77	−0.17	−0.13	+0.68	+0.37
August	+2.30	+0.93	+0.19	−0.14	−0.16	−1.55	−0.78	−0.69	−0.38	−0.66	+0.45	+0.45
September	+1.13	+1.47	−0.21	−0.05	−0.61	−1.15	−0.31	+1.05	+1.10	−0.18	+0.12	−0.34
October	−0.22	+0.42	−0.02	+0.22	−0.41	−0.59	−0.78	+0.38	−0.02	+1.04	+1.10	+1.01
November	−0.91	−1.12	−0.71	−0.57	−0.76	+0.03	−0.01	+0.45	−0.77	+0.06	+0.62	+2.57
December[5]	−0.26	−0.74	−0.21	−0.44	−1.14	−0.33	−0.41	−0.18	+0.14	+0.33	+0.36	+0.60

☽'s Lower-transit.

1844.	12h.	13h.	14h.	15h.	16h.	17h.	18h.	19h.	20h.	21h.	22h.	23h.
	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.
January[4]	+0.32	+0.10	+0.31	−0.09	−0.61	−0.17	+0.84	+0.95	+0.32	−0.10	−0.80	−0.48
February	+0.44	0.00	+0.54	−0.26	−1.10	−0.49	−1.13	−0.39	−0.02	−0.05	−0.02	+0.84
March	+1.33	+0.52	−0.50	−0.21	−0.21	−0.68	−1.60	−0.50	−0.31	−0.48	+0.35	−0.10
April	+0.87	+0.70	+0.37	+0.64	−0.22	−0.54	−0.50	+0.05	−0.66	−0.42	−0.02	−1.63
May	+0.46	+0.09	+0.74	+0.43	+0.62	+0.06	−0.19	−1.10	−0.85	−1.17	+0.10	+0.06
June	+0.19	+0.48	−0.30	−0.36	−0.01	+0.35	+0.31	+0.29	+0.25	+0.20	+0.11	0.00
July	+1.27	+0.36	+0.46	−0.70	−0.51	−0.70	−1.03	−0.13	−0.31	−0.57	−0.12	+0.78
August	+0.50	+0.22	+0.84	−0.30	−0.19	−0.77	−1.06	−0.75	−0.34	−0.14	−0.43	+0.76
September	+0.25	+0.04	+0.73	−0.20	−0.03	−1.20	−1.89	−1.27	−1.33	−0.62	+0.13	+1.15
October	+0.56	+1.19	+0.78	−0.10	−0.36	−0.32	−0.03	−0.83	−0.58	−0.55	+0.06	+0.06
November	+0.36	+1.09	+0.67	+0.43	+0.05	+0.40	+0.07	−0.77	−0.68	+0.41	+0.15	−0.11
December[5]	+0.48	+0.64	+1.06	−0.12	−0.12	−0.64	−0.22	+0.23	+0.26	+0.68	+0.17	+0.42

[1] There are no observations in January, February, and March, of this year.

[2] Attention was paid to the shifting of the zero of the scale between the 9th and 10th.

[3] Commencement of the hourly series of observations.

[4] Proper attention was paid to the change in the zero of divisions after the 10th.

[5] The half-monthly normals were used.

☽'s Upper transit.					Moon's hour-angle.							
1845.	0h.	1h.	2h.	3h.	4h.	5h.	6h.	7h.	8h.	9h.	10h.	11h.
	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.
January	—0.46	—1.65	—1.52	—1.65	—1.63	—0.24	+0.11	+1.41	+0.96	+1.83	+0.91	+1.11
February	—0.13	+0.48	—0.26	—1.15	—0.56	—0.81	—0.39	—0.28	+0.18	+1.03	+0.98	+1.28
March	—0.42	—0.47	—0.26	—0.48	—0.25	—0.75	—0.81	—0.25	+0.20	+0.39	+0.79	+0.91
April	+0.45	+0.54	+0.07	+0.52	—0.21	—0.47	—0.27	—0.07	—0.25	—0.03	+0.27	+1.08
May	+0.53	+0.49	+0.01	+0.16	—0.21	—0.22	—0.66	—0.25	—0.88	+0.04	+0.92	+0.43
June	+1.77	+1.63	+0.90	+1.24	+0.86	+0.54	—0.66	—1.09	—0.75	—0.93	—0.83	—0.31
☽'s Lower transit.												
1845.	12h.	13h.	14h.	15h.	16h.	17h.	18h.	19h.	20h.	21h.	22h.	23h.
	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.
January	+0.02	—0.28	—1.07	—0.60	—0.30	+0.14	+1.09	+0.29	+0.86	+0.34	+0.39	+0.38
February	+1.70	+0.67	—0.13	+0.40	+0.03	—0.76	—0.92	—1.26	—0.46	+0.17	—0.05	—0.09
March	+1.15	+0.95	+1.79	+0.35	+0.86	—0.08	—0.83	—1.27	—0.56	+0.37	—0.39	—0.73
April	+0.54	+0.56	0.00	+0.76	+1.01	—0.30	—1.00	—1.67	—1.62	—0.97	+0.37	—0.78
May	+0.53	+0.03	—0.63	—0.01	—0.24	—0.48	—0.70	—0.30	—0.40	—0.53	+1.16	+0.63
June	+0.01	+0.86	+0.30	+0.18	—0.33	—1.27	—0.82	—0.59	—0.92	+0.05	+0.74	+0.64

Value of a scale division 0′.453.

One of the first questions to determine is how many of these residuals must be used to give a definite result, and another one is whether numbers deduced from different parts of the series would give harmonious results. To test both of these the observations were formed into three groups—one containing 4,900 in 19 months of 1840, '41; another, 6,715 results in 21 months of 1842, '43; and a third, 10,029 results in 18 months of 1844, '45. In all, 21,644 results.

The following table contains the result for each group. Group II includes three months of the hourly series of observations treated as if only equal in weight to the bi-hourly series.

The sign Σ indicates the algebraic sum of the values in the preceding tables for the months comprised in each group, and for every hour-angle of the moon. The lines headed I, II, III, contain the preceding values divided by their respective number of months and expressed in minutes of arc, or the lunar diurnal variation.

☽'s Upper transit.					Moon's hour-angle.							
	0h.	1h.	2h.	3h.	4h.	5h.	6h.	7h.	8h.	9h.	10h.	11h.
	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.
Σ of group I	+10.27	+8.07	+7.52	+11.17	—4.32	—1.46	— 7.06	— 5.49	—14.63	—1.61	—1.70	+ 0.30
Σ " II	+ 6.59	+7.35	—0.23	— 2.38	+0.87	—5.95	—10.90	—10.83	— 6.35	—2.78	+2.48	+ 7.65
Σ " III	+ 7.96	+6.93	+1.77	— 0.53	—5.91	—7.48	— 7.60	— 4.05	— 2.01	+2.00	+7.77	+10.98
I	+0′.24	+0′.19	+0′.18	+0′.27	—0′.10	—0′.04	—0′.17	—0′.13	—0′.35	—0′.04	—0′.04	+0′.01
II	+0.14	+0.16	+0.00	—0.05	+0.02	—0.13	—0.24	—0.23	—0.14	—0.06	+0.05	+0.16
III	+0.20	+0.17	+0.05	—0.02	—0.15	—0.19	—0.20	—0.10	—0.05	+0.05	+0.20	+0.28
☽'s Lower transit.					Moon's hour-angle.							
	12h.	13h.	14h.	15h.	16h.	17h.	18h.	19h.	20h.	21h.	22h.	23h.
	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.
Σ of group I	+11.91	— 2.18	+4.54	+6.00	—7.22	— 3.54	— 9.43	—8.71	—0.71	+3.14	+1.58	+7.60
Σ " II	+13.46	+16.15	+4.52	+3.86	—6.64	—11.24	—14.67	—4.68	—6.90	+2.79	+1.53	+8.73
Σ " III	+10.98	+ 8.22	+5.96	+0.24	—1.66	— 7.45	— 9.61	—9.02	—7.35	—3.38	+1.90	+1.80
I	+0′.29	—0′.05	+0′.11	+0′.14	—0′.17	—0′.09	—0′.23	—0′.21	—0′.04	+0′.08	+0.04	+0′.18
II	+0.29	+0.35	+0.10	+0.08	—0.14	—0.24	—0.32	—0.10	—0.14	+0.06	+0.03	+0.19
III	+0.28	+0.21	+0.15	0.00	—0.04	—0.19	—0.24	—0.23	—0.19	—0.08	+0.05	+0.05

+ indicates west, — east, deflection from the normal position.

These results, I, II, III, when expressed analytically by means of Bessel's form of periodic functions, and when treated by the method of least squares, are represented by the following equations, in which the moon's hour-angle θ is reckoned from the upper transit westwards at the rate of 15° to each hour. $\Delta_{☾}$ represents the lunar diurnal variation.

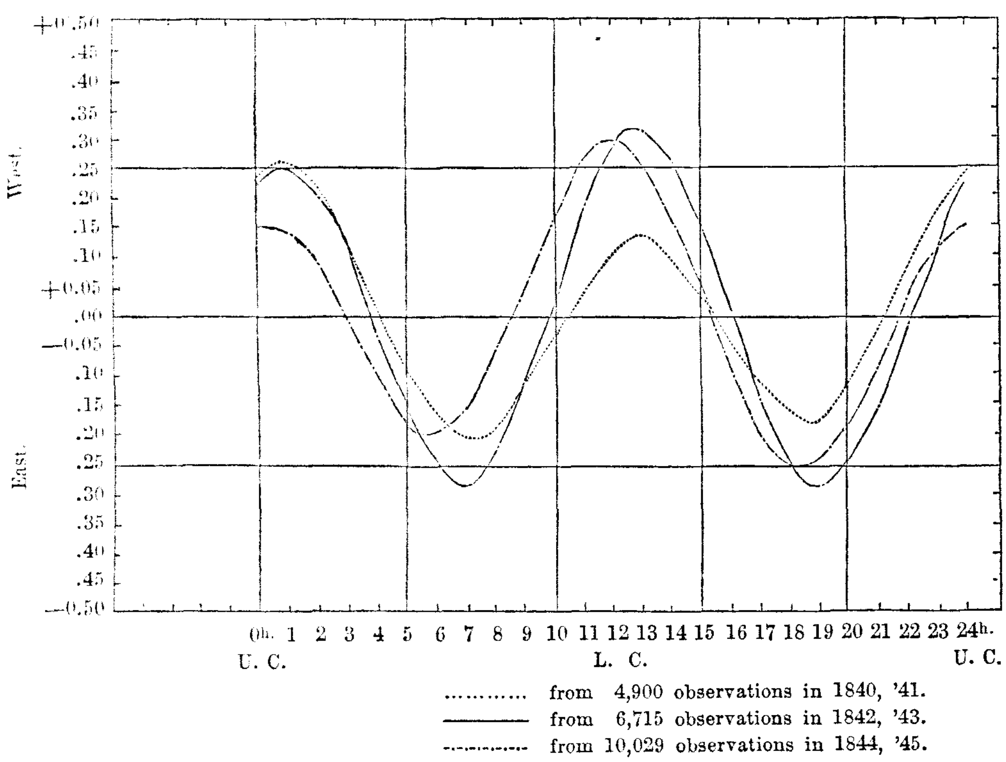
Group I, 1840–'41. $\Delta_{☾} = +0'.003 + 0'.063 \; sin. \; (\theta + 92^\circ) + 0'.189 \; sin. \; (2\theta + 67^\circ)$

" II, 1842–'43. $\Delta_{☾} = -0'.006 + 0'.030 \; sin. \; (\theta + 263^\circ) + 0'.282 \; sin. \; (2\theta + 63^\circ)$

" III, 1844–'45. $\Delta_{☾} = 0'.000 + 0'.075 \; sin. \; (\theta + 292^\circ) + 0'.219 \; sin. \; (2\theta + 88^\circ)$

The numerical results from these equations are presented graphically on the following diagram.

LUNAR-DIURNAL VARIATION OF THE MAGNETIC DECLINATION.



The curves all agree in their distinctive characters, and show two east and two west deflections in a lunar day, the maxima W. and E. occurring about the upper and lower culminations, and the minima at the intermediate six hours. The total range hardly reaches 0'.5. These results agree generally with those obtained for Toronto and Prague.

From 8,000 to 10,000 observations seem to be required to bring out the results satisfactorily, and the best results are derived from the use of all the groups.

The following table contains annual sums of deflections for each hour, and the resulting lunar-diurnal variation from the 21,644 observations available for the purpose :—

Upper curve. Westerly hour-angles.

YEAR.	0h.	1h.	2h.	3h.	4h.	5h.	6h.	7h.	8h.	9h.	10h.	11h.	
	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	
Σ for 1840	+1.77	+2.05	+2.57	+4.97	—0.94	+2.63	—3.85	—2.18	—3.99	+1.56	—0.76	—0.40	
" 1841	+8.50	+6.02	+4.95	+6.20	—3.38	—4.09	—3.21	—3.31	—10.64	—3.17	—0.94	+0.70	
" 1842	+3.92	+6.15	+1.04	—1.93	+2.50	—3.25	—7.27	—8.82	—2.07	—1.61	+2.55	+2.54	
" 1843 (*a*)	+1.51	+1.14	—0.19	+1.00	+0.08	—0.52	—3.32	—0.85	—3.49	—2.66	—1.32	+2.78	
" 1843 (*b*)	+1.16	+0.06	—1.08	—1.45	—1.71	—2.18	—0.31	—1.16	—0.79	+1.49	+1.25	+2.33	
" 1844	+6.22	+5.91	+2.83	+0.83	—3.91	—5.53	—4.92	—3.52	—1.47	—0.33	+4.73	+6.48	
" 1845	+1.74	+1.02	—1.06	—1.36	—2.00	—1.95	—2.68	—0.53	—0.54	+2.33	+3.04	+4.50	
Mean $\frac{\Sigma}{79}$	+0.43	+0.37	+0.12	+0.08	—0.21	—0.31	—0.42	—0.32	—0.33	+0.01	+0.23	+0.41	
Same in arc	+0′.19	+0.17	+0.05	+0.04	—0.10	—0.14	—0.19	—0.14	—0.15	+0.01	+0.10	+0′.19	

Lower curve. Easterly hour-angles.

YEAR.	12h.	13h.	14h.	15h.	16h.	17h.	18h.	19h.	20h.	21h.	22h.	23h.	Mos.
	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	
Σ for 1840	+4.66	—3.50	—0.66	+1.32	—3.57	+3.85	—4.02	—0.81	—0.16	+0.81	—2.89	+3.36	7
" 1841	+7.25	+1.32	+5.20	+4.68	—3.65	—7.39	—5.41	—7.90	—0.55	+2.33	+4.47	+4.24	12
" 1842	+6.51	+7.70	+1.10	—0.26	—3.07	—6.81	—5.06	—3.56	—5.83	+2.29	+1.46	+5.40	12
" 1843 (*a*)	+4.69	+5.99	+1.85	+3.22	—2.60	—2.41	—7.02	+0.15	—1.59	—1.46	—0.07	+2.47	6
" 1843 (*b*)	+2.26	+2.46	+1.57	+0.90	—0.97	—2.02	—2.59	—1.27	+0.52	+1.96	+0.14	+0.86	3
" 1844	+7.03	+5.43	+5.70	—0.84	—2.69	—4.70	—6.43	—4.22	—4.25	—2.81	—0.32	+1.75	12
" 1845	+3.95	+2.79	+0.26	+1.08	+1.03	—2.75	—3.18	—4.80	—3.10	—0.57	+2.22	+0.05	6
Mean $\frac{\Sigma}{79}$	+0.63	+0.42	+0.29	+0.14	—0.23	—0.40	—0.58	—0.42	—0.27	+0.01	+0.09	+0.26	37 21
Same in arc	+0′.29	+0.19	+0.13	+0.06	—0.10	—0.18	—0.26	—0.19	—0.12	+0.01	+0.04	+0′.12	21
													79

The two values for 1843, marked (*a*) and (*b*), exhibit the separate sums for the bi-hourly and the hourly observations, and were required to give proper weights to each. There are 37 months of bi-hourly, and 21 months of hourly observations —the latter having double weight, as found from a consideration of the probable errors derived respectively from all the results of the years 1842 and 1844. The probable error of any single monthly mean for any hour in the year 1842 was found $= \pm\ 0^{d}.60$, and the same for the year 1844 was $= \pm\ 0^{d}.40$. Hence the weights for a resulting value in the bi-hourly series is to the weight for a value in the hourly series nearly as 1 : 2, or the weights are nearly proportional to the number of observations—a result which indicates that no constant errors influence the result. The accordance among themselves of the values for the easterly hour-angles is somewhat better than the corresponding values for the westerly hour-angles—a circumstance which seems to connect itself with another phenomenon to be mentioned presently. Giving, therefore, double weight to months of the hourly series, the lunar-diurnal variation resulted as given above. When expressed analytically, it takes the form

$$\triangle_{☾} = +0'.001 + 0'.029 \sin (\theta + 295^\circ) + 0'.207 \sin (2\theta + 85^\circ)$$

which may also be written

$$\triangle_{☾} = 0''.0 + 1''.7 \sin (15n + 295^\circ) + 12''.4 \sin (30n + 85^\circ)$$

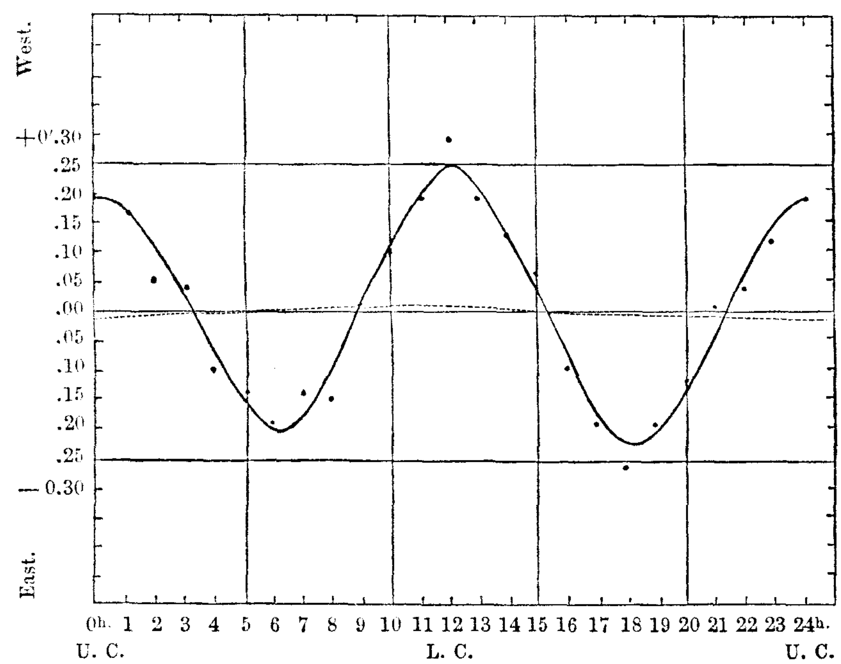
where θ represents the moon's hour-angle, reckoned from the upper culmination, or n the number of hours after the same epoch: + indicates west, and — east deflection.

The constant in Bessel's formula comes out zero, and hence it is inferred that the moon has no specific action in deflecting the magnet by a constant quantity. The coefficient of the first term of the formula is small, and it is from the second term that the distinctive features of the double-crested curve result. These results are all represented by curves.

Both the east and west deflections are well marked, those occurring when the moon is east of the meridian being greater than those when west.

It is not at all necessary to take in the third or higher terms. The progression of the hourly values is systematic, and the agreement between the computed and observed values is deemed satisfactory. The following diagram represents the curve resulting from the above equation, the observed values being indicated by dots.

LUNAR-DIURNAL VARIATION.



From 21,644 observations at Philadelphia, from 1840 to 1845.

The principal western maximum occurs 6 minutes after the lower culmination of the moon, and amounts to 0'.23. The secondary maximum occurs 14 minutes after the upper culmination, and amounts to 0'.18. The principal minimum occurs at $6^{h.}$ $17^{m.}$ after the lower culmination, the easterly deflection being 0'.22. The secondary minimum at $6^{d.}$ $03^{m.}$ after the upper culmination, with a deflection of 0'.19. The greatest range is 27", and the secondary 22". The epochs of the maxima and minima are found from the formula to be at a mean 10 minutes after culmination. The probable error of a single computed value of the lunar declination is $\pm$ 1".32. The Toronto observations gave $\pm$ 1".37 from more than twice the number of observations, so that the Philadelphia results are worthy of every confidence.

At Toronto, from the second investigation, embracing about 44,000 observations, the western and eastern deflections balanced, giving for the range 38".3. The

Prague observations also confirm the nearly equal deflections (mean) to the west and east. The epochs of the maxima and minima were found from the four roots of the equation $0 = 0.029 \cos(\theta + 295°) + 0.414 \cos(2\theta + 85°)$, which gave 10 minutes as the mean time elapsed between the moon's passing the meridian, and the time of maxima deflections. If we take the four phases into account, the lunar action seems to be retarded 10 minutes, which quantity may be termed the *lunar-magnetic interval* for the Philadelphia station. At Toronto the intervals are not so regular.

The secondary range exists at Toronto, and is a marked feature in the Prague result.

The following table contains the observed and computed values and their differences:—

	Upper Curve.				Lower Curve.		
	Observed.	Computed.	Difference.		Observed.	Computed.	Difference.
0h.	+0′.19	+0′.18	+0′.01	12h.	+0′.29	+0′.23	+0′.06
1	+0.17	+0.17	0.00	13	+0.19	+0.21	—0.02
2	+0.05	+0.10	—0.05	14	+0.13	+0.13	0.00
3	+0.04	+0.01	+0.03	15	+0.06	+0.03	+0.03
4	—0.10	—0.09	—0.01	16	—0.10	—0.08	—0.02
5	—0.14	—0.16	+0.02	17	—0.18	—0.18	0.00
6	—0.19	—0.19	0.00	18	—0.26	—0.22	—0.04
7	—0.14	—0.17	+0.03	19	—0.19	—0.21	+0.02
8	—0.15	—0.09	—0.06	20	—0.12	—0.14	+0.02
9	+0.01	+0.01	0.00	21	+0.01	—0.05	+0.06
10	+0.10	+0.12	—0.02	22	+0.04	+0.06	—0.02
11	+0.19	+0.20	—0.01	23	+0.12	+0.14	—0.02

The formula or curve enables us to divide the observed curve so as to show the diurnal and semi-diurnal part of the observed variations. The decomposition of the curve is made on the diagram where the resulting curve for the diurnal period is given.

The lunar-diurnal variation seems to be subject to an inequality depending on the solar year, for the investigation of which the preceding results were rearranged in two groups, one containing the hourly values for the summer months (April to September), the other the values for the winter months (October to March). For the summer season we have 11,087 observations, and for the winter 10,557.

Hourly Sums of the Lunar Variation for the Summer Season.

Moon's hour-angle.

	0h.	1h.	2h.	3h.	4h.	5h.	6h.	7h.	8h.	9h.	10h.	11h.	
Σ 1840–3	+9.29	+14.15	+3.45	+7.20	—2.93	—2.23	—15.73	—6.18	—12.04	—4.57	—1.49	+12.38	
Σ 1844–5	+8.62	+8.26	+3.92	+3.95	+0.05	—4.36	—4.64	—4.87	—3.81	—3.87	+1.51	+2.13	
$\frac{\Sigma}{40}$	+0.66	+0.77	+0.28	+0.38	—0.07	—0.27	—0.63	—0.40	—0.49	—0.31	+0.04	+0.42	
Same in arc	+0′.30	+0.35	+0.13	+0.17	—0.03	—0.13	—0.28	—0.18	—0.22	—0.14	+0.02	+0.19	
	12h.	13h.	14h.	15h.	16h.	17h.	18h.	19h.	20h.	21h.	22h.	23h.	Mo's.
Σ 1840–3	+17.02	+11.44	+5.18	+13.14	—4.94	—7.97	—16.72	—10.27	—12.44	+0.11	—5.49	+10.12	22
Σ 1844–5	+4.62	+3.32	+2.51	+0.44	+0.10	—4.85	—6.88	—5.47	—6.18	—4.17	+2.04	+1.61	9
$\frac{\Sigma}{40}$	+0.66	+0.45	+0.26	+0.35	—0.12	—0.44	—0.76	—0.53	—0.62	—0.21	—0.04	+0.33	
Same in arc	+0′.30	+0.20	+0.12	+0.16	—0.05	—0.20	—0.34	—0.24	—0.28	—0.09	—0.02	+0.15	

HOURLY SUMS OF THE LUNAR VARIATION FOR THE WINTER SEASON.

Moon's hour-angle.

	0h.	1h.	2h.	3h.	4h.	5h.	6h.	7h.	8h.	9h.	10h.	11h.	
Σ 1840–2	+6.42	+1.21	+4.92	+3.04	+1.19	—3.00	—1.92	—8.98	—8.15	—1.31	+1.02	—6.76	
Σ 1843–5	+0.50	—1.27	—3.23	—5.93	—7.67	—5.30	—3.27	—0.34	+1.02	+7.36	+7.51	+11.18	
$\frac{\Sigma}{39}$	+0.19	—0.04	—0.04	—0.23	—0.36	—0.35	—0.22	—0.25	—0.16	+0.35	+0.41	+0.40	
Same in arc	+0′.09	—0.02	—0.02	—0.10	—0.16	—0.16	—0.10	—0.11	—0.07	+0.16	+0.18	+0.18	
	12h.	**13h.**	**14h.**	**15h.**	**16h.**	**17h.**	**18h.**	**19h.**	**20h.**	**21h.**	**22h.**	**23h.**	**Mo's.**
Σ 1840–2	+6.09	+0.09	+2.31	—4.18	—7.95	—4.79	—4.79	—1.85	+4.31	+3.86	+8.46	+5.35	15
Σ 1843–5	+8.62	+7.34	+5.02	+0.70	—2.73	—4.62	—5.32	—4.82	—0.65	+2.75	0.00	+1.05	12
$\frac{\Sigma}{39}$	+0.60	+0.38	+0.32	—0.07	—0.35	—0.37	—0.40	—0.29	+0.08	+0.25	+0.22	+0.19	
Same in arc	+0′.27	+0.17	+0.14	—0.03	—0.16	—0.17	—0.18	—0.13	+0.04	+0.11	+0.10	+0.09	

Expressed analytically, the lunar-diurnal variation in the two seasons is as follows :—

In summer,

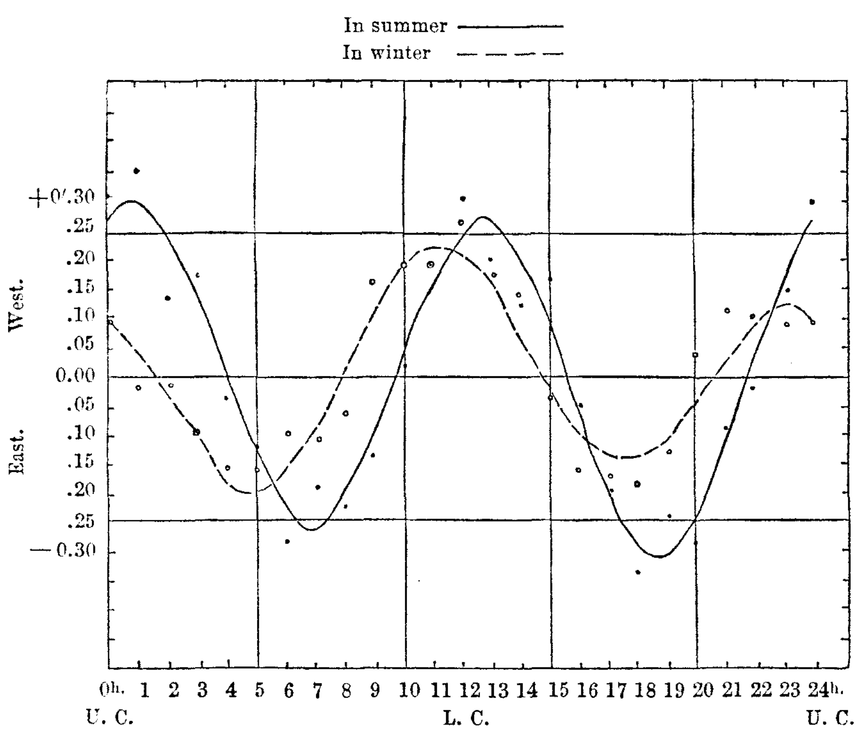
$$\triangle_{☾} = -0'.006 + 0'.028\ sin\ (\theta + 18^\circ) + 0'.278\ (2\theta + 67^\circ)$$

In winter,

$$\triangle_{☾} = +0'.005 + 0'.058\ sin\ (\theta + 264^\circ) + 0'.173\ (2\theta + 115^\circ)$$

The characteristic feature of the annual inequality in the lunar-diurnal variation is, therefore, a much smaller amplitude in winter than in summer. Kreil, indeed,

LUNAR-DIURNAL VARIATION.



Summer curve from 11,087 observations (.)
Winter " " 10,557 " (°)

inferred from the ten-year series of the Prague observations, that in winter the lunar-diurnal variation either disappears, or is entirely concealed by irregular fluctuations, requiring a long series for their diminution. The method of reduction which he employed was, however, less perfect than that now used. The second characteristic of the inequality consists in the earlier occurrence of the maxima and minima in winter than in summer. The winter curve precedes the summer curve by about one and three-quarter hours. Both these features are well expressed in the above diagram. At Toronto, the same shifting in the maxima and minima epochs was noticed, but the other inequality in the amount of deflection is not exhibited. It seems probable that the Philadelphia results are more typical in form than those either of Prague or Toronto. It is also apparent that the smaller deflection at the upper culmination in the annual mean, when compared with the deflection at the lower culmination, is entirely produced by the feeble lunar action in winter. The maximum west deflection in summer occurs actually near the upper culmination. At the same season the maximum east deflection is still retained (as in the annual curve) about six hours after the lower culmination. In the winter season this last mentioned maximum east deflection is actually the smaller of the two. We have—

Maximum summer range	35″.4,	Secondary,	31″.8
" winter "	25 .2,	"	15 .6
Difference	10 .2,		16 .2

At Prague the maximum summer range was 44″.

Next I proceed to examine whether the phases of the moon, the declination, or parallax, have any sensible effect upon the magnetic declination. Mr. Kreil found, from a ten years' series of observations at Prague, that there was no specific change in the position of the magnet depending upon the moon's phases and parallax, but that the declination was 6″.8 greater when the moon was at the greatest northern declination than when at the greatest southern declination. On the contrary, Mr. Broun, from the Makerstoun observations, a much shorter series than the one at Prague, inferred that there was a maximum of declination two days after the full moon. He also found a maximum corresponding to the greatest northern declination of the moon, but does not appear to have investigated the effect of distance. The residuals which we have been treating enable us at once to examine these several points.

Beginning with the lunar phases, the daily means for the day of full and new moon, and for two succeeding days, were compared with the monthly mean declination. In case any of the hours were disturbed, the monthly normal for the hour was substituted for the disturbed observation before the mean was taken. If one-half or more of the hourly readings were disturbed, the daily mean was altogether omitted. Accidental omissions of hourly observations were supplied by the hourly normal. The half-monthly normals were then compared with the half-monthly means. In the table of differences thus formed, equal weight is given to the bi-hourly and hourly observations. The daily mean having been subtracted from the monthly mean, the positive sign indicates a western deflection, and the negative sign an eastern one, as compared with the normal position. The following table contains the result:—

	Sum of deflections.	Number.	Deflection.		
Full moon ☉	+11.6	52	+0d..22	+0′.10	±0′.07
1st day after	—7.1	51	—0.14	—0.06	
2d day after	—9.3	48	—0.19	—0.08	
New moon ●	—11.5	43	—0.27	—0.12	±0′.09
1st day after	+1.5	47	+0.03	+0.01	
2d day after	+4.4	49	+0.09	+0.04	

The effect is very small, scarcely much beyond the probable error, but the table indicates that the north end of the magnet is deflected to the westward 0′.1 at the full, and as much to the eastward at the change day, the range between full and new moon being 0′.2. A more definite result could hardly be expected from a series of observations extending over but five years.

Treating the subject of the effect of the moon's variation in declination in precisely the same manner, we obtain the following result:—

	Mean deflection.	
One day before	—0′.20	from 54 days of observation.
At moon's max. declination . .	—0.10	" 53 " "
One day after	—0.09	" 55 " "
Mean . . .	—0′.13	" 162 " "
One day before	—0′.04	" 54 " "
At moon's min. declination . .	—0.07	" 52 " "
One day after	+0.14	" 52 " "
Mean . . .	+0′.01	" 158 " "

These results do not positively fix a deflection of the magnet as depending on the moon's greatest north and south declination, the amount resulting from the comparisons being of nearly the same magnitude as its probable error.

A similar investigation, with respect to the moon's distance from the earth, gives the following results:—

	Mean deflection.	
One day before	—0′.18	from 50 days of observation.
At moon's perigee	—0.18	" 41 " "
One day after	0.00	" 59 " "
Mean . . .	—0′.12	" 150 " "
One day before	—0′.02	" 55 " "
At moon's apogee	—0.20	" 53 " "
One day after	—0.13	" 47 " "
Mean . . .	—0′.12	" 155 " "

The differences being of the same order of magnitude as the probable errors, no conclusion as to the effect of distance can be drawn from them.

I propose hereafter to extend the discussion of the moon's effect on the declination to the effect on the earth's magnetic force.

SMITHSONIAN CONTRIBUTIONS TO KNOWLEDGE.

162

DISCUSSION

OF THE

MAGNETIC AND METEOROLOGICAL OBSERVATIONS

MADE AT THE GIRARD COLLEGE OBSERVATORY, PHILADELPHIA, IN 1840, 1841, 1842, 1843, 1844, AND 1845.

SECOND SECTION,

COMPRISING PARTS IV, V, AND VI. HORIZONTAL FORCE.

INVESTIGATION OF THE ELEVEN (OR TEN) YEAR PERIOD AND OF THE DISTURBANCES OF THE HORIZONTAL COMPONENT OF THE MAGNETIC FORCE, WITH AN INVESTIGATION OF THE SOLAR DIURNAL VARIATION, AND OF THE ANNUAL INEQUALITY OF THE HORIZONTAL FORCE; AND OF THE LUNAR EFFECT ON THE SAME.

BY

A. D. BACHE, LL.D., F.R.S.,

MEM. CORR. ACAD. SC. PARIS; SUPERINTENDENT U. S. COAST SURVEY.

[ACCEPTED FOR PUBLICATION, JUNE, 1862.]

PART IV.

INVESTIGATION

OF THE

ELEVEN (OR TEN) YEAR PERIOD AND OF THE DISTURBANCES OF THE HORIZONTAL COMPONENT OF THE MAGNETIC FORCE.

INVESTIGATION

OF THE

ELEVEN (OR TEN) YEAR PERIOD, AND OF THE DISTURBANCES OF THE HORIZONTAL COMPONENT OF THE MAGNETIC FORCE.

VOLUME XI of the Smithsonian Contributions to Knowledge contained a discussion, in three parts, of the observations for magnetic declination. The first part referring to the eleven (or ten) year period in the amplitude of the solar diurnal variation, and of the disturbances of the magnetic declination; the second, to the annual inequality of the solar diurnal variation, and the third, to the influence of the moon on the magnetic declination. The present discussion refers to the changes of horizontal force, and will be carried on in the same order as the former, so as to dispense with explanations in the mode of treatment, unless in those portions involving the peculiarities of the horizontal force instrument and record. Charles A. Schott, Esq., has rendered me the same assistance in this work, stated in the introduction to Part I.

The horizontal force instrument was one of Gauss's large bifilar magnetometers, made by Meyerstein, of Göttingen, the weight of the magnetic bar being about twenty-five pounds, and its length being thirty-six inches and five-eighths. The suspension wires were slightly inclined, the smaller distances between them being above the larger. The value of one division of the scale in parts of the horizontal force was determined to be:—

in May, 1840, ...	0.000035
in June, 1841, ...	0.000038

The mean, or 0.0000365 is the value used throughout the series. The sensibility of the instrument was thus very considerable. The instrument having been properly adjusted with the bar at right angles to the mean magnetic meridian, the torsion angle Z was found to be $71° \; 43'$. The relation $k = a \; cotan. \; Z$ expresses the value of one scale division k in parts of the horizonal force, a being the value of a scale division in parts of the radius, or $0.00011 = 0'.38$, and Z the angle of torsion. Increase of readings on the scale corresponded to decrease of horizontal force.

The instruments were placed in position by the equations deduced by Professor Lloyd, for the case of the declinometer in equilibrium with the horizontal and vertical force magnetometers, the position of instable equilibrium being taken necessarily from the form and position of the observatory. The effect of the small vertical force bar at first used, upon the bifilar was quite insensible, and that of the declinometer bar affected the value of the scale but slightly, the effect of both instruments changing the value of the scale divisions only in the ratio of 1 to 0.9956.

A thermometer, by Francis, of Philadelphia, divided to half degrees of Fahrenheit's scale, and easily read to tenths, was placed in the box of the horizontal force magnetometer and as near as practicable to the bar.

After the bifilar was set up, a motion commenced in the direction indicating decrease of force; it was progressive though not steadily so. After a time an extra scale was required on occasions of auroral, or other disturbances, and finally the ordinary readings were upon this extra scale. On the occasion of the change of the vertical force magnetometer, in January, 1841, by the substitution of Saxton's balance magnetometer for Lloyd's, the magnetism of the horizontal force bar was examined and found to have sensibly decreased; its force amounted to 0.9601 of its original force, in May, 1840. The experiments were made by means of deflections with a subsidiary declinometer bar, the only means then available. A further experiment of the loss of force was made in June, 1841, when the instrument was accidentally disturbed by one of the observers. The loss of magnetism then found, by means of a new determination of the angle Z, was 0.0314 of its amount in January, 1841. To ascertain the change of magnetism of the bars of the magnetometers, vibrations were also made use of, but they led to no satisfactory result. The progressive change of the scale readings from the change of the horizontal force and loss of magnetism of the bar, will be investigated further on.

The observations, between June, 1840, and September, 1843, were made bi-hourly, and from October, 1843, to the close of the series, hourly. The series extending over five years is not quite continuous; no observations were made on eleven days in January, 1841, on the occasion of the introduction of a new vertical force magnetometer, and the consequent necessity of readjusting the instruments; in January, February, and March, 1843, the work was reduced to but a single reading a day, by circumstances elsewhere stated; there are also some minor disturbances at other times when the difference in the readings, however, were ascertained and allowed for. Full statements bearing on the continuity of the series will be given in subsequent pages.

The reduction proper, necessarily commences with the operation of bringing all the readings to the same standard temperature, to render them comparable among themselves.

Correction of the Readings of the Bifilar Magnetometer for Changes of Temperature.

The care bestowed on the experiments to ascertain the effect of the temperature on the instrument, and the perseverance with which they were carried out were not rewarded with a corresponding degree of agreement in the results obtained, by the various processes employed. This it will be recollected was also the case at other observatories. The subject of the co-efficient of temperature for the bifilar magnet is fully treated in the preface to the three volumes containing the record, and it will, therefore, in this place only be necessary to recapitulate in general the results and to state the nature of the experiments there described.

The first observations for the temperature co-efficient were made on July 16, 1840. Oscillations were observed alternately at the ordinary temperature and near the freezing point, obtained by surrounding the box containing the magnet with ice; at the same time comparative oscillations of a bar in another building were observed to furnish the necessary data to correct the bifilar results for any change in the horizontal force during the progress of the experiments. The value deduced was 2.8 scale divisions for a change of 1° Fahrenheit. No reliance was placed on this result on account of the comparatively rude indications of the subsidiary instrument, and also on account of an irregularity at a certain point in the curve representing the connection of change of force with change of temperature.

The method of deflections was tried, and abandoned on account of the small amount of deflection at a distance sufficiently great to prevent the chance of permanent changes from the mutual action of the bars.

On the 22d of February, 1841, comparisons by vibrations were again resorted to, but with no better success, the correction for change of force during the interval being unsatisfactory. The result deduced was 3.0 scale divisions for 1° Fahr.

Applying the results to the readings of the bar when mounted on the bifilar suspension wires in the observatory, they were so little satisfactory that it was determined to get the change of intensity of the bar by heating and cooling the observatory while the bar remained in situ.

In January and February, 1842, a continuous series of observations was made by allowing the observatory to attain the winter temperature on one day, and obtaining thus a result by comparison with the preceding and succeeding days, when the room was artificially warmed. The value found was 1.55 scale divisions for 1° Fahr. At this time the observatory was warmed by a soap-stone stove with copper fixtures.

About the close of the year 1842 an efficient set of subsidiary instruments was mounted in one of the College buildings, the bifilar magnet being about nine inches in length. After the relative value of the scales of the instruments had been ascertained, comparative observations were made, six each day, in the morning and afternoon. These observations and results are given in a table extending over eleven months, in 1843, and over eleven months, in 1844. The results were fluctuating, and the discrepancies proved conclusively, that other causes were at work which would not be accounted for. The changes in the force were generally small. In the course of these experiments I found, beyond a doubt, that instruments of the same dimensions were required to give comparative results. During an aurora the small instrument in the College gave by no means the same results as the large instrument in the observatory; there were numerous comparisons determining this. I had reason also to believe that the large bar had its induced magnetism easily disturbed, and not regularly renewing itself, so that the correction for temperature may be supposed compound, one part permanent and cne part temporary. The following results were obtained:—

Observations between	February and June, 1843,	2.50 scale divisions
" "	July and December, 1843,	2.28 " "
" "	January and June, 1844,	1.94 " "
" "	July and December, 1844,	2.00 " "

for 1° Fahr. It may also be stated that no reasonable supposition in regard to differences of temperature between the indications of the thermometer and magnetic bar, or to changes in the co-efficient varying with the temperature, will explain all the cases of discrepancies. In these comparisons, always near each other in time, small differences in intensity, as shown by the subsidiary instrument, were allowed for, but the corrections for temperature of this latter instrument were neglected, as the changes of temperature in the building where it was placed were small.

Another method, not quite so unobjectionable as the preceding one, was tried; it consisted in taking the results corresponding to the highest temperatures during each winter, and comparing them with those corresponding to the lowest temperatures, a correction being made to reduce the changes of force by means of the secondary instrument. These comparisons were liable to be affected by the unequal distribution of the results used over the different parts of the month. The result was: for combinations and comparisons, from

January, 1844, to June, 1844,	2.03
July, 1844, to December, 1844,	2.29

scale divisions for each degree of Fahrenheit's scale.

The mean value of all the results obtained by the various processes explained, is 2.6 scale divisions, and as a preliminary measure, it was supposed that the co-efficient was changeable, and hence a correction for change of temperature was applied, varying from 3.2 scale divisions, in 1840, to 2.0 scale divisions, in 1844.

On resuming the discussion it was thought desirable to deduce a value for this co-efficient directly from the entire mass of observations, as this could not fail to satisfy the whole series. For this purpose it was indispensable to make the series of observations continuous, or, in other words, to refer the readings, extending over five consecutive years, to the same initial division of the scale. This is, therefore, a proper place for stating all cases when the instrument suffered any disturbance and the amount of scale correction required. All necessary explanations are given in the record.

The first break in the series occurred August 27, 1840, at $12^h\ 22^m$ (Philadelphia time), when the mirror was accidentally deranged. The observed numbers from this date to September 22, at $12^h\ 22^m$ have been brought to comparison with former numbers by the mean position of the bar for six previous days (in some cases seven) and by the hours, from $0^h\ 22^m$ to $22^h\ 22^m$ inclusive. This correction is already applied in the record, its probable error is given as 3.3 scale divisions.

On September 22, 1840, the instrument was readjusted.

An interruption of eleven days occurred, in January, 1841, owing to the introduction of a reflecting vertical force magnetometer, and requiring a new arrangement of the instruments. The horizontal force magnetometer was left in its place. The mean values for January, viz: 944.6 divisions for the bifilar, and 36°.5 for the corresponding temperature, as given in volume I of the record, may be reduced to the true mean by the interpolation of values, between December 31 and January 12. The daily mean (at 32°), on December 31, was 842.3, and on January 12, 913.0, hence, omitting the readings for January 3d, and 10th, as Sundays, the complete monthly mean should be 18.6 divisions less or equal 926.0.

The observations were resumed on the 12th, and continued to February 8th at $22^h\ 49\frac{1}{2}^m$, when the wires were found to have been slightly deranged, two days previously, February 6, $18^h\ 22^m$ (Philadelphia time), a great change in the position was noticed; on re-arranging the instrument it did not return to its former readings. A correction of + 116 has been applied (in the record) to the previous *mean* readings only in this month, and in consequence + 116 divisions should be added to each individual reading from the commencement of the series; but on account of another disturbance of the instrument, on the 22d, at $16^h\ 22^m$ (Philadelphia time), a further correction of + 92.8 scale divisions should be applied. The total correction is therefore + 208.8. Besides these corrections the readings on the 22d from $0^h\ 22^m$ (Philadelphia time) to $10^h\ 22^m$ (Philadelphia time), inclusive, should be increased by + 25.1 divisions, the alhidade of the instrument having been disturbed.[1]

On the 2d of June, 1841, the suspension wires were struck accidentally, deranging the instrument; the readings were then near the end of the subsidiary scale, and in rearranging the instrument the new readings were brought near the middle of the scale. The total difference between the old and new scale readings, the latter commencing with the first of the month, is 900 scale divisions. The means between June 1st and 5th are already corrected in the record, but the individual bi-hourly readings require a correction of + 213[2] scale divisions to produce these means. It was thought best not to apply this correction of − 900 divisions to the observations between June, 1840, and June, 1841, but simply to state the quantity since it can be applied easily to any result hereafter.

At the close of 1842 the regular observations were discontinued for three months, during January, February, and March, 1843; a daily reading was taken at $14^h\ 22^m$ (Philadelphia time), in order to keep up a continuity in the series. By means of the reduced readings in the same months in the other years, it was found that a correction of $-\ 3^d.4$ $-\ 3^d.7$ and $+\ 1^d.5$ for January, February, and March, respectively, was required to refer the mean at $14^h\ 22^m$ to the mean of a complete bi-hourly daily series. Applying these corrections, the corrected monthly means become:—

---

[1] The corrected daily means for the month of February, 1841, should, therefore, read as follows:—

1st	1163.5	10th	1131.1	19th	1127.9
2d	1144.8	11th	1103.8	20th	1130.0
3d	1141.9	12th	1082.5	22d	1182.9
4th	1133.0	13th	1083.5	23d	1182.6
5th	1138.1	15th	1100.0	24th	1128.0
6th	1138.6	16th	1122.1	25th	1107.7
8th	1181.2	17th	1139.7	26th	1144.6
9th	1150.6	18th	1137.0	27th	1162.3
Mean					1135.7

[2] For the first day only + 142, according to the mean in the record.

For January, 1843, ...	803^d.7 at 59°.2
For February, 1843, ...	798^d.9 at 51°.9
For March, 1843, ...	815^d.1 at 48°.7

On the 15th of April, 1843, the instrument was carefully examined and found in adjustment.

At 6^h 50^m on May 4, 1843, the bifilar was disturbed, but readjusted on May 5, before the regular observation at 2^h 21^m P. M. A correction of − 16 divisions during the interval is to be applied to the readings. After this date the instrument remained undisturbed.

We have, therefore, for discussion the following continuous series of monthly means of the readings of the bifilar magnetometer with its corresponding mean temperature. The series extends over five years and one month. To obtain a better view of the series, the correction of − 900 divisions for the first twelve months has been applied, it gives a negative value to the June mean of 1840.

TABLE 1.—RECAPITULATION OF MONTHLY MEAN READINGS OF THE BIFILAR MAGNETOMETER, CORRECTED SO AS TO PRESENT A CONTINUOUS SERIES.

	1840–41.	1841–42.	1842–43.	1843–44.	1844–45.
June	− 85.4	+432.3	+663.5	+901.0	+1092.0
July	+ 90.1	463.9	710.2	946.5	1126.6
August . . .	146.2	511.6	718.1	956.3	1149.5
September . .	162.1	537.9	740.3	985.4	1124.8
October . . .	149.4	515.6	768.8	988.6	1140.7
November . .	136.8	503.1	777.8	983.7	1135.1
December . .	156.0	535.4	775.9	986.1	1191.3
January . . .	234.8	561.0	803.7	988.3	1227.2
February . . .	235.7	576.4	798.9	1018.1	1221.6
March	248.9	572.1	815.1	1052.1	1235.3
April	266.5	606.7	869.5	1067.6	1257.3
May	307.8	625.1	873.6	1072.4	1250.8
June	...	...	...	...	1291.7
	Temperature of the bifilar magnet.				
June	+72°.1	+74°.1	+71°.3	+75.1	+72.9
July	75.6	77.3	76.8	76.8	77.8
August . . .	75.5	75.4	74.7	77.2	75.8
September . .	65.0	70.6	72.5	73.1	71.5
October . . .	58.7	53.7	67.9	66.3	68.8
November . .	47.4	47.1	61.8	60.5	61.5
December . .	35.7	55.4	57.3	57.7	57.4
January . . .	36.5	61.5	59.2	51.7	58.8
February . . .	34.7	60.5	51.9	54.6	53.6
March	43.5	64.1	48.7	62.8	58.2
April	50.5	65.5	67.4	63.8	64.1
May	60.3	68.3	68.4	68.9	64.3
June	...	...	...	...	74.8

Under the supposition of a uniform progression in the change of the mean monthly readings (due to change in the horizontal force and loss of magnetism of the bar) the bifilar readings for a given period may be represented by the form:—

$$B = B_m + \triangle e\, x + \triangle t\, y$$

where B_m a mean bifilar reading for the period.

x the change during a period.

y the change in the reading due to a change of 1° Fahr.

$\triangle e =$ difference between any single period and the mean epoch.

$\triangle t =$ " " any temperature and the mean temperature.

The formula was first applied to the monthly means resulting from five years of observation; it gave $y = +$ 1.0 scale division; but the remaining differences showed that the irregular changes between June and July, and December and January, of the years 1840–41, had an undue effect on the result, the first year's observations were, therefore, omitted, and the process repeated for the remaining four years. The twelve conditional equations gave the normal equations:—

$$+ 2143.15 = + 143x - 200.4\,y.$$
$$- 2549.73 = - 200.4x + 711.1\,y.$$

whence $x=$ monthly effect of the progression $= +$ 16.5 scale divisions.

$y=$ temperature correction for 1° Fahr. $= +$ 1.8 " "

An examination of the observed and computed values showed that the introduction of a term $\triangle e^2 z$ would improve the agreement, solving the three normal equations we found

$$x = + 17.6$$
$$y = + \ \ 1.62$$
$$z = - \ \ 0.31$$

The following table shows the comparison of the observed and computed monthly mean readings of the bifilar:—

1841–1845.	Mean temperature.	Mean observed bifilar reading.	Mean computed.	Difference c. — o.	C. — o. + 3.5.
June	73.3	772.2	779.2	+ 7.0	+10.5
July	77.2	811.8	806.2	— 5.6	— 2.1
August . . .	76.5	833.9	824.7	— 9.2	— 5.7
September . .	71.9	847.1	837.0	—10.1	— 6.6
October . . .	64.2	853.4	843.3	—10.1	— 6.6
November . .	57.7	849.9	851.4	+ 1.5	+ 5.0
December. . .	57.0	872.2	867.8	— 4.4	— 0.9
January . . .	57.8	895.0	886.0	— 9.0	— 5.5
February . . .	55.2	903.8	897.9	— 5.9	— 2.4
March	58.5	918.6	919.3	+ 0.7	+ 4.2
April	65.2	950.3	945.4	— 4.9	— 1.4
May	67.5	955.5	963.5	+ 8.0	+11.5
Mean	65.17	872.0			

Adding + 3.5 scale divisions to the mean value of B_m the above differences will balance. According to the above results, the annual progressive change is $+ 17.6 \times 12 = 211.2$ scale divisions, and the change in magnetic moment of the bar for a change of 1° Fahr. in the temperature, or $q = + 1.62 \times 0.0000365 = 0.0000591$. This agrees with the best direct determination, being the one in which the observatory was alternately heated and cooled.

To test these results, a combination of the six warmest months with the six coldest months, by alternate means furnished several values for q depending merely on the assumption of a gradual regular progressive change during each year and a half, for which separate results were deduced; this series commences with May, 1841, and ends with April, 1845, and contains, therefore, the same number of months as the first combination, excluding at the same time the two defective portions noticed above. This combination also possesses the advantage of showing the variations in the values of q.

COMBINATION BY ALTERNATE MEANS OF THE WARMER MONTHS, FROM MAY TO OCTOBER INCLUSIVE, WITH THE COLDER MONTHS, FROM NOVEMBER TO APRIL INCLUSIVE.

	Bifilar.	Temperature.	Alternate Means.		Δd	Δt	q in scale divisions.
May, 1841 to Oct., 1841	461.5	68.57					
Nov., 1841 to April, 1842	559.1	59.05	582.9	70.25	23.8	11.20	+2.1
May, 1842 to Oct., 1842	704.3	71.92	683.0	58.38	21.3	13.54	+1.6
Nov., 1842 to April, 1843	806.8	57.72	823.1	72.37	16.3	14.65	+1.1
May, 1843 to Oct., 1843	941.9	72.82	911.4	58.12	30.5	14.70	+2.1
Nov., 1843 to April, 1844	1016.0	58.52	1029.8	72.72	13.8	14.20	+1.0
May, 1844 to Oct., 1844	1117.7	72.62	1113.6	58.72	4.1	13.90	+0.3
Nov., 1844 to April, 1845	1211.3	58.93					
				Sum	109.8	82.19	+1.3

The result from this combination + 1.3 confirms the preceding value, the result, according to weight or + 1.5 scale divisions or q=0.0000548 in parts of the horizontal force has, therefore, been adopted in the reduction of the bifilar readings to a standard temperature, for which + 63°.0 Fahr. has been determined upon as the mean temperature of the magnetic bar during the five years series of observations.

The difference in the resulting value for q, when obtained from deflections or vibrations, and from combinations of the bifilar readings themselves, has been remarked before, and no satisfactory explanation has as yet been given of it. Thus, for instance, at Toronto, the two respective values were 2.69 and 1.63 scale divisions, as shown in General Sabine's remarks (Vol. III.) The existence of a similar discrepancy in the case of the Makerstoun bifilar has been detected by Mr. Broun. Whatever may be the cause of the difference, there can be no hesitation in saying that the result derived from the bifilar observations themselves is the one to be preferred. At St. Helena (Vol. II., London, 1860), the two values were 1.45 and 0.98, the half yearly comparisons at this station even show a less value, viz., 0.88 scale divisions; 0.98 (for convenience 1.0) was adopted in the reduction. Dr. Lamont, in his Handbook of Terrestrial Magnetism (p. 206, edition of 1849), says: "It deserves to be remarked that the value obtained by comparing monthly mean readings of the bifilar at high and low temperatures is smaller than that obtained by direct observation."

In the present discussion the value $\frac{q}{k} = \frac{0.0000548}{0.0000365} = 1.5$ has been adopted. At Toronto this value was $\frac{q}{k} = \frac{0.000142}{0.000087} = 1.63$, and at St. Helena $\frac{q}{k} = \frac{0.00019}{0.00019} = 1.0$.

It will be seen from these values that the Philadelphia bifilar magnetometer was very sensitive; its scale value in parts of the horizontal force is but four-tenths of the Toronto value, and only two-tenths of that of the St. Helena instrument.

In the computations which follow the tenths of scale readings have been omitted (keeping only the nearest unit) as contributing nothing to the accuracy of the results, and merely increasing the labor of reduction. The uncertainty in the readings arising from the uncertainty in the value of q probably affects the units, and the same may be said of the declination changes, so that in extreme (individual) cases the next higher figure may be affected.

The next step of the reduction consisted in transcribing the whole body of the observations after correcting them individually for differences of temperature; the adopted standard temperature being 63° Fahr.

The following table contains the monthly means of the bifilar readings reduced to the standard temperature; the series has been made continuous by the application of certain corrections explained before.

The readings are in scale divisions of 0.0000365 parts of the horizontal force; increasing numbers denote decrease of force. The time is Observatory mean time, counted to twenty-four hours for convenience sake.

TABLE II.—MONTHLY MEANS OF THE BIFILAR READINGS TAKEN AT INTERVALS OF TWO HOURS AND REDUCED TO THE STANDARD TEMPERATURE 63° FAHRENHEIT.

Philadelphia time (A. M.)						(P. M.)						
	$0^h\ 22^m$	$2^h\ 22^m$	$4^h\ 22^m$	$6^h\ 22^m$	$8^h\ 22^m$	$10^h\ 22^m$	$12^h\ 22^m$	$14^h\ 22^m$	$16^h\ 22^m$	$18^h\ 22^m$	$20^h\ 22^m$	$22^h\ 22^m$
1840.	Div's.	Div's.	Div's.	Div's.	Div's.	Div's.	Div's.	Div's.	Div's.	Div's.	Div's.	Div's.
June	—96	—98	—101	—113	—102	— 79	— 94	—115	—117	— 96	— 88	— 90
July	+74	+67	+ 63	+ 60	+ 86	+100	+ 81	+ 52	+ 41	+ 74	+ 79	+ 79
August	129	117	117	113	146	157	137	113	110	129	126	133
Sept.	158	147	143	138	169	201	183	157	152	153	157	153
October	155	149	137	140	153	179	177	161	155	157	158	152
Nov.	160	157	149	141	153	171	179	165	167	159	160	164
Dec.	203	192	184	178	184	210	218	206	192	196	202	202
1841.	$0^h\ 22^m$	$2^h\ 22^m$	$4^h\ 22^m$	$6^h\ 22^m$	$8^h\ 22^m$	$10^h\ 22^m$	$12^h\ 22^m$	$14^h\ 22^m$	$16^h\ 22^m$	$18^h\ 22^m$	$20^h\ 22^m$	$22^h\ 22^m$
*January	296	287	286	276	272	294	322	306	289	298	294	298
Feb.	279	270	265	256	261	286	303	295	276	283	289	275
March	276	273	267	260	272	298	299	272	279	281	282	280
April	285	278	268	265	287	312	314	282	273	280	289	286
May	311	312	311	303	318	335	323	304	298	307	312	315
June	420	417	414	405	418	427	406	402	408	416	426	427
July	444	440	435	436	447	457	449	429	430	442	453	448
August	490	490	485	481	499	515	500	479	481	496	501	497
Sept.	517	520	517	514	534	561	538	522	521	528	523	524
October	528	520	517	518	532	540	545	535	529	530	531	530
Nov.	528	529	522	515	525	535	539	525	523	525	528	529
Dec.	545	541	537	534	539	551	562	550	547	553	553	551
1842.	$0^h\ 21\frac{1}{2}^m$	$2^h\ 21\frac{1}{2}^m$	$4^h\ 21\frac{1}{2}^m$	$6^h\ 21\frac{1}{2}^m$	$8^h\ 21\frac{1}{2}^m$	$10^h\ 21\frac{1}{2}^m$	$12^h\ 21\frac{1}{2}^m$	$14^h\ 21\frac{1}{2}^m$	$16^h\ 21\frac{1}{2}^m$	$18^h\ 21\frac{1}{2}^m$	$20^h\ 21\frac{1}{2}^m$	$22^h\ 21\frac{1}{2}^m$
January	560	558	557	554	553	575	579	564	559	568	565	565
Feb.	582	576	574	568	570	580	593	582	578	583	589	582
March	573	564	561	561	567	580	577	567	568	574	576	577
April	605	599	598	593	601	618	612	596	592	605	607	607
May	618	614	609	609	624	632	622	607	609	618	620	622
June	652	655	649	641	652	664	654	642	639	652	655	656
July	684	689	682	683	695	710	698	681	674	687	693	697
August	702	695	695	693	712	722	703	689	690	700	704	703
Sept.	721	723	719	712	732	746	734	722	718	729	730	727
October	757	750	747	747	755	774	778	772	766	764	765	762
Nov.	780	774	772	769	778	791	786	782	778	778	781	785
Dec.	783	780	778	776	779	793	800	791	780	781	784	785
1843.	$0^h\ 21\frac{1}{2}^m$	$2^h\ 21\frac{1}{2}^m$	$4^h\ 21\frac{1}{2}^m$	$6^h\ 21\frac{1}{2}^m$	$8^h\ 21\frac{1}{2}^m$	$10^h\ 21\frac{1}{2}^m$	$12^h\ 21\frac{1}{2}^m$	$14^h\ 21\frac{1}{2}^m$	$16^h\ 21\frac{1}{2}^m$	$18^h\ 21\frac{1}{2}^m$	$20^h\ 21\frac{1}{2}^m$	$22^h\ 21\frac{1}{2}^m$
January								813				
Feb.								819				
March								835				
April	860	859	853	853	867	880	875	860	859	863	866	859
May	866	864	862	860	875	877	862	855	856	863	873	8[illegible]0
June	884	883	879	876	886	895	887	873	873	884	887	888
July	924	921	921	920	933	940	932	920	916	921	931	929
August	932	931	931	928	950	957	944	924	925	930	936	935
Sept.	968	967	962	957	977	990	981	966	968	970	970	966

Hourly Series.

	$0^h 21\frac{1}{2}^m$	$1^h 21\frac{1}{2}^m$	$2^h 21\frac{1}{2}^m$	$3^h 21\frac{1}{2}^m$	$4^h 21\frac{1}{2}^m$	$5^h 21\frac{1}{2}^m$	$6^h 21\frac{1}{2}^m$	$7^h 21\frac{1}{2}^m$	$8^h 21\frac{1}{2}^m$	$9^h 21\frac{1}{2}^m$	$10^h 21\frac{1}{2}^m$	$11^h 21\frac{1}{2}^m$
1843.	Div's.	Div's.	Div's.	Div's.	Div's.	Div's.	Div's.	Div's.	Div's.	Div's.	Div's.	Div's.
October	983	978	980	978	976	978	977	980	984	987	991	992
Nov.	988	987	986	984	983	981	981	984	988	992	994	994
Dec.	996	994	993	992	990	988	988	988	992	992	993	998
	$12^h 21\frac{1}{2}^m$	$13^h 21\frac{1}{2}^m$	$14^h 21\frac{1}{2}^m$	$15^h 21\frac{1}{2}^m$	$16^h 21\frac{1}{2}^m$	$17^h 21\frac{1}{2}^m$	$18^h 21\frac{1}{2}^m$	$19^h 21\frac{1}{2}^m$	$20^h 21\frac{1}{2}^m$	$21^h 21\frac{1}{2}^m$	$22^h 21\frac{1}{2}^m$	$23^h 21\frac{1}{2}^m$
October	991	989	985	983	983	983	985	985	985	984	983	984
Nov.	992	990	988	987	985	986	987	987	988	988	989	989
Dec.	1000	999	997	994	992	991	993	996	996	996	998	999
1844.	$0^h 21\frac{1}{2}^m$	$1^h 21\frac{1}{2}^m$	$2^h 21\frac{1}{2}^m$	$3^h 21\frac{1}{2}^m$	$4^h 21\frac{1}{2}^m$	$5^h 21\frac{1}{2}^m$	$6^h 21\frac{1}{2}^m$	$7^h 21\frac{1}{2}^m$	$8^h 21\frac{1}{2}^m$	$9^h 21\frac{1}{2}^m$	$10^h 21\frac{1}{2}^m$	$11^h 21\frac{1}{2}^m$
January	1009	1007	1006	1004	1002	1002	1001	1001	1004	1007	1010	1013
Feb.	1031	1031	1031	1029	1026	1026	1026	1028	1029	1030	1034	1036
March	1050	1048	1047	1046	1045	1044	1045	1046	1051	1058	1060	1062
April	1067	1066	1065	1062	1059	1059	1062	1062	1067	1075	1079	1079
May	1066	1066	1064	1063	1063	1062	1062	1065	1069	1075	1076	1071
June	1080	1079	1078	1079	1079	1077	1075	1079	1082	1084	1086	1083
July	1103	1104	1106	1107	1107	1106	1105	1105	1110	1117	1119	1115
August	1129	1130	1130	1130	1129	1127	1126	1131	1139	1148	1149	1143
Sept.	1108	1108	1108	1109	1105	1107	1106	1113	1123	1129	1133	1129
October	1132	1128	1127	1123	1122	1124	1125	1130	1137	1143	1146	1141
Nov.	1136	1135	1133	1132	1131	1127	1128	1129	1134	1141	1147	1149
Dec.	1203	1201	1198	1196	1194	1192	1188	1191	1192	1196	1207	1215
	$12^h 21\frac{1}{2}^m$	$13^h 21\frac{1}{2}^m$	$14^h 21\frac{1}{2}^m$	$15^h 21\frac{1}{2}^m$	$16^h 21\frac{1}{2}^m$	$17^h 21\frac{1}{2}^m$	$18^h 21\frac{1}{2}^m$	$19^h 21\frac{1}{2}^m$	$20^h 21\frac{1}{2}^m$	$21^h 21\frac{1}{2}^m$	$22^h 21\frac{1}{2}^m$	$23^h 21\frac{1}{2}^m$
January	1011	1008	1005	1001	1000	1002	1004	1005	1005	1006	1007	1009
Feb.	1035	1032	1028	1028	1032	1031	1032	1033	1034	1033	1034	1033
March	1067	1063	1056	1049	1052	1054	1054	1053	1051	1052	1052	1051
April	1074	1069	1063	1059	1061	1059	1065	1067	1068	1069	1066	1069
May	1065	1058	1054	1054	1052	1055	1060	1064	1065	1065	1064	1064
June	1079	1074	1069	1067	1067	1069	1073	1075	1077	1079	1079	1080
July	1107	1101	1097	1094	1093	1094	1097	1100	1102	1103	1104	1105
August	1134	1125	1117	1115	1117	1123	1130	1131	1132	1131	1132	1131
Sept.	1119	1108	1102	1100	1101	1105	1108	1110	1111	1111	1112	1116
October	1139	1134	1128	1129	1128	1132	1133	1133	1135	1132	1133	1130
Nov.	1146	1145	1139	1137	1138	1138	1138	1143	1141	1138	1135	1139
Dec.	1215	1210	1205	1200	1195	1196	1197	1197	1197	1201	1201	1201
1845.	$0^h 21\frac{1}{2}^m$	$1^h 21\frac{1}{2}^m$	$2^h 21\frac{1}{2}^m$	$3^h 21\frac{1}{2}^m$	$4^h 21\frac{1}{2}^m$	$5^h 21\frac{1}{2}^m$	$6^h 21\frac{1}{2}^m$	$7^h 21\frac{1}{2}^m$	$8^h 21\frac{1}{2}^m$	$9^h 21\frac{1}{2}^m$	$10^h 21\frac{1}{2}^m$	$11^h 21\frac{1}{2}^m$
January	1233	1230	1231	1229	1227	1225	1224	1226	1230	1238	1244	1248
Feb.	1232	1234	1232	1230	1230	1227	1224	1228	1234	1238	1246	1249
March	1237	1237	1235	1236	1235	1235	1231	1234	1242	1250	1256	1262
April	1253	1250	1249	1247	1245	1243	1241	1247	1255	1270	1280	1279
May	1249	1248	1246	1245	1241	1238	1235	1242	1254	1264	1265	1263
June	1274	1274	1274	1273	1268	1267	1262	1266	1273	1284	1290	1289
	$12^h 21\frac{1}{2}^m$	$13^h 21\frac{1}{2}^m$	$14^h 21\frac{1}{2}^m$	$15^h 21\frac{1}{2}^m$	$16^h 21\frac{1}{2}^m$	$17^h 21\frac{1}{2}^m$	$18^h 21\frac{1}{2}^m$	$19^h 21\frac{1}{2}^m$	$20^h 21\frac{1}{2}^m$	$21^h 21\frac{1}{2}^m$	$22^h 21\frac{1}{2}^m$	$23^h 21\frac{1}{2}^m$
January	1245	1241	1238	1235	1233	1236	1237	1233	1232	1231	1231	1229
Feb.	1251	1247	1240	1236	1235	1233	1234	1236	1236	1232	1232	1233
March	1261	1254	1246	1240	1241	1243	1245	1242	1241	1238	1241	1240
April	1271	1267	1255	1253	1249	1251	1254	1257	1257	1254	1251	1252
May	1256	1248	1242	1242	1242	1246	1251	1251	1251	1253	1251	1245
June	1282	1278	1269	1267	1266	1269	1274	1278	1277	1276	1275	1275

The monthly means are contained in the following table:—

TABLE III.—MONTHLY MEANS OF THE PRECEDING BIFILAR READINGS REDUCED TO THE STANDARD TEMPERATURE 63° FAHRENHEIT.

	1840–41.	1841–42.	1842–43.	1843–44.	1844–45.
	Div's.	Div's.	Div's.	Div's.	Div's.
June	— 99				
July	+ 71	443	689	926	1104
August . . .	127	493	701	935	1130
September . .	159	527	726	970	1112
October . . .	156	530	761	984	1132
November . .	160	527	780	987	1138
December . .	197	547	784	994	1199
January . . .	[1]274	563	[3]808	1005	1233
February . . .	278	580	[3]814	1031	1235
March	278	570	[3]835	1052	1243
April	285	603	863	10 6	1255
May	312	617	865	10 4	1249
June	[2]415	651	883	10 7	1274

Correction for progressive change in the readings.—The observations having been referred to a uniform temperature, still require a correction for the effect of the progressive change during each month before Peirce's criterion can be applied for the purpose of separating the disturbances. We have seen that the mean monthly value of this change due to loss of magnetism of the bar and to change in the horizontal force itself, was 17.6 scale divisions; on the average, therefore, a correction must be applied to the observations on the first and last day of each month of + 8.8 and — 8.8 scale divisions, and in proportion for the intermediate days. At Toronto, also, the progressive change in some months was so great as to present a practical difficulty by its interference with the proper comparability of the observations, and in these cases new means at shorter intervals than a month were taken.

[1] The actual mean of 17 days was 293; to reduce this to the mean of 27 days, 19 scale divisions were subtracted, resulting from an interpolation between January 1st and January 12th; the mean of 7 days preceding and following the gap was made use of.

[2] Owing to causes already explained, the means of May and June differ so much as to affect the continuity of the series; the same is to be said of the differences between June and July, 1840, and between December, 1840, and January, 1841; the corresponding differences between the same months in the other four years furnish us with the means of correcting the series for the first year, as will be seen hereafter; it also appeared advisable to omit the readings in June, 1840, altogether, the instrument not having then been in stable adjustment.

[3] The numbers in table II have been slightly changed, to refer the mean of the hour of observation to the mean resulting from observation of 12 hours a day. Comparing the mean at $14^h\ 22^m$ in each month with the respective monthly means in the other four years, the above corrections became —5, —5 and 0 for January, February, and March.

The bar between September and October, 1843, separates the means from the bi-hourly and the hourly series.

In the application of the reduction for temperature no attempt whatever has been made at interpolation in the magnetic series, but whenever a temperature reading was accidentally omitted, it has been supplied by comparison with the observed temperature immediately preceding and following. No magnetic reading can be supplied by interpolation, however short the interval, as long as the law of the occurrence of the disturbances remains unknown.

At Philadelphia the progressive change is so large as to require a systematic correction throughout the series. In the manuscript tables used for the preparation of the monthly normals and containing the observations reduced to 63° Fahr., the readings corrected for progressive change were written in blue ink underneath each observation. If the monthly differences are taken from Table No. III., it is apparent that the change is irregular, and in three cases at least it is certain that other causes were in operation, which produced larger monthly differences than could be attributed to the gradual loss of magnetism. These cases are the following (already noticed in the preceding temperature discussion): between June and July, 1840, a difference of 170 divisions; between December and January, 1840–41, a difference of 77; and between May and June, 1841, a difference of 103 divisions. They require separate treatment, as will be presently explained. For the correction of the progressive change the mean reading from *one* month's series was made out for the first, middle, and last of each month. By this process of taking the mean from 14 days preceding and 14 days following each of the epochs the lunar effect on the solar variation is practically eliminated from the resulting mean value.[1] These means corresponding in time to the beginning, the middle, and the end of each month, furnish the *rate* of change for the first and second half of the month, and by simple interpolation give the correction for progressive change for each day. If the rates for the first and second half of the month are different, the monthly means of each hour (from the blue figures) will differ by a small but *constant* quantity from the former monthly means. Thus, for instance, for the month of June, 1842, the monthly mean is 651 divisions, corresponding in time to the middle of the month, the mean of the readings (at 63°) for the second half of May and the first half of June is 641, corresponding in time to the first of June, and the mean of the readings (at 63°) of the second half of June and the first half of July is 673, corresponding in time to the last of June; the correction applied to the bi-hourly readings (at 63°) on June 1st was + 10, and to the readings on June 30th was — 22 divisions. At the middle of the month the correction is zero, and for the intermediate days it is in proportion to their respective distances from the middle. The algebraic sum of the daily corrections divided by the number of days of observation is — 3, which gives the new monthly mean 648, as corrected for irregularity in the progressive change. In the exceptional case of a break, or beginning and termination, the required rate of change for half the month was found by a similar process, using half monthly and quarterly means.

The following table, No. IV., contains the monthly means of the bi-hourly and hourly readings of the bifilar magnetometer referred to a uniform temperature (63° Fahr.), and corrected for irregularity in the progressive change. It is here inserted for the purpose of comparing it with the monthly normals, showing the change produced by the exclusion of the disturbances. The means in the month of June, 1840, are suppressed, and the readings between June 1 and June 5, 1841, were not used.

---

[1] In connection with this subject, the first part of an interesting paper by Mr. Broun may be consulted, viz.: "On the lunar diurnal variation of the magnetic declination at the magnetic equator." —*Proceedings Royal Society*, vol. X., No. 39, 1860.

TABLE IV.—MONTHLY MEANS OF THE BI-HOURLY AND HOURLY READINGS OF THE BIFILAR MAGNETOMETER, REDUCED TO A UNIFORM TEMPERATURE AND CORRECTED FOR IRREGULARITY IN THE PROGRESSIVE CHANGE.

Philadelphia time (A. M.) (P. M.)

	$0^h\ 22^m$	$2^h\ 22^m$	$4^h\ 22^m$	$6^h\ 22^m$	$8^h\ 22^m$	$10^h\ 22^m$	$12^h\ 22^m$	$14^h\ 22^m$	$16^h\ 22^m$	$18^h\ 22^m$	$20^h\ 22^m$	$22^h\ 22^m$
1840.	Div's.	Div's.	Div's.	Div's.	Div's.	Div's.	Div's.	Div's.	Div's.	Div's.	Div's.	Div's.
July	90	83	79	76	102	116	97	68	57	90	95	95
August	130	118	118	114	147	158	139	115	112	130	127	134
Sept.	161	150	146	141	172	204	186	160	155	156	160	156
October	153	147	135	138	151	177	175	159	153	155	156	150
Nov.	155	152	144	136	148	166	174	160	162	154	155	159
Dec.	202	191	183	177	183	209	217	205	191	195	201	201
1841.	$0^h\ 22^m$	$2^h\ 22^m$	$4^h\ 22^m$	$6^h\ 22^m$	$8^h\ 22^m$	$10^h\ 22^m$	$12^h\ 22^m$	$14^h\ 22^m$	$16^h\ 22^m$	$18^h\ 22^m$	$20^h\ 22^m$	$22^h\ 22^m$
[1]January	300	291	290	280	276	298	326	310	293	302	298	302
Feb.	279	270	265	256	261	286	303	295	276	2 3	289	275
March	276	273	267	260	272	298	299	272	279	281	282	280
April	283	275	265	262	284	309	311	279	270	277	286	283
May	307	308	307	299	314	331	319	300	294	303	308	312
[2]June	392	390	389	383	400	406	390	380	386	392	402	400
July	445	441	436	437	448	458	450	430	431	443	454	449
August	492	492	487	483	501	517	502	481	483	498	503	499
Sept.	519	522	519	516	536	562	540	524	523	530	525	526
October	527	519	516	517	531	539	544	534	528	529	530	529
Nov.	525	526	519	512	522	532	536	522	520	522	525	526
Dec.	546	542	538	535	540	552	563	551	548	554	554	552
1842.	$0^h\ 21\frac{1}{2}^m$	$2^h\ 21\frac{1}{2}^m$	$4^h\ 21\frac{1}{2}^m$	$6^h\ 21\frac{1}{2}^m$	$8^h\ 21\frac{1}{2}^m$	$10^h\ 21\frac{1}{2}^m$	$12^h\ 21\frac{1}{2}^m$	$14^h\ 21\frac{1}{2}^m$	$16^h\ 21\frac{1}{2}^m$	$18^h\ 21\frac{1}{2}^m$	$20^h\ 21\frac{1}{2}^m$	$22^h\ 21\frac{1}{2}^m$
January	558	556	555	552	551	573	577	562	557	566	563	563
Feb.	585	579	577	571	573	583	596	585	581	586	592	585
March	569	560	557	557	563	576	573	563	564	570	572	573
April	610	604	603	598	606	623	617	601	597	610	612	612
May	614	610	606	605	621	629	618	604	606	615	617	619
June	649	652	645	638	649	661	651	639	636	649	652	653
July	687	692	685	686	698	713	700	684	677	690	696	700
August	701	694	695	692	711	721	702	688	689	699	703	702
Sept.	723	725	720	713	734	748	736	724	720	731	732	729
October	761	754	751	751	759	778	782	776	770	768	769	766
Nov.	779	773	771	768	777	790	785	781	777	777	780	784
Dec.	780	777	775	773	776	790	797	788	777	778	781	782
1843.	$0^h\ 21\frac{1}{2}^m$	$2^h\ 21\frac{1}{2}^m$	$4^h\ 21\frac{1}{2}^m$	$6^h\ 21\frac{1}{2}^m$	$8^h\ 21\frac{1}{2}^m$	$10^h\ 21\frac{1}{2}^m$	$12^h\ 21\frac{1}{2}^m$	$14^h\ 21\frac{1}{2}^m$	$16^h\ 21\frac{1}{2}^m$	$18^h\ 21\frac{1}{2}^m$	$20^h\ 21\frac{1}{2}^m$	$22^h\ 21\frac{1}{2}^m$
January								818				
Feb.								819				
March								831				
April	863	862	856	856	870	883	878	863	862	866	869	862
May	865	863	861	859	874	876	861	854	855	862	872	869
June	881	880	876	873	883	892	884	870	870	881	884	885
July	927	924	924	923	936	943	935	923	919	924	934	932
August	931	930	930	927	949	956	943	923	924	929	935	934
Sept.	971	970	965	960	980	993	984	969	971	973	973	969

[1] The mean of 17 days is given; to refer it to a complete month subtract 19 divisions.
[2] The mean of 19 days is given; to refer it to a complete month add 8 divisions.

Hourly Series.

	$0^h 21\frac{1}{2}^m$	$1^h 21\frac{1}{2}^m$	$2^h 21\frac{1}{2}^m$	$3^h 21\frac{1}{2}^m$	$4^h 21\frac{1}{2}^m$	$5^h 21\frac{1}{2}^m$	$6^h 21\frac{1}{2}^m$	$7^h 21\frac{1}{2}^m$	$8^h 21\frac{1}{2}^m$	$9^h 21\frac{1}{2}^m$	$10^h 21\frac{1}{2}^m$	$11^h 21\frac{1}{2}^m$
1843.	Div's.	Div's.	Div's.	Div's.	Div's.	Div's.	Div's.	Div's.	Div's.	Div's.	Div's.	Div's.
October	983	978	980	978	976	978	977	980	984	987	991	992
Nov.	987	986	985	983	982	980	980	983	987	991	993	993
Dec.	995	993	992	991	989	987	987	987	991	991	992	997
	$12^h 21\frac{1}{2}^m$	$13^h 21\frac{1}{2}^m$	$14^h 21\frac{1}{2}^m$	$15^h 21\frac{1}{2}^m$	$16^h 21\frac{1}{2}^m$	$17^h 21\frac{1}{2}^m$	$18^h 21\frac{1}{2}^m$	$19^h 21\frac{1}{2}^m$	$20^h 21\frac{1}{2}^m$	$21^h 21\frac{1}{2}^m$	$22^h 21\frac{1}{2}^m$	$23^h 21\frac{1}{2}^m$
October	991	989	985	983	983	983	985	985	985	984	983	984
Nov.	991	989	987	986	984	985	986	986	987	987	988	988
Dec.	999	998	996	993	991	990	992	995	995	995	997	998
1844.	$0^h 21\frac{1}{2}^m$	$1^h 21\frac{1}{2}^m$	$2^h 21\frac{1}{2}^m$	$3^h 21\frac{1}{2}^m$	$4^h 21\frac{1}{2}^m$	$5^h 21\frac{1}{2}^m$	$6^h 21\frac{1}{2}^m$	$7^h 21\frac{1}{2}^m$	$8^h 21\frac{1}{2}^m$	$9^h 21\frac{1}{2}^m$	$10^h 21\frac{1}{2}^m$	$11^h 21\frac{1}{2}^m$
January	1007	1005	1004	1002	1000	1000	999	999	1002	1005	1008	1011
Feb.	1031	1031	1031	1029	1026	1026	1026	1028	1029	1030	1034	1036
March	1051	1049	1048	1047	1046	1045	1046	1047	1052	1059	1061	1063
April	1070	1069	1068	1065	1062	1062	1065	1065	1070	1078	1082	1082
May	1065	1065	1063	1062	1062	1061	1061	1064	1068	1074	1075	1070
June	1078	1077	1076	1077	1077	1075	1073	1077	1080	1082	1084	1081
July	1102	1103	1105	1106	1106	1105	1104	1104	1109	1116	1118	1114
August	1133	1134	1134	1134	1133	1131	1130	1135	1143	1152	1153	1147
Sept.	1102	1102	1102	1103	1099	1101	1100	1107	1117	1123	1127	1123
October	1136	1132	1131	1127	1126	1128	1129	1134	1141	1147	1150	1145
Nov.	1132	1131	1129	1128	1127	1123	1124	1125	1130	1137	1143	1145
Dec.	1205	1203	1200	1198	1196	1194	1190	1193	1194	1198	1209	1217
	$12^h 21\frac{1}{2}^m$	$13^h 21\frac{1}{2}^m$	$14^h 21\frac{1}{2}^m$	$15^h 21\frac{1}{2}^m$	$16^h 21\frac{1}{2}^m$	$17^h 21\frac{1}{2}^m$	$18^h 21\frac{1}{2}^m$	$19^h 21\frac{1}{2}^m$	$20^h 21\frac{1}{2}^m$	$21^h 21\frac{1}{2}^m$	$22^h 21\frac{1}{2}^m$	$23^h 21\frac{1}{2}^m$
January	1009	1006	1003	999	998	1000	1002	1003	1003	1004	1005	1007
Feb.	1035	1032	1028	1028	1032	1031	1032	1033	1034	1033	1034	1033
March	1068	1064	1057	1050	1053	1055	1055	1054	1052	1053	1053	1052
April	1077	1072	1066	1062	1064	1062	1068	1070	1071	1072	1069	1072
May	1064	1057	1053	1053	1051	1054	1059	1063	1064	1064	1063	1063
June	1077	1072	1067	1065	1065	1067	1071	1073	1075	1077	1077	1078
July	1106	1100	1096	1093	1092	1093	1096	1099	1101	1102	1103	1104
August	1138	1129	1121	1119	1121	1127	1134	1135	1136	1135	1136	1135
Sept.	1113	1102	1096	1094	1095	1099	1102	1104	1105	1105	1106	1110
October	1143	1138	1132	1133	1132	1136	1137	1137	1139	1136	1137	1134
Nov.	1142	1141	1135	1133	1134	1134	1134	1139	1137	1134	1131	1135
Dec.	1217	1212	1207	1202	1197	1198	1199	1199	1199	1203	1203	1203
1845.	$0^h 21\frac{1}{2}^m$	$1^h 21\frac{1}{2}^m$	$2^h 21\frac{1}{2}^m$	$3^h 21\frac{1}{2}^m$	$4^h 21\frac{1}{2}^m$	$5^h 21\frac{1}{2}^m$	$6^h 21\frac{1}{2}^m$	$7^h 21\frac{1}{2}^m$	$8^h 21\frac{1}{2}^m$	$9^h 21\frac{1}{2}^m$	$10^h 21\frac{1}{2}^m$	$11^h 21\frac{1}{2}^m$
January	1234	1231	1232	1230	1228	1226	1225	1227	1231	1239	1245	1249
Feb.	1231	1233	1231	1229	1229	1226	1223	1227	1233	1237	1245	1248
March	1236	1236	1234	1235	1234	1234	1230	1233	1241	1249	1255	1261
April	1255	1252	1251	1249	1247	1245	1243	1249	1257	1272	1282	1281
May	1244	1243	1241	1240	1236	1233	1230	1237	1249	1259	1260	1258
June	1281	1281	1281	1280	1275	1274	1269	1273	1280	1291	1297	1296
	$12^h 21\frac{1}{2}^m$	$13^h 21\frac{1}{2}^m$	$14^h 21\frac{1}{2}^m$	$15^h 21\frac{1}{2}^m$	$16^h 21\frac{1}{2}^m$	$17^h 21\frac{1}{2}^m$	$18^h 21\frac{1}{2}^m$	$19^h 21\frac{1}{2}^m$	$20^h 21\frac{1}{2}^m$	$21^h 21\frac{1}{2}^m$	$22^h 21\frac{1}{2}^m$	$23^h 21\frac{1}{2}^m$
January	1246	1242	1239	1236	1234	1237	1238	1234	1233	1232	1232	1230
Feb.	1250	1246	1239	1235	1234	1232	1233	1235	1235	1231	1231	1232
March	1260	1253	1245	1239	1240	1242	1244	1241	1240	1237	1240	1239
April	1273	1269	1257	1255	1251	1253	1256	1259	1259	1256	1253	1254
May	1251	1243	1237	1237	1237	1241	1246	1246	1246	1248	1246	1240
June	1289	1285	1276	1274	1273	1276	1281	1285	1284	1283	1282	1282

TABLE V.—MONTHLY MEANS OF THE PRECEDING BIFILAR READINGS REFERRED TO A UNIFORM TEMPERATURE AND CORRECTED FOR IRREGULARITY IN THE PROGRESSIVE CHANGE.

The column 1840–41 contains a double set of figures, the first are the monthly means directly obtained from Table IV, the second contains the means when the series is made continuous for the two breaks already noticed. The mean difference between May and June (from four years) is 25 scale divisions, and between December and January it is 22 scale divisions; these corrections were applied in the second set of figures.

	1840–1841.		1841–1842.	1842–1843.	1843–1844.	1844–1845.	Monthly Means of Series.
	Div's.	Div's.	Div's.	Div's.	Div's.	Div's.	
July	87	215	444	692	929	1103	677
August	128	256	495	700	934	1134	704
September	162	290	529	728	973	1106	725
October	154	282	529	765	984	1136	739
November	155	283	524	779	986	1134	741
December	196	324	548	781	993	1201	769
January	297—19	346	561	813	1003	1234	791
February	278	346	583	814	1031	1234	802
March	278	346	566	831	1053	1242	808
April	282	350	608	866	1069	1257	830
May	308	376	614	864	1063	1244	832
June	393+8	401	648	880	1075	1281	857
Annual Means		318	554	793	1008	1192	773

The differences in the successive annual means indicate that the progressive change may be assumed to have been uniform from year to year, and applying the usual method we find an annual progressive change of 220 scale divisions.

Introduction of the Horizontal Intensity in absolute measure and separation of the effect of the loss of Magnetism of the Bifilar bar from the effect due to the secular change of the Horizontal Intensity.—Although some experiments were made to determine the gradual loss of magnetism of the bar, as, for instance, in January, 1841, when the amount was found to be 0.9601 of the force in May, 1840, and again in June, 1841, when the amount was 0.9686 of its amount in January, 1841, yet the experiments do not extend over the whole period of observation, and consequently we are obliged to deduce the effect of the secular change of the horizontal intensity from other independent means, and, after converting it into scale divisions, we can assign the proper proportion of what is due to secular change and to loss of magnetism, in the whole progressive change of 220 scale divisions in a year.

In connection with the operations of the U. S. Coast Survey, Assistant Schott has investigated[1] the secular change of the horizontal intensity at a number of stations on the Atlantic and Pacific coasts. At several stations the results were subsequently improved by a discussion of my observations for intensity, made in part in connection with a magnetic survey of Pennsylvania, and also extending into adjoining States, and, in one of the journeys, into Canada. From the complete material the values in the following table of observed horizontal and total intensities have been collected. The horizontal intensity X and the total intensity ϕ are expressed in absolute measure (grains and feet).

[1] Report to Superintendent, dated January 19, 1861.

No.	Year.	Observer.	Reference from which the values were derived or taken.	X.	φ.
1	1835.0	Bache and Courtenay.	Trans. Amer. Phil. Soc., Vol. V, 1837.	4.195	13.58
2	1836.7	Bache.		4.159	13.46
3	1839.5	Loomis.	Trans. Amer. Phil. Soc., Vol. VIII.	4.149	13.41
4	1840.9	Bache.		......	13.41
5	1841.5	Locke.	Phil. Trans. Roy. Soc., 1846.	4.172	13.51
6	1841.8	Bache.		......	13.46
7	1842.5	Locke.	Phil. Trans. Roy. Soc., 1846.	4.174	13.52
8	1842.8	Lefroy.	" " " " "	4.176	13.50
9	1843.6	Bache.		4.172	13.46
10	1844.5	Locke.	Phil. Trans. Roy. Soc., 1846.	4.162	13.47
11	1846.4	Locke.	U. S. Coast Survey Records.	4.143	13.42
12	1855.7	Schott.	" " " "	4.226	13.89
[1]13	1862.6	Schott.	" " " "	4.088	13.30

The first three observations were not made at the Girard College grounds; and it appears from Prof. Loomis' observation when compared with Dr. Locke's, that a correction of 0.023 in the value of X should be added to these; to the twelfth observation I have assigned only half weight; it was probably made during a disturbance. From the general discussion an annual diminution in the horizontal force of 0.0011 parts was deduced for a number of stations on the Atlantic coast. At Toronto (vol. III of General Sabine's Discussion) the annual decrease was found 0.0010 in parts of the horizontal force. Being somewhat guided by these results, after several trials, the following combination of the results in the table has been adopted, as perhaps best representing the values for the time during which the Girard College observations were made, these latter being merely of a differential character:—

Combination.	Mean epoch.	Mean horiz'l int. X.
1, 2, 3	1837.1	4.191
5, 7, 8, 9	1842.6	4.174
10, 11, 12, 13	1852.3	4.145

The annual diminution of X is 0.0030, or, when expressed in parts of the horizontal force, = 0.0007; its equivalent in scale divisions is 19.2. The total annual change was found to be 220 scale divisions; hence, 200.8 scale divisions of annual change is due to loss of magnetism of the bar.

The mean epoch is 1844.0, and the corresponding mean $X = 4.170$; the mean epoch of the observation taken at the Girard College, is January, 1843, for which, therefore, the mean value of $X = 4.173$. This value has been adopted whenever it was desirable to introduce the horizontal force in absolute measure.

Separation of the Larger Disturbances.—The observations having been referred to a uniform temperature, and corrected for progressive change, Peirce's criterion was applied separately to each month. For this purpose, a systematic application was made extending over the whole series of observations, commencing with the hour 0 and the month of July, next with the hour 2 and August, followed by hour 4 and September, and so on in regular progression. This process eliminates from the result the diurnal variation and the annual variation of the disturbances themselves. The value for 0^h in July, 1840, was omitted as affected by two very large disturbances. The following table shows the limiting value of difference from the

[1] Added while this paper is passing through the press.

mean (the monthly mean for the respective hour), also the number of observations in each year subjected to the process:—

LIMITS OF REJECTION BY PEIRCE'S CRITERION.

		Div's.	
1840–41	$ex =$	53	$n =$ 241
1841–42	"	44	312
1842–43	"	37	309
1843–44	"	28	313
1844–45	"	33	313
Mean value		39	Sum 1488

The limiting value derived from nearly 1,500 observations is 39 scale divisions, and the separate annual values show plainly the effect of the eleven (ten?) year period, the year 1843–4 being a minimum year. Certain limits in the adoption of a separating value are allowable, and upon trial as to the actual number of disturbances separated, the value 33 scale divisions was finally adopted. Any observation differing 33 divisions or more from its respective monthly mean, was therefore marked and excluded from the mean. 33 divisions equal 0.0012 parts of the horizontal force, and in the value of the absolute scale it amounts to 0.005. At Toronto the limiting value was 14 divisions, = 0.0012 parts of the horizontal force, equal to 0.004 in the absolute scale. (Vol. III of the Toronto Obser's.)

TABLE VI.—SHOWS THE NUMBER OF OBSERVATIONS AND THE NUMBER OF THE LARGER DISTURBANCES SEPARATED BY THE VALUE 33, AS THE LIMIT, FOR EACH MONTH, YEAR, AND THE WHOLE PERIOD.

MONTH.	1840—1841.		1841—1842.		1842—1843.		1843—1844.		1844—1845.	
	Obser's.	Dist's.	Obser's.	Dist's.	Obser's.	Dist's.	Obser's.	Dist's.	Obser's.	Dist's.
July	323	165	323	26	308	24	312	15	648	0
August	308	73	312	17	321	3	324	11	648	4
September	312	54	310	41	308	44	312	16	600	27
October	323	68	308	28	310	53	624	3	648	32
November	293	49	312	32	312	15	624	1	624	42
December	321	120	323	26	323	5	624	0	624	46
January	201[1]	23	311	14	26[3]	0	646	3	648	27
February	288	50	287	37	24[4]	1	600	5	576	18
March	320	62	323	26	27[5]	1	624	29	624	3
April	309	48	309	38	300	14	624	16	624	33
May	310	46	300	29	324	25	648	3	648	19
June	225[2]	13	311	16	312	4	600	0	600	56
Sums	3533	770	3729	330	2895	189	6562	102	7512	307
Ratio	1 dist. in 4.6 ob's.		1 dist. in 11.3 ob's.		1 dist. in 15.3 ob's.		1 dist. in 64.3 ob's.		1 dist. in 24.4 ob's.	

Total number of observations 24,231
Total number of disturbances 1,698

The limiting value separated, therefore, one in every 14.3 observations. At Toronto one in every 12.5 was marked as a disturbance.

[1] In 17 days.
[2] In 19 days.
[3] One observation a day.
[4] One observation a day.
[5] One observation a day.

The larger disturbances having been excluded, new monthly means were taken, and the process was repeated several times, when required, until all readings differing 33 scale divisions or more had been excluded; the final means constitute the normals as given in the following table:—

TABLE VII.—MONTHLY NORMAL OF THE BI-HOURLY AND HOURLY READINGS OF THE BIFILAR MAGNETOMETER REDUCED TO A NORMAL TEMPERATURE AND CORRECTED FOR IRREGULARITY IN THE PROGRESSIVE CHANGE.

Philadelphia time (A. M.)							(P. M.)					
	$0^h 22^m$	$2^h 22^m$	$4^h 22^m$	$6^h 22^m$	$8^h 22^m$	$10^h 22^m$	$12^h 22^m$	$14^h 22^m$	$16^h 22^m$	$18^h 22^m$	$20^h 22^m$	$22^h 22^m$
1840.	Div's.	Div's.	Div's.	Div's.	Div's.	Div's.	Div's.	Div's.	Div's.	Div's.	Div's.	Div's.
July	113	97	89	50	112	116	94	59	52	92	93	108
August	108	112	117	106	138	153	134	103	111	114	121	126
September	155	147	139	141	180	202	177	155	153	158	157	150
October	142	137	122	138	153	166	159	158	148	146	148	151
November	155	150	144	133	144	154	175	157	151	148	144	160
December	196	188	176	166	178	208	217	193	182	185	200	194
1841.	$0^h 22^m$	$2^h 22^m$	$4^h 22^m$	$6^h 22^m$	$8^h 22^m$	$10^h 22^m$	$12^h 22^m$	$14^h 22^m$	$16^h 22^m$	$18^h 22^m$	$20^h 22^m$	$22^h 22^m$
January[1]	298	300	294	284	281	302	326	311	289	296	301	302
February	269	261	264	257	265	288	297	289	275	274	275	272
March	268	272	267	257	271	294	286	267	266	282	264	272
April	273	271	262	262	283	317	315	279	268	271	283	280
May	311	305	306	297	306	323	313	301	294	306	309	313
June[2]	392	390	392	386	400	401	395	382	385	392	402	392
July	442	442	435	435	447	458	449	428	430	444	448	439
August	490	494	487	482	501	518	502	483	483	497	500	495
September	510	514	515	508	531	542	537	516	519	520	515	515
October	521	517	518	514	526	537	547	530	525	527	529	528
November	519	517	515	509	518	529	531	514	518	513	516	518
December	546	541	538	535	537	548	562	549	545	547	550	552
1842.	$0^h 21\frac{1}{2}^m$	$2^h 22\frac{1}{2}^m$	$4^h 21\frac{1}{2}^m$	$6^h 21\frac{1}{2}^m$	$8^h 21\frac{1}{2}^m$	$10^h 21\frac{1}{2}^m$	$12^h 21\frac{1}{2}^m$	$14^h 21\frac{1}{2}^m$	$16^h 21\frac{1}{2}^m$	$18^h 21\frac{1}{2}^m$	$20^h 21\frac{1}{2}^m$	$22^h 21\frac{1}{2}^m$
January	561	556	555	558	553	573	577	559	554	564	563	564
February	580	573	572	567	568	582	589	578	578	580	590	578
March	565	559	557	554	563	574	575	561	565	571	567	566
April	595	598	597	594	604	620	618	603	598	607	608	611
May	614	610	611	605	621	630	622	606	607	615	618	619
June	649	652	646	638	649	659	650	639	638	649	648	650
July	692	686	682	678	695	708	700	680	677	690	694	700
August	699	694	695	692	711	721	700	688	689	701	703	702
September	726	733	722	717	739	750	737	730	727	737	737	734
October	764	759	757	757	764	781	783	776	776	768	769	764
November	774	770	771	768	777	789	787	781	778	775	776	776
December	780	777	775	773	776	790	795	786	773	776	781	782
1843.	$0^h 21\frac{1}{2}^m$	$2^h 21\frac{1}{2}^m$	$4^h 21\frac{1}{2}^m$	$6^h 21\frac{1}{2}^m$	$8^h 21\frac{1}{2}^m$	$10^h 21\frac{1}{2}^m$	$12^h 21\frac{1}{2}^m$	$14^h 21\frac{1}{2}^m$	$16^h 21\frac{1}{2}^m$	$18^h 21\frac{1}{2}^m$	$20^h 21\frac{1}{2}^m$	$22^h 21\frac{1}{2}^m$
January								818				
February								817				
March								829				
April	861	861	854	854	868	883	878	863	860	861	865	859
May	864	862	858	857	875	872	864	855	856	862	867	863
June	881	879	876	873	883	894	884	870	870	881	881	885
July	927	924	924	923	935	941	934	923	916	221	928	931
August	931	930	931	927	947	954	938	921	924	929	932	933
September	974	967	965	960	980	992	985	972	972	975	974	973

[1] The mean of 17 days. [2] The mean of 19 days.

Hourly Series.

	0h 21½m	1h 21½m	2h 21½m	3h 21½m	4h 21½m	5h 21½m	6h 21½m	7h 21½m	8h 21½m	9h 21½m	10h 21½m	11h 21½m
1843.	Div's.	Div's.	Div's.	Div's.	Div's.	Div's.	Div's.	Div's.	Div's.	Div's.	Div's.	Div's.
October	983	978	983	978	976	978	977	980	983	987	991	992
November	987	986	985	983	982	980	980	983	987	991	992	993
December	995	993	992	991	989	987	987	987	991	991	992	997
	12h 21½m	13h 21½m	14h 21½m	15h 21½m	16h 21½m	17h 21½m	18h 21½m	19h 21½m	20h 21½m	21h 21½m	22h 21½m	23h 21½m
October	991	989	985	983	983	983	985	985	985	986	983	984
November	991	989	987	986	984	985	986	986	987	987	988	988
December	999	998	996	993	991	990	992	995	995	995	997	998
1844.	0h 21½m	1h 21½m	2h 21½m	3h 21½m	4h 21½m	5h 21½m	6h 21½m	7h 21½m	8h 21½m	9h 21½m	10h 21½m	11h 21½m
January	1006	1005	1004	1002	1000	1000	999	999	1002	1005	1008	1011
February	1031	1031	1031	1029	1026	1026	1026	1028	1029	1030	1034	1034
March	1048	1047	1046	1046	1046	1043	1043	1047	1050	1054	1057	1063
April	1070	1069	1068	1065	1063	1062	1064	1061	1067	1074	1078	1078
May	1065	1065	1063	1062	1062	1061	1061	1064	1068	1076	1077	1070
June	1078	1077	1076	1077	1077	1075	1073	1077	1080	1082	1084	1081
July	1102	1103	1105	1106	1106	1105	1104	1104	1109	1116	1118	1114
August	1133	1134	1134	1133	1133	1131	1130	1135	1143	1152	1153	1147
September	1106	1104	1107	1105	1101	1101	1100	1107	1117	1125	1128	1125
October	1133	1132	1131	1127	1124	1125	1129	1134	1141	1149	1152	1145
November	1131	1130	1127	1126	1125	1123	1122	1125	1130	1135	1138	1142
December	1213	1202	1200	1198	1196	1194	1190	1193	1194	1197	1209	1217
	12h 21½m	13h 21½m	14h 21½m	15h 21½m	16h 21½m	17h 21½m	18h 21½m	19h 21½m	20h 21½m	21h 21½m	22 21½m	23 21½m
January	1009	1006	1003	999	998	1000	1002	1003	1003	1004	1004	1005
February	1035	1032	1028	1028	1030	1031	1032	1033	1034	1033	1032	1030
March	1063	1061	1057	1050	1051	1050	1050	1052	1050	1048	1050	1048
April	1077	1071	1066	1062	1064	1062	1068	1068	1071	1071	1068	1069
May	1064	1057	1053	1053	1051	1054	1059	1063	1064	1064	1065	1063
June	1077	1072	1067	1065	1065	1067	1071	1073	1075	1077	1077	1078
July	1106	1100	1096	1093	1092	1093	1096	1099	1101	1102	1103	1104
August	1138	1129	1121	1119	1121	1127	1134	1135	1135	1134	1135	1135
September	1115	1104	1097	1095	1095	1100	1102	1104	1104	1108	1107	1108
October	1145	1137	1134	1130	1132	1134	1135	1137	1138	1134	1137	1135
November	1136	1133	1129	1127	1124	1131	1128	1129	1130	1130	1131	1131
December	1220	1212	1209	1202	1201	1201	1203	1198	1200	1204	1206	1206
1845.	0h 21½m	1h 21½m	2h 21½m	3h 21½m	4h 21½m	5h 21½m	6h 21½m	7h 21½m	8h 21½m	9h 21½m	10h 21½m	11h 21½m
January	1233	1228	1231	1230	1228	1226	1225	1226	1231	1241	1248	1252
February	1230	1230	1231	1229	1229	1226	1223	1227	1231	1236	1243	1244
March	1236	1236	1234	1235	1234	1234	1231	1233	1241	1249	1255	1261
April	1252	1250	1249	1247	1245	1243	1241	1244	1253	1268	1278	1281
May	1244	1243	1241	1239	1236	1233	1229	1236	1251	1261	1262	1258
June	1280	1281	1281	1281	1275	1271	1266	1273	1282	1293	1295	1292
	12h 21½m	13h 21½m	14h 21½m	15h 21½m	16h 21½m	17h 21½m	18h 21½m	19h 21½m	20h 21½m	21h 21½m	22h 21½m	23h 21½m
January	1249	1242	1239	1233	1229	1230	1233	1231	1230	1230	1229	1229
February	1250	1242	1238	1231	1233	1229	1231	1233	1235	1231	1231	1232
March	1260	1253	1245	1239	1240	1242	1244	1240	1239	1237	1240	1239
April	1268	1267	1255	1252	1248	1253	1256	1254	1254	1253	1250	1254
May	1253	1244	1238	1236	1237	1239	1245	1246	1246	1248	1247	1242
June	1286	1280	1272	1269	1269	1273	1278	1281	1280	1277	1279	1280

Increase of scale readings corresponds to decrease of force. Value of one division of the scale = 0.0000365 parts of the horizontal force, or in the absolute scale equal to 0.0001523.

Investigation of the Eleven Year (also called Ten Year) Period, as shown in the Changes of the Amplitude of the Solar Diurnal Variation of the Horizontal Force.— The variation in the amplitude of the diurnal motion of the horizontal force is subject to the same inequality of about eleven years as the declination, and the means of investigation will be analogous to those used in Part I of this discussion. For greater convenience, the preceding monthly normals were united into annual means and the results put into an analytical form, using Bessel's function applicable to periodical phenomena, and determining the numerical quantity by the application of the method of least squares.

In the following table of the regular solar diurnal variation of the horizontal force the means for 1842–43 depend only on nine months of observation; the correction given to refer them to twelve months of observation depends on the mean difference between the results of the same nine months and twelve months of the preceding and following year; this correction is nearly constant and the same within one scale division for the adjacent years. In the second corrected column for 1842–43 the effect of the annual inequality is thus eliminated. In the year 1843–44 the results from nine months of observation at the odd hours were reduced to twelve months by means of corresponding differences in the series of even hours; thus (omitting the minutes) at hour 2, mean of 12 months = 1006, mean of 9 months = 1028; at hour 3 for the same 9 months, mean = 1026, or 2 divisions less; at hour 3 for 12 months the mean is therefore 1004, and the same result is found by comparing with the following hour 4; the mean is given in case of a difference in the two results.

TABLE VIII.—REGULAR SOLAR DIURNAL VARIATION OF THE HORIZONTAL FORCE FOR EACH YEAR OF OBSERVATION EXPRESSED IN SCALE DIVISIONS.

Increased numbers indicate decrease of force. The minutes at the head of each column are to be added to the hours given in the first vertical column. Each year commences with the month of July.

Hour of the day.	1840–41. 22^m	1841–42. $21\frac{3}{4}^m$	1842–43 (9 m'ths). $21\frac{1}{2}^m$	Correction.	1842–43. $21\frac{1}{2}^m$	1843–44. $21\frac{1}{2}^m$	1844–45. $21\frac{1}{2}^m$
	Div's.	Div's.	Div's.	Div's.	Div's.	Div's.	Div's.
0 (A. M.)	223	549	782	+6	788	1008	1191
1						1007	1189
2	219	548	780	+6	786	1006	1189
3						1004	1188
4	214	545	777	+6	783	1003	1186
5						1002	1184
6	206	542	774	+6	780	1002	1182
7						1005	1186
8	226	552	788	+5	793	1010	1194
9						1013	1202
10	244	564	799	+5	804	1017	1206
11						1016	1207
12 (P. M.)	241	563	792	+6	798	1014	1202
13						1010	1195
14	221	547	781	+7	788	1005	1189
15						1002	1186
16	215	547	778	+7	785	1002	1185
17						1004	1188
18	222	553	783	+7	790	1006	1190
19						1008	1191
20	225	554	786	+7	793	1008	1191
21						1009	1191
22	227	553	785	+6	791	1008	1191
23						1008	1191
Mean.	223.5	551.5			789.9	1007.4	1191.4

(Philadelphia local time, counted from midnight to midnight, 24 hours.)

The preceding mean diurnal variations were put in the following analytical form, in which the angle θ counts from midnight at the rate of 15° an hour.

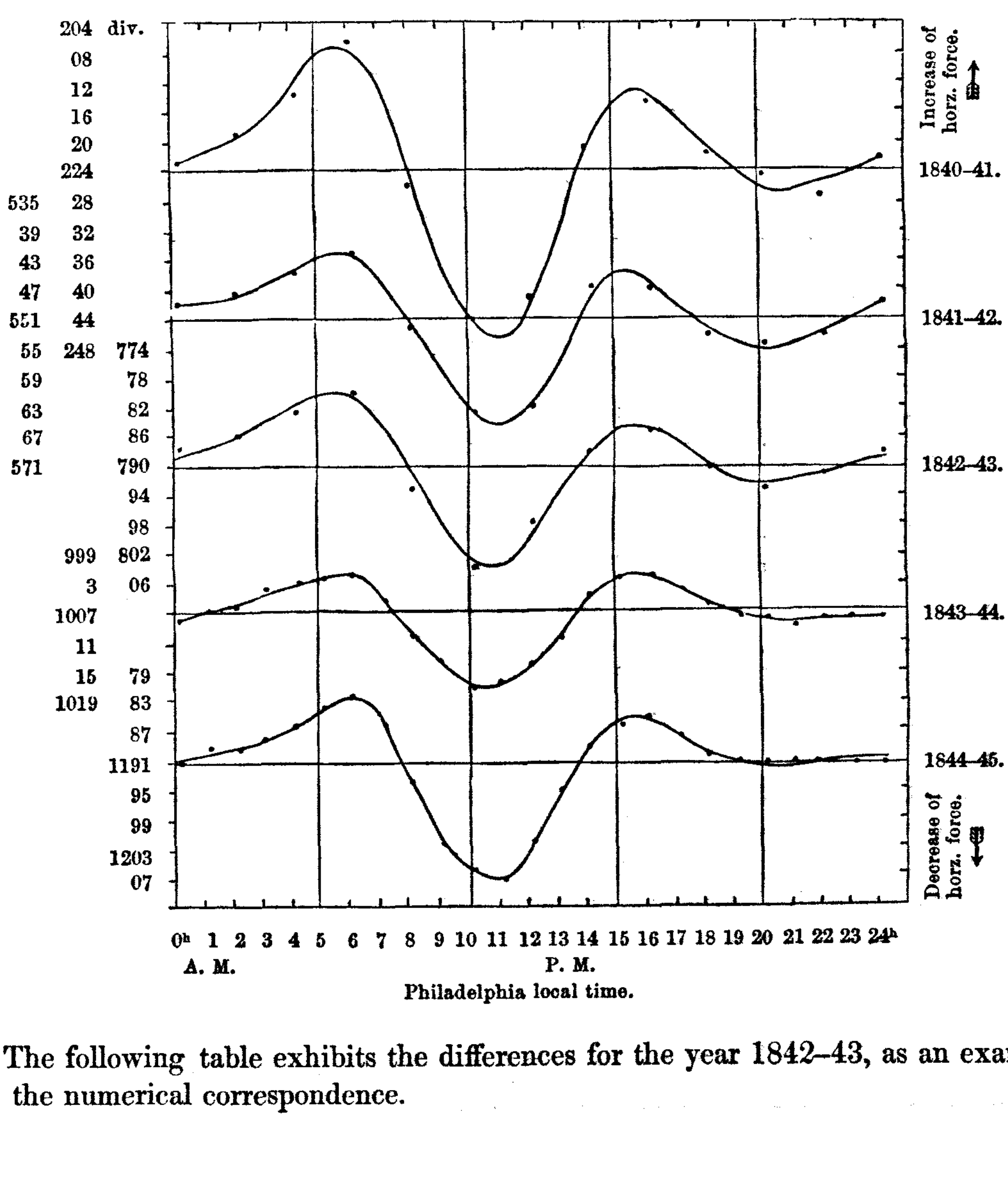
Year 1840–41	H =	$223^d.5 + 5^d.98 \sin(\theta + 252°\ 14') + 11^d.68 \sin(2\theta + 121°\ 16') + 5^d.89 \sin(3\theta + 314°\ 42')$
" 1841–42	H =	$551.5 + 4.03 \sin(\theta + 244\ 07) + 6.58 \sin(2\theta + 131\ 32) + 4.48 \sin(3\theta + 312\ 19)$
" 1842–43	H =	$789.9 + 4.14 \sin(\theta + 250\ 06) + 7.07 \sin(2\theta + 132\ 24) + 3.74 \sin(3\theta + 323\ 06)$
" 1843–44	H =	$1007.4 + 2.14 \sin(\theta + 273\ 55) + 5.09 \sin(2\theta + 128\ 58) + 2.35 \sin(3\theta + 317\ 58)$
" 1844–45	H =	$1191.4 + 4.40 \sin(\theta + 271\ 13) + 6.86 \sin(2\theta + 123\ 25) + 4.11 \sin(3\theta + 321\ 26)$

To show the degree of correspondence in the formulæ when deduced from the observations of the even and odd hours separately, the results for the last year have been added, viz:—

Even hours	H =	$1191^d.3 + 4^d.20 \sin(\theta + 271°\ 28') + 6^d.98 \sin(2\theta + 122°\ 36') + 4^d.11 \sin(3\theta + 322°\ 35')$
Odd hours	H =	$1191.5 + 4.60 \sin(\theta + 270\ 59) + 6.73 \sin(2\theta + 124\ 13) + 4.12 \sin(3\theta + 320\ 17)$

The close agreement between the observed and computed values is shown generally in the annexed diagram.

(A).—Inequality in the Diurnal Variation of the Horizontal Intensity.



The following table exhibits the differences for the year 1842–43, as an example of the numerical correspondence.

A. M.	Computed.	Observed.	C—O.	P. M.	Computed.	Observed.	C—O.
$0^m\ 21\frac{1}{2}^h$	788.7	788	+0.7	$12^m\ 21\frac{1}{2}^h$	799.5	798	+1.5
2 "	786.6	786	+0.6	14 "	787.6	788	—0.4
4 "	781.3	783	—1.7	16 "	784.5	785	—0.5
6 "	781.2	780	+1.2	18 "	790.2	790	+0.2
8 "	792.5	793	—0.5	20 "	792.9	793	—0.1
10 "	803.3	804	—0.7	22 "	720.5	791	—0 5

The differences, using three terms in the equations, are within the uncertainty of the observed values. The probable error of a single representation is $\pm$ 0.6 scale divisions, or $\pm$ 0.00009 in the absolute scale.

The curves show a double progression in the daily motion, with a principal maximum of horizontal force in the morning, a principal minimum before noon, and a secondary maximum in the afternoon; the precise epochs (to the nearest five minutes) and extreme values were computed by means of the preceding formulæ.

Year. From July to July.	Principal A. M. Maximum of hor. force.		Principal A. M. minimum of hor. force.		Diurnal range in			Secondary P. M. maximum of hor. force.		Less than A. M. max. by div's.
	Epoch.	Amount. Div's.	Epoch.	Amount. Div's.	Scale div's.	Parts of horizontal force.	Value in absol. scale.	Epochs.	Amount. Div's.	
1840–41	$5^h\ 45^m$	207.3	$11^h\ 0^m$	246.1	38.8	0.00142	0.0059	$4^h\ 05^m$	213.5	6.2
1841–42	5 50	541.7	11 5	565.5	23.8	0.00087	0.0036	3 50	545.1	3.4
1842–43	5 30	779.8	10 55	803.9	24.1	0.00088	0.0037	3 50	784.0	4.2
1843–44	5 40	1001.7	10 50	1016.9	15.2	0.00055	0.0023	4 0	1002.0	0.3
1844–45	5 40	1182.4	10 50	1206.6	24.2	0.00088	0.0037	4 0	1184.8	2.4
Mean	5 41		10 56				0.0038	3 57		

The secondary maximum is reached about $8^h\ 30^m$ P. M. with a comparatively small range.

The mean value of the force is attained about $7^h\ 55^m$ A. M., and again about $1^h\ 55^m$ P. M., with considerable regularity; it is again reached at $6\frac{3}{4}^h$ and $11\frac{1}{2}^h$ P. M., though with less regularity.

At Toronto (see Vol. II. of the Toronto Observations) the diurnal variation of the horizontal force has a principal maximum at a little after 4 P. M., and a principal minimum at 10 or 11 A. M.; the secondary maximum occurs about 6 A. M. There is, therefore, this specific difference in the diurnal motion at these two stations: in that at Philadelphia the morning maximum is the higher of the two, while at Toronto it is the afternoon maximum. The difference between the two maxima, as shown above, is almost nothing in the minimum year 1843–44, but increases before (and after) this epoch in proportion to the interval. At Toronto the daily range seems to be slightly greater. The secondary minimum at Toronto occurs about 2 or 3 A. M., or about six hours later than at Philadelphia; this is a second though less significant point of difference.

The minimum daily range occurs in 1843–44; its value is then less than one-half what it was in 1840–41.

The following equation expresses the mean diurnal range in scale divisions:—

$$R = + 19.68 - 3.78\,(t - 1843) + 2.77\,(t - 1843)^2.$$

It represents the observed values as follows:—

	Observed range.	Computed range.
January, 1841	38.8	38.3
" 1842	23.8	26.2
" 1843	24.1	19.7
" 1844	15.2	18.7
" 1845	24.2	23.2

The minimum range as given by the formula is in September, 1843. In Part I. of the discussion we found the minimum range of the declination in May, 1843, and the minimum from the disturbances of the declination in August, 1843.

Before proceeding to the discussion of the disturbances in the horizontal force, the formulæ given for the diurnal variation require to be put in a different form for future use and for convenience of comparison with other places.

The scale divisions were multiplied by the value of one division of the scale (0.0000365), and again by the value of X found for the year; the numerical constant was replaced by X and the angular quantities were changed by 180° so as to make increasing numbers correspond to increase of force; we then obtain in absolute measure the following expressions for the regular solar-diurnal variation of the horizontal force at the Girard College:—

Year 1840–41 $H = 4.178 + 0.00091 \sin(\theta + 72°\,14') + 0.00178 \sin(2\theta + 301°\,16') + 0.00090 \sin(3\theta + 134°\,42')$
" 1841–42 $H = 4.175 + 0.00061 \sin(\theta + 64\;\;07\;) + 0.00100 \sin(2\theta + 311\;\;32\;) + 0.00069 \sin(3\theta + 132\;\;19\;)$
" 1842–43 $H = 4.173 + 0.00063 \sin(\theta + 70\;\;06\;) + 0.00108 \sin(2\theta + 312\;\;24\;) + 0.00057 \sin(3\theta + 143\;\;06\;)$
" 1843–44 $H = 4.170 + 0.00033 \sin(\theta + 93\;\;55\;) + 0.00078 \sin(2\theta + 308\;\;58\;) + 0.00036 \sin(3\theta + 137\;\;58\;)$
" 1844–45 $H = 4.168 + 0.00067 \sin(\theta + 91\;\;13\;) + 0.00104 \sin(2\theta + 303\;\;25\;) + 0.00063 \sin(3\theta + 141\;\;26\;)$

The angle θ counts from midnight; the middle epoch to which each equation refers is January.

Investigation of the Eleven (Ten?) Year Inequality in the Disturbances of the Horizontal Magnetic Force.—In Table VI. the number of disturbances in each month has been given as found from the observations; these numbers are, however, not directly comparable with one another, first, on account of some omissions in the record, and secondly, on account of the change from a bi-hourly to an hourly series. For any incomplete month the number of disturbances for the whole month is obtained by simple proportion from the number during the part of the month recorded; for January, 1841, the total number becomes 35, for June, 1841 the total number is 18. For January, February, and March, 1843, the mean total number of the disturbances, as found in the same months in the preceding and following year, was substituted; this mean gave 8, 20, and 20, respectively. The number of disturbances after October, 1843, were halved to make them comparable with the bi-hourly series. There were two anomalous months, July and December, 1840, in which the disturbances amount to 165 and 120, with an annual mean of 64, whereas in the same months in the following year they only amount to 26 and 26 respectively, with an annual mean of 27; the mean annual difference 37 was applied to the numbers found in 1841, which give 63 and 63 as a substitute for the anomalous values in July and December, 1840. This anomaly does not exist in the phenomenon itself, but is unquestionably due to the irregularity in the progressive change.

Table IX. contains the number of disturbances as distributed over the several years and months, all referred to a uniform series of bi-hourly observations. To this table the monthly means and their ratio, when compared with the annual mean, have been added; also, for comparison, the corresponding ratios found in Part I. of the discussion of the disturbances of the declination.

MONTH.	1840–41.	1841–42.	1842–43.	1843–44.	1844–45.	Mean.	Hor. force. Ratio.	Declination. Ratio.
July	(63)	26	24	15	0	26	1.09	0.86
August	73	17	3	11	2	21	0.89	1.59
September	54	41	44	16	13	34	1.43*	1.36
October	68	28	53	2	16	33	1.39	2.12*
November	49	32	15	0	21	24	1.00	1.08
December	(63)	26	5	0	23	23	0.97	1.00
January	35	14	8	1	13	14	0.59	0.77
February	50	37	20	3	9	24	1.00	0.52
March	61	25	20	14	2	25	1.06	0.68
April	48	38	14	8	16	25	1.06	0.91
May	46	30	25	2	10	23	0.97	0.58
June	18	16	4	0	28	13	0.55*	0.53*
Sums	628	330	235	72	153	285	12.00	12.00
Mean	52	28	20	6	13	24		

In the columns of ratios the principal maxima and minima are indicated by an asterisk.

The annual means exhibit plainly the eleven year inequality; they have been represented by the formula:—

$$N = + 14.4 - 10.2\,(t - 1843) + 4.8\,(t - 1843)^2.$$

	Observed N.	Computed N.
January, 1841	52	54
" 1842	28	29
" 1843	20	14
" 1844	6	9
" 1845	13	13

According to the formula, the minimum occurs in January, 1844.

We have next to consider the eleven year inequality in the magnitude of the disturbances of the horizontal force. Table X. contains the aggregate amount of the disturbances expressed in scale divisions, and also their mean amount obtained by application of the number of disturbances already given in Table VI.

For reasons already explained, the amount of disturbances in July, 1840, equal to 10761 scale divisions, has been diminished in the ratio of 165 : 63. The ratio of each monthly mean to the mean amount of the year is also given, together with a column of corresponding ratios derived from the disturbances of the declination, as made out in Part I. of the discussion.

Table X.—Aggregate and Mean Amount of the Disturbances of the Horizontal Force. Expressed in Scale Divisions.

Month.	1840–41.	1841–42.	1842–43.	1843–44.	1844–45.	Mean Amount.	Hor. force. Ratio.	Declination. Ratio.
July	(4089)	1157	1295	669	0	56	1.10	0.87
August	4084	755	131	471	142	52	1.03	1.61
September	3092	3075	2099	660	1228	56	1.11*	1.56
October	3720	1284	2399	169	1412	49	0.97	2.06*
November	2390	1991	915	34	2173	54	1.06	1.06
December	6515	1225	239	0	2283	52	1.03	1.00
January	1186	601	0	111	1402	49	0.97	0.72
February	2664	1822	44	200	806	50	0.99	0.54
March	3112	1176	39	1412	127	49	0.97	0.66
April	2138	2075	676	861	1604	49	0.97	0.94
May	2456	1211	1187	131	789	47	0.93	0.56
June	560	794	164	0	2390	44	0.87*	0.42*
Mean amount	53.9	52.0	48.6	46.3	46.8	50.6	1.00	1.00

Maxima and minima in the columns of ratios are marked with an asterisk.

The inequality in the mean amount of the horizontal force disturbances in each year, indicates the year 1843-44 as the minimum year.

From the preceding results, we may assume the month of November, 1843, as the epoch for the minimum of the eleven (ten?) year inequality, as far as indicated by the differential observations of the horizontal force.

Further Analysis of the Disturbances of the Horizontal Force.—The distribution of the disturbances in number and mean amount over the several months of the year has been given in Tables IX. and X. From Table IX. we learn that the disturbances are greatest in number in September and March or April, or about the time of the equinoxes, and least in number about January and June, or about the time of the solstices. At the autumnal equinox the numbers exceed those of the vernal equinox; the same law was found at Toronto; also the numbers are smaller at the summer solstice than at the winter solstice, in perfect accordance with the result found at Toronto. These results are shown graphically on the annexed diagram, which contains also the ratio of the disturbances for the declination in which the same law is apparent.

(B).—Distribution of the Number of Disturbances in the several Months of the Year.

Full line for horizontal force. Dotted line for declination.

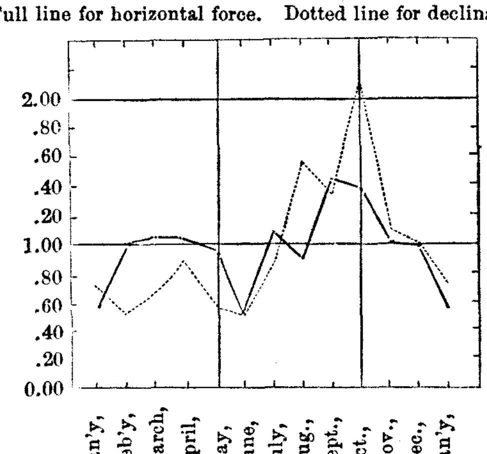


Table X. shows that, in reference to the average magnitude of the disturbances, the same law holds good, viz: the greatest relative magnitude occurring about the time of the equinoxes; the greatest amount corresponding to the autumnal equinox, and the least to about the time of the solstices, the smaller amount occurring near the summer solstice. The average magnitude of the disturbances of the declination was found subject to the same law.

If we separate the disturbances which increase the force from those which decrease it, we may form the two following tables of the distribution of the disturbances in number and average amount over the several months of the years.

TABLE XI.—ANNUAL INEQUALITY IN THE NUMBER OF DISTURBANCES, INCREASING AND DECREASING THE HORIZONTAL FORCE.

	1840–41.		1841–42.		1842–43.		1843–44.		1844–45.		Sum.		Ratios.	
	Inc.	Dec.	Inc.	Dec.	Inc.	Dec.	Inc.	Dec.	Inc.	Dec.	Inc.	Dec.	Inc.	Dec.
July	(38)	(25)	6	20	5	19	1	14	0	0	50	78	1.2	1.0
August	18	55	6	11	1	2	2	9	0	2	27	79	0.7	1.0
September	25	29	5	36	38	6	11	5	9	4	88	80	2.1*	1.1
October	18	50	11	17	37	16	1	1	8	8	75	92	1.8	1.2
November	13	36	1	31	4	11	0	0	0	21	18	99	0.4	1.3*
December	(25)	(38)	8	18	0	5	0	0	15	8	48	69	1.1	0.9
January	19	16	6	8	3	5	0	1	3	10	31	40	0.8	0.6
February	15	35	4	33	2	18	0	3	0	9	21	98	0.5	1.2
March	17	44	10	16	3	17	0	14	1	1	31	92	0.8	1.2
April	18	30	14	24	1	13	1	7	0	16	34	90	0.8	1.2
May	24	22	16	13	10	15	1	1	5	5	56	56	1.3	0.7
June	9	9	6	10	1	3	0	0	7	21	23	43	0.5*	0.6*
Sum	239	389	93	237	105	130	17	55	48	105	502	916	12.0	12.0

In each year the number of disturbances increasing the force is less than the number which decreases it; the numbers of increase are to the numbers of decrease as 1 : 1.8. The numbers of the monthly ratio for the increasing disturbances exhibit the same law as found in Table IX.: with respect to the numbers for the decreasing force the law is apparently less distinctly marked; the maximum seems to occur about two months later (before the winter solstice), at a time when the number for increasing force is apparently at its minimum. This indistinctness in the law may possibly be due to an irregular distribution in reference to the hours of the day, and could only disappear through a longer series of observations.

TABLE XII.—ANNUAL INEQUALITY IN THE MEAN AMOUNT OF THE DISTURBANCES OF THE HORIZONTAL FORCE. AGGREGATE AMOUNT FOR INCREASING AND DECREASING DISTURBANCES, EXPRESSED IN SCALE DIVISIONS.

Month.	1840–41.		1841–42.		1842–43.		1843–44.		1844–45.		1840–45.		Aver. am't.		Ratios.	
	Inc.	Dec.	Inc.	Dec.	Inc.	Dec.	Inc.	Dec.	Inc.	Dec.	Inc.	Dec.	Inc.	Dec.	Inc.	Dec
July	(2202)	(1887)	214	943	292	1003	41	628	0	0	2749	4461	55d	57d	1.2	1.1
August	794	3290	261	494	51	80	69	402	0	142	1175	4408	44	54	1.0	1.0
Sept.	1082	2010	186	2889	1857	242	452	208	873	355	4450	5704	45	56	1.0	1.1
October	726	2994	421	863	1685	714	128	41	691	721	3651	5333	44	53	1.0	1.0
Nov.	520	1870	35	1956	185	730	0	34	0	2173	740	6763	41	56	0.9	1.1
Dec.	2204	4311	289	936	0	239	0	0	1483	800	3976	6286	47	56	1.0	1.1
January	723	463	231	370	0	0	0	111	302	1100	1256	2044	48	50	1.1	0.8
Feb.	649	2015	140	1682	0	44	0	200	0	806	789	4747	42	52	1.0	1.0
March	643	2469	415	761	0	39	0	412	37	90	1095	4771	39	52	0.9	1.0
April	732	1406	550	1525	54	622	75	786	41	1563	1452	5902	40	52	0.9	1.0
May	1000	1456	696	515	412	775	83	48	398	391	2589	3185	42	52	1.0	1.0
June	307	253	284	510	50	114	0	0	604	1786	1245	2663	44	44	1.0	0.8
Sum	11582	24424	3722	13444	4586	4602	848	3870	4429	9927	25167	56267			12.0	12.0
Number	254	414	93	237	97	92	20	82	96	211	560	1036				
Mean	46	59	40	57	47	50	42	47	46	47	45	54				

The average amount of a disturbance increasing the horizontal force is 45 scale divisions, or 0.0069 in absolute measure; the average amount of a disturbance decreasing the same is 54 scale divisions, or 0.0082 in absolute value. The ratio of these numbers is as 1 : 1.2, whereas at Toronto the ratio is 1 : 6.4.

The law of the monthly inequality for amount of increasing or decreasing disturbances is, as in the preceding case, very indistinct and further obscured by the small absolute amount of variation.

In the following Table, XIII., the larger disturbances have been distributed over the different hours of their occurrence; in this combination the bi-hourly series (of the even hours) of observation has been used throughout.

Hour.	Aggregate amount in sc. div.	Number of occurrence.	Average amount.	Ratio of numbers.
0 (Midnight)	8116	142	57	1.12
2	5967	109	55	0.86
4	4961	93	53	0.73*
6	4751*	94	51	0.74
8	5562	104	53	0.83
10	7721*	146	53	1.15
12 (Noon)	6825	161	42	1.27*
14	6636	127	52	1.00
16	6634	135	49	1.07
18	6894	132	52	1.05
20	7574	139	55	1.09
22	7358	139	53	1.09

Directing our attention to the columns of aggregate amount and of ratios of number of occurrence, we find a principal maximum about 11 A. M., which seems to correspond to the *secondary* maximum of corresponding ratios at Toronto occurring about three hours earlier; the principal minimum occurs about 5 A. M., which corresponds to the *secondary* minimum at Toronto occurring between 5 and 6 A. M.; again, at Philadelphia, the secondary maximum at midnight is about two hours earlier than the *principal* maximum at Toronto, and the secondary minimum about

4 P. M. corresponds in time to the *principal* minimum at Toronto occurring between 2 and 6 P. M. Thus, the curves at the two stations, representing the diurnal variation of the disturbances (irrespective of increase or decrease) of the horizontal force, is double crested with an exchange of the principal and secondary maximum and also of the principal and secondary minimum.

In the next Table, XIV., the diurnal variation of the disturbances is exhibited separately for disturbances increasing and disturbances decreasing the horizontal force.

Hour.	Disturbances Increasing Horizontal Force.			Disturbances Decreasing Horizontal Force.			Excess of aggregate decrease over aggregate increase.
	Number of occurrences.	Aggregate amount.	Ratio.	Number of occurrences.	Aggregate amount.	Ratio.	
0 (Midn't)	57	2878	1.28	85	5238	1.21	2360
2	44	2173	0.97	65	3794	0.87	1621
4	42	1998	0.89	51	2963*	0.68	965
6	28	1213*	0.54	66	3538	0.81	2325
8	48	2345	1.04	56	32 7	0.74	872
10	61	2732	1.22	85	4989	1.15	2257
12 (Noon)	74	3134*	1.39	87	3691	0.85	557
14	48	2239	1.00	79	4397	1.01	2158
16	49	2200	0.98	86	4434	1.03	2234
18	45	2005	0.89	87	4889	1.13	2884
20	39	1758	0.78	100	5816*	1.34	4058
22	50	2296	1.02	89	5062	1.18	2766
Sums.	585	26971	12.00	936	52028	12.00	25057

The disturbances increasing and those decreasing the horizontal force evidently follow different laws; at Toronto they were found completely opposed; they are less so at Philadelphia. The principal maximum of increasing disturbances (at noon) seem to be contemporaneous with a secondary minimum of the decreasing disturbances; again the principal maximum of the decreasing disturbances (at 8 P. M.) corresponds to a secondary minimum of the increasing disturbances. In reference to the main feature, the maximum disturbance of those increasing the force and of those decreasing the force, the Philadelphia ratios show even a greater resemblance to the results at St. Helena and the Cape of Good Hope than to those at Toronto. At the two southern stations the maximum in the disturbances which increase occurs at 11 A. M. and the maximum in the disturbances which decrease occurs about 6 or 7 P. M. (See Vol. II. of the St. Helena Observations.)

Table XIV. contains also the hourly excess of the aggregate amount of the disturbances which decrease the horizontal force over those which increase the same. If we divide the numbers by the whole number of days of observation (nearly 1500) we obtain the diurnal disturbance variation expressed in scale divisions.

Table XV.—Diurnal Disturbance Variation.

Hour.	S. D.	In absolute measure.	Hour.	S. D.	In absolute measure.
0 (Midn't)	1.6	0.00024	12 (Noon)	0.4	0.00006
2	1.1	17	14	1.4	21
4	0.7	11	16	1.5	23
6	1.6	24	18	2.0	30
8	0.6	09	20	2.8	43
10	1.5	23	22	1.9	29

The average amount by which the disturbances tend to decrease the diurnal variation of the horizontal force is 1.4 scale divisions or 0.00021 in the absolute scale. The maximum effect takes place at 8 P. M., at exactly the same hour when the declination disturbances reach their greatest effect.

In the preceding Tables, XIII., XIV., and XV., to the hours indicated 21½ minutes should be added, the observations being made so much later than the even hours.

The preceding discussion shows that for two stations, even at a comparatively short distance, as for Philadelphia and Toronto, there are, generally speaking, some close coincidences in the laws derived from independent observations; but there are also certain differences in other results; yet it must not be forgotten that for a strict comparability we require, if not simultaneous observations, at least observations extending over similar parts or the whole of an eleven year period. The Philadelphia series includes a minimum year of that inequality, with the greater extent of observations before that epoch, whereas at Toronto the series begins after the minimum epoch and barely extends to a maximum year.

For the purpose of obtaining a better view of the absolute amount of the disturbances and their frequency of occurrence,[1] they were classified in nine groups of equal differences of 20 scale divisions; the number of disturbances in each was found as follows:—

Limits Adopted.			Number of disturbances.
In scale divisions.	In parts of horizontal force.	In the absolute scale.	
33 to 53	0.0012 to 0.0019	0.005 to 0.008	1159
53 " 73	19 " 27	08 " 11	348
73 " 93	27 " 34	11 " 14	93
93 " 113	34 " 41	14 " 17	45
113 " 133	41 " 48	17 " 20	27
133 " 153	48 " 55	20 " 23	14
153 " 173	55 " 62	23 " 26	4
173 " 193	62 " 70	26 " 29	6
193 " 213	0.0070 " 0.0077	0.029 " 0.032	2
Beyond.	....	....	0

The numbers in the last column cannot be considered as entirely independent of the eleven year period, and in attempting to apply the theory of probabilities in

[1] A table analogous to that given above, showing the distribution of the disturbances in *declination*, is here added for comparison:—

Limits Adopted.		Number of disturbances.
In scale divisions.	In minutes of arc.	
8 to 16	3'.6 to 7'.2	1856
16 " 24	7.2 " 10.8	333
24 " 32	10.8 " 14.4	105
32 " 40	14.4 " 18.1	42
40 " 48	18.1 " 21.7	16
48 " 56	21.7 " 25.3	2
56 " 64	25.3 " 29.0	2
64 " 72	29.0 " 32.6	1
Beyond	....	0

reference to the number of disturbances which ought to occur between the assigned limits, it became apparent that the larger disturbances greatly preponderate, a fact no doubt intimately connected with the difficulty in correctly allowing for the progressive change during the first year of observation.

PART V.

INVESTIGATION

OF THE

SOLAR-DIURNAL VARIATION AND OF THE ANNUAL INEQUALITY OF THE HORIZONTAL COMPONENT OF THE MAGNETIC FORCE.

INVESTIGATION

OF THE

SOLAR-DIURNAL VARIATION, AND OF THE ANNUAL INEQUALITY OF THE HORIZONTAL COMPONENT OF THE MAGNETIC FORCE.

THE discussion of the diurnal and annual variations of the horizontal force is based on the resulting monthly normal values for each observation hour as given in the preceding part (IV.), in which the horizontal force has been discussed in relation to the ten or eleven year period, and which also contains the investigation of the disturbances; in the same part all necessary statements are given relating to the instrumental data and the absolute values of the horizontal force.

The normals, as has been shown, are referred to a uniform standard temperature; they are corrected for irregularity in the progressive change, and are necessarily freed from all the larger disturbances. The use of the normals instead of the simple means of the readings (corrected for difference of temperature) will insure greater regularity in the variations of the horizontal force, now under consideration.

The diurnal variation requires an arrangement of the five year series of monthly normals according to the months of the year and hours of the day; in general, the method of interpolation for an occasional omission in either a month or hour, is the same as that used in Part II. of the discussion of the Girard College observations; there is, however, this difference in the tabulation of the monthly values, that in the present case the results are consolidated in a five years' arrangement, and in consequence the year commences with the month of July. This arrangement was preferred, particularly since it was found desirable to make no use of the observations in the first month of the series.

Tabulation of monthly normals for each observing hour and each observing year, beginning and ending with July. The individual values are taken from Table VII. of the preceding Part IV.

After applying the corrections of — 19 scale divisions to the normals for January, 1841, and of +8 scale divisions to those of June, 1841, to allow for defective number of observations in these months, a further correction of + 68 scale divisions was applied to all values between July, 1840, and May, 1841, inclusive, and of + 60 to all values between July, 1840, and December, 1840, inclusive, to allow for defects in the regularity of the progressive change, thus making the total correction for the latter months = 128 scale divisions. The above corrections, when divided by 5, in order to give the correction to the means derived from five years, become, therefore: for months between July and December inclusive, + 26; for January + 10; for February, March, April, and May + 14; for June + 2. These corrections are constant for each hour of the day in any one month, and consequently do not affect the diurnal variation; but they have nevertheless been applied at once to facilitate subsequent deductions. Their origin has also been explained in the remarks accompanying Table V. of the preceding part.

The following example of the process of interpolation for the odd hour values will suffice for all similar cases: Required the mean normal from the 5 year series for $5^h\ 21\frac{1}{2}^m$ A. M. in June (see tabular values and results below). The mean normals for the two last years at $4^h\ 21\frac{1}{2}^m$, $5^h\ 21\frac{1}{2}^m$, and $6^h\ 21\frac{1}{2}^m$, are 1176, 1173, and 1169 respectively; the mean at $5^h\ 21\frac{1}{2}^m$ is therefore 3 divisions less than the mean at $4^h\ 21\frac{1}{2}^m$, and since the mean of the 5 year series at $4^h\ 21\frac{1}{2}^m$ is 853, the result for $5^h\ 21\frac{1}{2}^m$ becomes 849; again, adding 4 divisions to 847, the mean at $6^h\ 21\frac{1}{2}^m$, we find 851; the mean of the two values, or 850, is that given in the table, to which + 2 has been added, making the final result 852. The means of the odd hours, thus found from the adjacent even hours, in general, do not differ by as much as a scale division.

The time given in the tables of the normals is mean local time, counting from midnight to midnight to twenty-four hours. The observations were taken (on the average) $21\frac{1}{2}$ minutes after the full hours, as indicated in the tables. Increase of scale readings indicates decrease of horizontal force; the value of a scale division equals 0.0000365 parts of the horizontal force, or 0.0001523 in absolute measure, the mean horizontal force being 4.173 (in absolute measure). Proper weights have been given to the normals of the even and odd hours, in proportion to the number of observations, as will be seen hereafter. Other special remarks will be found at the end of the month to which they refer.

Tabulation of the hourly normals for each month and the mean of the five year series, expressed in scale division readings and reduced to the standard temperature of 63° (Fahrenheit's scale), also corrected for all irregularities in the progressive change. The regular progressive and secular change, therefore, remains in the tabular quantities.

Normals of the Horizontal Force for July.

Year.	0^h	1^h	2^h	3^h	4^h	5^h	6^h	7^h	8^h	9^h	10^h	11^h	$+21\frac{1}{2}^m$
1840	113		97		89		50		112		116		
1841	442		442		435		435		447		458		
1842	692		686		682		678		695		708		
1843	927		924		924		923		935		941		
1844	1102	1103	1105	1106	1106	1105	1104	1104	1109	1116	1118	1114	
Mean	605		651		647		638		660		668		
Referred mean		653		649		642		647		666		664	
Constant correction + 26													
Normals	681	679	677	675	673	668	664	673	686	692	694	690	

Year.	12^h Noon.	13^h	14^h	15^h	16^h	17^h	18^h	19^h	20^h	21^h	22^h	23^h	$+21\frac{1}{2}^m$
1840	94		59		52		92		93		108		
1841	449		428		430		444		448		439		
1842	700		680		677		690		694		700		
1843	934		923		916		921		928		931		
1844	1106	1100	1096	1093	1092	1093	1096	1099	1101	1102	1103	1104	
Mean	657		637		633		649		653		656		
Referred mean		646		634		640		651		655		657	
Constant correction + 26													
Normals	683	672	663	660	659	666	675	677	679	681	682	683	

Monthly mean normal from the even hours ($+21\frac{1}{2}^m$) 676.3, weight 5.
" " " odd " " 676.3, " 1.

Normals of the Horizontal Force for August.

Year.	0^h	1^h	2^h	3^h	4^h	5^h	6^h	7^h	8^h	9^h	10^h	11^h	$+21\frac{1}{2}^m$
1840	108		112		117		106		138		153		
1841	490		494		487		482		501		518		
1842	699		694		695		692		711		721		
1843	931		930		931		927		947		954		
1844	1133	1134	1134	1133	1133	1131	1130	1135	1143	1152	1153	1147	
Mean	672		673		673		667		688		700		
Referred mean		673		672		669		676		698		692	
Constant correction + 26													
Normals	698	699	699	698	699	695	693	702	714	724	726	718	

Year.	12^h Noon.	13^h	14^h	15^h	16^h	17^h	18^h	19^h	20^h	21^h	22^h	23^h	$+21\frac{1}{2}^m$
1840	134		103		111		114		121		126		
1841	502		483		483		497		500		495		
1842	700		688		689		701		703		702		
1843	938		921		924		929		932		933		
1844	1138	1129	1121	1119	1121	1127	1134	1135	1135	1134	1135	1135	
Mean	682		663		666		675		678		678		
Referred mean		672		662		670		677		677		676	
Constant correction + 26													
Normals	708	698	689	688	692	696	701	703	704	703	704	702	

Monthly mean normal from the even hours ($+21\frac{1}{2}^m$) 702.2, weight 5.
" " " odd " " 702.2, " 1.

Normals of the Horizontal Force for September.

Year.	0^h	1^h	2^h	3^h	4^h	5^h	6^h	7^h	8^h	9^h	10^h	11^h	$+ 21\frac{1}{2}^m$
1840	155		147		139		141		180		202		
1841	510		514		515		508		531		542		
1842	726		733		722		717		739		750		
1843	974		967		965		960		980		992		
1844	1106	1104	1107	1105	1101	1101	1100	1107	1117	1125	1128	1125	
Mean	694		694		688		685		709		723		
Referred mean		692		692		687		695		718		720	
Constant correction + 26													
Normals	720	718	720	718	714	713	711	721	735	744	749	746	

Year.	12^h Noon.	13^h	14^h	15^h	16^h	17^h	18^h	19^h	20^h	21^h	22^h	23^h	$+ 21\frac{1}{2}^m$
1840	177		155		153		158		157		150		
1841	537		516		519		520		515		515		
1842	737		730		727		737		737		734		
1843	985		972		972		975		974		973		
1844	1115	1104	1097	1095	1095	1100	1102	1104	1104	1108	1107	1108	
Mean	710		694		693		698		697		696		
Referred mean		700		692		697		699		699		696	
Constant correction + 26													
Normals	736	726	720	718	719	723	724	725	723	725	722	722	

Monthly mean normal from the even hours ($+ 21\frac{1}{2}^m$) 724.4, weight 5.
" " " odd " " 724.9, " 1.

Normals of the Horizontal Force for October.

Year.	0^h	1^h	2^h	3^h	4^h	5^h	6^h	7^h	8^h	9^h	10^h	11^h	$+ 21\frac{1}{2}^m$
1840	142		137		122		138		153		166		
1841	521		517		518		514		526		537		
1842	764		759		757		757		764		781		
1843	983	978	983	978	976	978	977	980	983	787	991	992	
1844	1133	1132	1131	1127	1124	1125	1129	1134	1141	1149	1152	1145	
Mean	709		705		699		703		713		725		
Referred mean		705		701		702		708		720		724	
Constant correction + 26													
Normals	735	731	731	727	725	728	729	734	739	746	751	750	

Year.	12^h Noon.	13^h	14^h	15^h	16^h	17^h	18^h	19^h	20^h	21^h	22^h	23^h	$+ 21\frac{1}{2}^m$
1840	159		158		148		146		148		151		
1841	547		530		525		527		529		528		
1842	783		776		776		768		769		764		
1843	991	989	985	983	983	983	985	985	985	986	983	984	
1844	1145	1137	1134	1130	1132	1134	1135	1137	1138	1134	1137	1135	
Mean	725		717		713		712		714		713		
Referred mean		721		714		713		713		713		712	
Constant correction + 26													
Normals	751	747	743	740	739	739	738	739	740	739	739	738	

Monthly mean normal from the even hours ($+ 21\frac{1}{2}^m$) 738.3, weight 5.
" " " odd " " 738.2, " 2.

Normals of the Horizontal Force for November.

Year.	0^h	1^h	2^h	3^h	4^h	5^h	6^h	7^h	8^h	9^h	10^h	11^h	$+ 21\frac{1}{2}^m$
1840	155		150		144		133		144		154		
1841	519		517		515		509		518		529		
1842	774		770		771		768		777		789		
1843	987	986	985	983	982	980	980	983	987	991	992	993	
1844	1131	1130	1127	1126	1125	1123	1122	1125	1130	1135	1138	1142	
Mean	713		710		707		702		711		720		
Referred mean		712		708		704		706		717		725	
Constant correction + 26													
Normals	739	738	736	734	733	730	728	732	737	743	746	751	

Year.	12^h Noon.	13^h	14^h	15^h	16^h	17^h	18^h	19^h	20^h	21^h	22^h	23^h	$+ 21\frac{1}{2}^m$
1840	175		157		151		148		144		160		
1841	531		514		518		513		516		518		
1842	787		781		778		775		776		776		
1843	991	989	987	986	984	985	986	986	987	987	988	988	
1844	1136	1133	1129	1127	1124	1131	1128	1129	1130	1130	1131	1131	
Mean	724		714		711		710		711		715		
Referred mean		720		712		713		710		713		714	
Constant correction + 26													
Normals	750	746	740	738	737	739	736	736	737	739	741	740	

Monthly mean normal from the even hours ($+ 21\frac{1}{2}^m$) 738.3, weight 5.
" " " odd " " 738.8 " 2.

Normals of the Horizontal Force for December.

Year.	0^h	1^h	2^h	3^h	4^h	5^h	6^h	7^h	8^h	9^h	10^h	11^h	$+ 21\frac{1}{2}^m$
1840	196		188		176		166		178		208		
1841	546		541		538		535		537		548		
1842	780		777		775		773		776		790		
1843	995	993	992	991	989	987	987	987	991	991	992	997	
1844	1213	1202	1200	1198	1196	1194	1190	1193	1194	1197	1209	1217	
Mean	746		740		735		730		735		749		
Referred mean		741		738		733		732		740		757	
Constant correction + 26													
Normals	772	767	766	764	761	759	756	758	761	766	775	783	

Year.	12^h Noon.	13^h	14^h	15^h	16^h	17^h	18^h	19^h	20^h	21^h	22^h	23^h	$+ 21\frac{1}{2}^m$
1840	217		193		182		185		200		194		
1841	562		549		545		547		550		552		
1842	795		786		773		776		781		782		
1843	999	998	996	993	991	990	992	995	995	995	997	998	
1844	1220	1212	1209	1202	1201	1201	1203	1198	1200	1204	1206	1206	
Mean	759		747		738		741		745		746		
Referred mean		752		742		739		742		746		745	
Constant correction + 26													
Normals	785	778	773	768	764	765	767	768	771	772	772	771	

Monthly mean normal from the even hours ($+ 21\frac{1}{2}^m$) 768.6, weight 5.
" " " odd " " 768.2, " 2.

NORMALS OF THE HORIZONTAL FORCE FOR JANUARY.

Year.	0h	1h	2h	3h	4h	5h	6h	7h	8h	9h	10h	11h	+ 21½m
1841	298		300		294		284		281		302		
1842	561		556		555		558		553		573		
1843	(820)		(817)		(814)		(815)		(814)		(827)		
1844	1006	1005	1004	1002	1000	1000	999	999	1002	1005	1008	1011	
1845	1233	1228	1231	1230	1228	1226	1225	1226	1231	1241	1248	1252	
Mean	784		782		778		776		776		792		
Referred mean		782		780		777		774		785		798	
Constant correction + 10													
Normals	794	792	792	790	788	787	786	784	786	795	802	808	

Year.	12h Noon.	13h	14h	15h	16h	17h	18h	19h	20h	21h	22h	23h	+ 21½m
1841	326		311		289		296		301		302		
1842	577		559		554		564		563		564		
1843	(830)		818		(813)		(820)		(820)		(821)		
1844	1009	1006	1003	999	998	1000	1002	1003	1003	1004	1004	1005	
1845	1249	1242	1239	1233	1229	1230	1233	1231	1230	1230	1229	1229	
Mean	798		786		777		783		783		784		
Referred mean		791		780		780		784		784		786	
Constant correction + 10													
Normals	808	801	796	790	787	790	793	794	793	794	794	796	

Monthly mean normal from the even hours 793.3, weight 4.
" " " odd " 793.4, " 2.

The values for 1843 within brackets are interpolated by means of the continued readings at $14^h\ 21\frac{1}{2}^m$; at this hour the difference of reading from the preceding year is 259, which added to the values of 1842 gave resulting normals for 1843; in the same manner the reading in 1843 at $14^h\ 21\frac{1}{2}^m$ when compared with the reading in the following year (1844) leaves the difference 185, which quantity when subtracted from each hourly value in 1844 gives a second determination for the year 1843; the mean of the two determinations for each hour has been inserted above.

NORMALS OF THE HORIZONTAL FORCE FOR FEBRUARY.

Year.	0^h	1^h	2^h	3^h	4^h	5^h	6^h	7^h	8^h	9^h	10^h	11^h	$+ 21\frac{1}{2}^m$
1841	269		261		264		257		265		288		
1842	580		573		572		567		568		582		
1843	(820)		(816)		(813)		(810)		(812)		(822)		
1844	1031	1031	1031	1029	1026	1026	1026	1028	1029	1030	1034	1034	
1845	1230	1230	1231	1229	1229	1226	1223	1227	1231	1236	1243	1244	
Mean	786		782		781		777		781		794		
Referred mean		784		782		779		779		786		796	
Constant correction + 14													
Normals	800	798	796	796	795	793	791	793	795	800	808	810	

Year.	12^h Noon.	13^h	14^h	15^h	16^h	17^h	18^h	19^h	20^h	21^h	22^h	23^h	$+ 21\frac{1}{2}^m$
1841	297		289		275		274		275		272		
1842	589		578		578		580		590		578		
1843	(826)		817		(818)		(820)		(826)		(819)		
1844	1035	1032	1028	1028	1030	1031	1032	1033	1034	1033	1032	1030	
1845	1250	1242	1238	1231	1233	1229	1231	1233	1235	1231	1231	1232	
Mean	799		790		787		787		792		786		
Referred mean		794		786		786		790		788		786	
Constant correction + 14													
Normals	813	808	804	800	801	800	801	804	806	802	800	800	

Monthly mean normal from even hours ($+ 21\frac{1}{2}^m$) 800.8, weight 4.
" " " " odd " " 800.4, " 2.

The values of 1843 inclosed in brackets are derived from the reading at $14^h\ 21\frac{1}{2}^m$ in the same manner as explained in the preceding month.

Normals of the Horizontal Force for March.

Year.	0h	1h	2h	3h	4h	5h	6h	7h	8h	9h	10h	11h	+ 21½m
1841	268		272		267		257		271		294		
1842	565		559		557		554		563		574		
1843	(827)		(823)		(822)		(819)		(827)		(836)		
1844	1048	1047	1046	1046	1046	1043	1043	1047	1050	1054	1057	1063	
1845	1236	1236	1234	1235	1234	1234	1231	1233	1241	1249	1255	1261	
Mean	789		787		785		781		790		803		
Referred mean		788		786		783		785		798		808	
Constant correction + 14													
Normals	803	802	801	800	799	797	795	799	804	812	817	822	

Year.	12h Noon.	13h	14h	15h	16h	17h	18h	19h	20h	21h	22h	23h	+ 21½m
1841	286		267		266		282		264		272		
1842	575		561		565		571		567		566		
1843	(839)		829		(828)		(831)		(828)		(828)		
1844	1063	1061	1057	1050	1051	1050	1050	1052	1050	1048	1050	1048	
1845	1260	1253	1245	1239	1240	1242	1244	1240	1239	1237	1240	1239	
Mean	805		792		790		796		790		791		
Referred mean		800		787		793		794		789		790	
Constant correction + 14													
Normals	819	814	806	801	804	807	810	808	804	803	805	804	

Monthly mean normal from the even hours (+ 21½m) 805.6, weight 4.
" " " " " odd " " 805.8, " 2.

The values for 1843 are interpolated as in the preceding two months

Normals of the Horizontal Force for April.

Year.	0h	1h	2h	3h	4h	5h	6h	7h	8h	9h	10h	11h	+ 21½m
1841	273		271		262		262		283		317		
1842	595		598		597		594		604		620		
1843	861		861		854		854		868		883		
1844	1070	1069	1068	1065	1063	1062	1064	1061	1067	1074	1078	1078	
1845	1252	1250	1249	1247	1245	1243	1241	1244	1253	1268	1278	1281	
Mean	810		809		804		803		815		835		
Referred mean		809		806		803		806		827		837	
Constant correction + 14													
Normals	824	823	823	820	818	817	817	820	829	841	849	851	

Year.	12h Noon.	13h	14h	15h	16h	17h	18h	19h	20h	21h	22h	23h	+ 21½m
1841	315		279		268		271		283		280		
1842	618		603		598		607		608		611		
1843	878		863		860		861		865		859		
1844	1077	1071	1066	1062	1064	1062	1068	1068	1071	1071	1068	1069	
1845	1268	1267	1255	1252	1248	1253	1256	1254	1254	1253	1250	1254	
Mean	831		813		808		813		816		814		
Referred mean		825		810		808		813		816		813	
Constant correction + 14													
Normals	845	839	827	824	822	822	827	827	830	830	828	827	

Monthly mean normal from the even hours (+ 21½m) 828.2, weight 5.
" " " " " odd " " 828.4, " 2.

Normals of the Horizontal Force for May.

Year.	0^h	1^h	2^h	3^h	4^h	5^h	6^h	7^h	8^h	9^h	10^h	11^h	$+21\frac{1}{2}^m$
1841	311		305		306		297		306		323		
1842	614		610		611		605		621		630		
1843	864		862		858		857		875		872		
1844	1065	1065	1063	1062	1062	1061	1061	1064	1068	1076	1077	1070	
1845	1244	1243	1241	1239	1236	1233	1229	1236	1251	1261	1262	1258	
Mean	820		816		815		810		824		833		
Referred mean		819		815		812		815		832		829	
Constant correction + 14													
Normals	834	833	830	829	829	826	824	829	838	846	847	843	

Year	12^h Noon.	13^h	14^h	15^h	16^h	17^h	18^h	19^h	20^h	21^h	22^h	23^h	$+21\frac{1}{2}^m$
1841	313		301		294		306		309		313		
1842	622		606		607		615		618		619		
1843	864		855		856		862		867		863		
1844	1064	1057	1053	1053	1051	1054	1059	1063	1064	1064	1065	1063	
1845	1253	1244	1238	1236	1237	1239	1245	1246	1246	1248	1247	1242	
Mean	823		811		809		817		821		821		
Referred mean		816		810		811		818		822		818	
Constant correction + 14													
Normals	837	830	825	824	823	825	831	832	835	836	835	832	

Monthly mean normal from the even hours ($+21\frac{1}{2}^m$) 832.3, weight 5.
" " " " " odd " " 832.1, " 2.

Normals of the Horizontal Force for June.

Year.	0^h	1^h	2^h	3^h	4^h	5^h	6^h	7^h	8^h	9^h	10^h	11^h	$+21\frac{1}{2}^m$
1841	392		390		392		386		400		401		
1842	649		652		646		638		649		659		
1843	881		879		876		873		883		894		
1844	1078	1077	1076	1077	1077	1075	1073	1077	1080	1082	1084	1081	
1845	1280	1281	1281	1281	1275	1271	1266	1273	1282	1293	1295	1292	
Mean	856		856		853		847		859		867		
Referred mean		856		856		850		853		865		864	
Constant correction + 2													
Normals	858	858	858	858	855	852	849	855	861	867	869	866	

Year.	12^h Noon.	13^h	14^h	15^h	16^h	17^h	18^h	19^h	20^h	21^h	22^h	23^h	$+21\frac{1}{2}^m$
1841	395		382		385		392		402		392		
1842	650		639		638		649		648		650		
1843	884		870		870		881		881		885		
1844	1077	1072	1067	1065	1065	1067	1071	1073	1075	1077	1077	1078	
1845	1286	1280	1272	1269	1269	1273	1278	1281	1280	1277	1279	1280	
Mean	858		846		845		854		857		857		
Referred mean		853		845		849		856		856		857	
Constant correction + 2													
Normals	860	855	848	847	847	851	856	858	859	858	859	859	

Monthly mean normal from the even hours ($+21\frac{1}{2}^m$) 856.6, weight 5.
" " " odd " " 857.0, " 2.

TABLE I.—RECAPITULATION OF THE HOURLY NORMALS OF THE HORIZONTAL FORCE (EXPRESSED IN SCALE DIVISIONS) FOR EACH MONTH OF THE YEAR.

Increase of scale readings denotes decrease of force.

1840–1845.	0^h	1^h	2^h	3^h	4^h	5^h	6^h	7^h	8^h	9^h	10^h	11^h	$+ 21\frac{1}{2}^m$
July . . .	681	679	677	675	673	668	664	673	686	692	694	690	
August . . .	698	699	699	698	699	695	693	702	714	724	726	718	
September . .	720	718	720	718	714	713	711	721	735	744	749	746	
October . .	735	731	731	727	725	728	729	734	739	746	751	750	
November . .	739	738	736	734	733	730	728	732	737	743	746	751	
December . .	772	767	766	764	761	759	756	758	761	766	775	783	
January . .	794	792	792	790	788	787	786	784	786	795	802	808	
February . .	800	798	796	796	795	793	791	793	795	800	808	810	
March . . .	803	802	801	800	799	797	795	799	804	812	817	822	
April . . .	824	823	823	820	818	817	817	820	829	841	849	851	
May	834	833	830	829	829	826	824	829	838	846	847	843	
June . . .	858	858	858	858	855	852	849	855	861	867	869	866	
Year . . .	771.5	769.8	769.1	767.4	765.7	763.7	761.9	766.7	773.7	781.3	786.1	786.5	
Summer . .	769.2	768.3	767.8	766.3	764.7	761.8	759.7	766.7	777.2	785.7	789.0	785.7	
Winter . . .	773.8	771.3	770.3	768.5	766.8	765.7	764.2	766.7	770.3	777.0	783.2	787.3	

1840–1845.	12^h Noon.	13^h	14^h	15^h	16^h	17^h	18^h	19^h	20^h	21^h	22^h	23^h	$+ 21\frac{1}{2}^m$
July . . .	683	672	663	660	659	666	675	677	679	681	682	683	
August . . .	708	698	689	688	692	696	701	703	704	703	704	702	
September .	736	726	720	718	719	723	724	725	723	725	722	722	
October . .	751	747	743	740	739	739	738	739	740	739	739	738	
November . .	750	746	740	738	737	739	736	736	737	739	741	740	
December . .	785	778	773	768	764	765	767	768	771	772	772	771	
January . .	808	801	796	790	787	790	793	794	793	794	794	796	
February . .	813	808	804	800	801	800	801	804	806	802	800	800	
March . . .	819	814	806	801	804	807	810	808	804	803	805	804	
April . . .	845	839	827	824	822	822	827	827	830	830	828	827	
May	837	830	825	824	823	825	831	832	835	836	835	832	
June . . .	860	855	848	847	847	851	856	858	859	858	859	859	
Year . . .	782.9	776.2	769.5	766.5	766.2	768.6	771.6	772.6	773.4	773.5	773.4	772.8	
Summer . .	778.2	770.0	762.0	760.2	760.3	763.8	769.0	770.3	771.7	772.2	771.7	770.8	
Winter . . .	787.7	782.3	777.0	772.8	772.0	773.3	774.2	774.8	775.2	774.8	775.2	774.8	

In the preceding table the normals for the summer half year comprise the months between April and September inclusive; those for the winter half year comprise the months between October and March inclusive.

The following table contains the mean values of the normals for each month and season.

TABLE II.

1840–1844.	Normal.	1841–1845.	Normal.	1840–1845.	Normal.
July	676.3	January . . .	793.3	Year	772.1
August . . .	702.2	February . . .	800.6	Summer	770.1
September . .	724.6	March	805.7	Winter	774.1
October . . .	738.2	April	828.3		
November . .	738.5	May	832 2		
December . . .	768.4	June	856.8		

Regular Solar-Diurnal Variation of the Horizontal Force.—If we subtract the hourly normals of Table I. from their respective monthly mean value as given in Table II., the difference (in scale divisions) will represent the regular solar-diurnal variation for each month in the year. In like manner we obtain the diurnal variation of the horizontal force—free of the larger disturbances—for the summer and winter half, and for the whole year. Table III. will exhibit these differences after their conversion from scale divisions into parts of the horizontal force (one scale division equalling 0.0000365 parts of the horizontal force). The tabular numbers are expressed in units of the sixth place of decimals. A plus sign indicates a greater force, a minus sign a less force than the mean value. Casting the eye over the vertical columns, we obtain also a view of the annual inequality of the diurnal variation, which will be examined further on.

TABLE III.—REGULAR SOLAR-DIURNAL VARIATION OF THE HORIZONTAL COMPONENT OF THE MAGNETIC FORCE EXPRESSED IN PARTS OF THE HORIZONTAL FORCE.

A plus sign indicates greater force than the mean. For convenience sake, the first three decimals (0.000) have been placed on the side of the table.

	1840–1845.	0^h	1^h	2^h	3^h	4^h	5^h	6^h	7^h	8^h	9^h	10^h	11^h	+ $21\frac{1}{2}^m$
0.000	July	—171	—098	—025	+047	+120	+303	+449	+120	—353	—572	—646	—499	
	August	+153	+116	+116	+153	+116	+262	+335	+007	—430	—795	—868	—576	
	September	+168	+241	+168	+241	+387	+423	+481	+131	—380	—708	—891	—781	
	October	+116	+262	+262	+408	+481	+372	+335	+153	—029	—284	—467	—430	
	November	—018	+018	+091	+164	+200	+310	+383	+237	+054	—164	—273	—456	
	December	—131	+051	+088	+161	+270	+343	+453	+380	+270	+088	—241	—533	
	January	—025	+047	+047	+120	+193	+230	+266	+339	+266	—061	—318	—536	
	February	+022	+095	+146	+168	+204	+277	+350	+277	+204	+022	—270	—343	
	March	+098	+134	+171	+207	+244	+317	+390	+244	+061	—230	—412	—595	
	April	+157	+193	+193	+303	+376	+412	+412	+303	—025	—463	—755	—828	
	May	—065	—029	+080	+116	+116	+226	+299	+116	—211	—503	—540	—394	
	June	—043	—043	—043	—043	+065	+175	+284	+065	—153	—372	—445	—335	
	Year	+022	+082	+108	+170	+231	+304	+370	+198	—060	—337	—511	—526	
	Summer	+033	+063	+082	+136	+197	+300	+377	+127	—259	—570	—691	—569	
	Winter	+010	+101	+134	+205	+265	+308	+363	+272	+138	—105	—330	—482	

	1840–1845.	12^h Noon.	13^h	14^h	15^h	16^h	17^h	18^h	19^h	20^h	21^h	22^h	23^h	+ $21\frac{1}{2}^m$
0.000	July	—244	+157	+485	+595	+631	+376	+047	—025	—098	—171	—244	—244	
	August	—211	+153	+481	+518	+372	+226	+043	—029	—065	—029	—065	+007	
	September	—416	—051	+168	+241	+204	+058	+022	—015	+058	—015	+095	+095	
	October	—467	—321	—175	—065	—029	—029	+007	—029	—065	—029	—029	+007	
	November	—419	—273	—054	+018	+054	—018	+091	+091	+054	—018	—091	—054	
	December	—606	—350	—168	+015	+161	+124	+051	+015	—095	—131	—131	—095	
	January	—536	—317	—098	+120	+230	+120	+011	—025	+011	—025	—025	—098	
	February	—453	—270	—124	+022	—015	+022	—015	—124	—197	—051	+022	+022	
	March	—485	—303	—011	+171	+061	—047	—157	—088	+061	+098	+025	+061	
	April	—609	—390	+047	+157	+230	+230	+047	+047	—061	—061	+011	+047	
	May	—175	+080	+262	+299	+335	+262	+043	+007	—102	—138	—102	+007	
	June	—116	+065	+321	+357	+357	+211	+029	—043	—080	—043	—080	—080	
	Year	—395	—152	+095	+204	+216	+128	+018	—019	—049	—051	—051	—027	
	Summer	—295	+002	+294	+361	+355	+227	+037	—009	—058	—076	—064	—028	
	Winter	—494	—306	—105	+047	+077	+029	—002	—027	—039	—026	—038	—026	

Table IV.

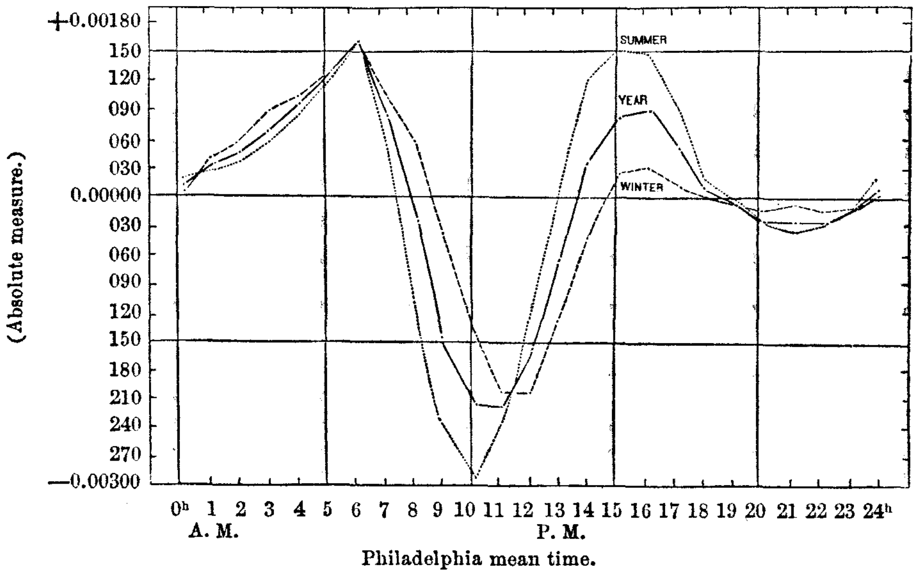
Table IV. is derived from Table III. by multiplication with the absolute value of the horizontal force (4.173); it contains, therefore, the regular solar-diurnal variation of the horizontal force in absolute measure. A plus sign indicates greater force than the mean. Two places of decimals have been placed on the side of the table.

	1840–1845.	0^h	1^h	2^h	3^h	4^h	5^h	6^h	7^h	8^h	9^h	10^h	11^h	$+21\frac{1}{2}^m$
	July	—071	—041	—010	+020	+050	+127	+188	+050	—147	—239	—270	—208	
	August	+064	+048	+048	+064	+048	+109	+140	+003	—180	—332	—362	—241	
	September	+070	+101	+070	+101	+162	+177	+201	+055	—159	—296	—372	—326	
	October	+048	+109	+109	+170	+201	+155	+140	+064	—012	—119	—195	—180	
	November	—008	+008	+038	+068	+083	+129	+160	+099	+022	—068	—114	—190	
	December	—055	+021	+037	+067	+113	+143	+189	+159	+113	+037	—101	—223	
0.00	January	—010	+020	+020	+050	+081	+096	+111	+141	+111	—025	—132	—224	
	February	+009	+040	+061	+070	+085	+116	+146	+116	+085	+009	—113	—143	
	March	+041	+056	+071	+086	+102	+132	+163	+102	+025	—096	—172	—248	
	April	+066	+081	+081	+127	+157	+172	+172	+127	—010	—193	—315	—346	
	May	—027	—012	+033	+048	+048	+094	+125	+048	—088	—210	—226	—165	
	June	—018	—018	—018	—018	+027	+073	+119	+027	—064	—155	—186	—140	
	Year	+009	+034	+045	+071	+096	+127	+155	+083	—025	—141	—213	—220	
	Summer	+014	+026	+034	+057	+082	+125	+157	+053	—108	—238	—289	—238	
	Winter	+004	+042	+056	+086	+111	+129	+152	+114	+058	—044	—138	—201	

	1840–1845.	12^h Noon	13^h	14^h	15^h	16^h	17^h	18^h	19^h	20^h	21^h	22^h	23^h	$+21\frac{1}{2}^m$
	July	—102	+065	+203	+248	+263	+157	+020	—010	—041	—071	—102	—102	
	August	—088	+064	+201	+216	+155	+094	+018	—012	—027	—012	—027	+003	
	September	—174	—021	+070	+101	+085	+024	+009	—006	+024	—006	+040	+040	
	October	—195	—134	—073	—027	—012	—012	+003	—012	—027	—012	—012	+003	
	November	—175	—114	—022	+008	+022	—008	+038	+038	+022	—008	—038	—022	
	December	—253	—146	—070	+006	+067	+052	+021	+006	—040	—055	—055	—040	
0.00	January	—224	—132	—041	+050	+096	+050	+005	—010	+005	—010	—010	—041	
	February	—189	—113	—052	+009	—006	+009	—006	—052	—082	—021	+009	+009	
	March	—203	—127	—005	+071	+025	—020	—065	—037	+025	+041	+010	+025	
	April	—254	—163	+020	+065	+096	+096	+020	+020	—025	—025	+005	+020	
	May	—073	+033	+109	+125	+140	+109	+018	+003	—043	—058	—043	+003	
	June	—048	+027	+134	+149	+149	+088	+012	—018	—033	—018	—033	—033	
	Year	—165	—063	+040	+085	+090	+053	+008	—008	—020	—021	—021	—011	
	Summer	—123	+001	+123	+151	+148	+095	+015	—004	—021	—032	—027	—012	
	Winter	—206	—128	—044	+020	+032	+012	—000	—011	—016	—011	—016	—010	

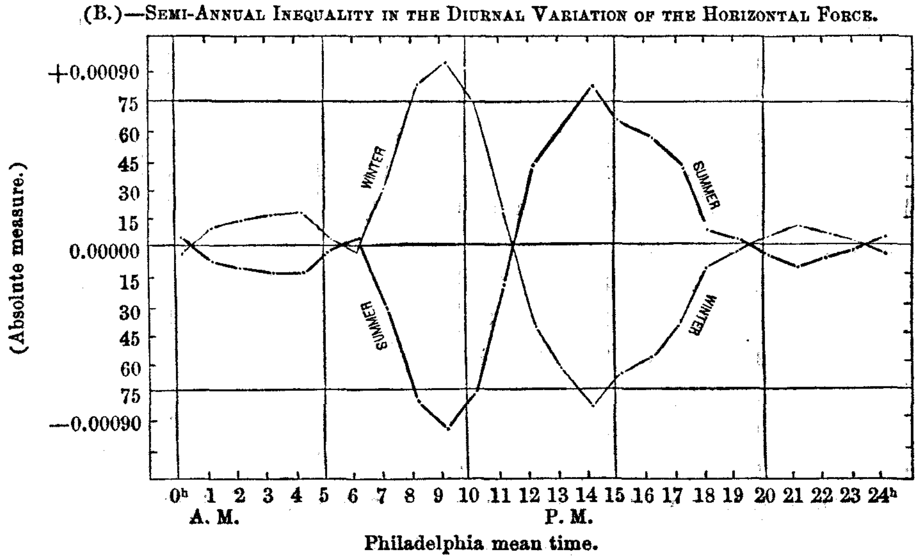
Annual Inequality in the Diurnal Variation of the Horizontal Force.—The distinctive feature of the diurnal variation is shown in the annexed diagram (A), constructed from the mean annual and half-yearly values given in the preceding table, IV. It exhibits in the annual mean, as its characteristic type, a maximum value about 6 A. M., a minimum value about 11 A. M., a secondary maximum value about $3\frac{1}{2}$ P. M., and a secondary minimum about 9 P. M. For the half year when the sun has north declination, the morning minimum becomes smaller and the afternoon maximum larger, thus increasing the diurnal range; the converse takes place in the other half of the year, when the sun has south declination. The 6 A. M. maximum remains nearly unchanged throughout the year. The average summer range (April to September inclusive) is 0.0046, and the average winter range (October to March inclusive) is 0.0025, both expressed in absolute measure. The range between the morning maximum and the morning minimum is 0.0045 in summer and 0.0036 in winter, as will be explained further on.

(A.)—Diurnal Variation of the Horizontal Force in Summer, Winter, and for the whole Year.



This semi-annual change in the diurnal amplitude is more conspicuously represented in the annexed diagram (B), derived from diagram (A) by straightening out the annual curve and using it as an axis of abscissæ for laying off the differences between the annual values and the summer and winter values at the same respective hours of the day.

(B.)—Semi-Annual Inequality in the Diurnal Variation of the Horizontal Force.



This diagram (B) may, with advantage, be compared with the analogous one representing the annual change of the diurnal variation of the declination as given in Part II. of this discussion. The construction is the same in either case.

At 6 A. M. there is hardly any change throughout the year. The maximum variation, in the course of a year, takes place at 9 A. M. (range 0.00194 in absolute measure); about 11½ A. M. there is an epoch of no variation; at 2 P. M. a second maximum is reached (range 0.00167); again at 7½ and 11 P. M. points of no variation are reached. Owing to the prominent annual variation near 2 P. M., the range of the diurnal variation between the morning minimum at 11 A. M. and the afternoon maximum at $3\frac{1}{2}$ P. M. is of more interest in the discussion of the diurnal fluctuation of the horizontal force than the 6 A. M. and 11 A. M. range, which latter range, as we have seen, is slightly greater than the first one.

To find the turning epochs of the annual variation, the monthly values for the hours 9 A. M. and 2 P. M., when it is best developed, were taken from Table IV., and each value was again compared with its annual mean.

TABLE V.—ANNUAL VARIATION AT THE HOURS 9 A. M. AND 2 P. M.

MONTH.	9 A. M. 0.00	Differences. 0.00	2 P. M. 0.00	Differences. 0.00	Mean difference. 0.00
January	—025	+116	—041	—081	+099
February	+009	+150	—052	—092	+121
March	—096	+045	—005	—045	+045
April	—193	—052	+020	—020	—016
May	—210	—069	+109	+069	—069
June	—155	—014	+134	+094	—054
July	—239	—098	+203	+163	—130
August	—332	—191	+201	+161	—176
September	—296	—155	+070	+030	—092
October	—119	+022	—073	—113	+046
November	—068	+073	—022	—062	+068
December	+037	+178	—070	—110	+144
Mean	—141		+040		

Casting the eye over the columns headed "differences," we see by the change of sign and the magnitude of the values that the transition from a positive to a negative value occurs some time after the equinoxes, and that the maximum variation is reached about the time of the solstices—a result in close correspondence with the conclusions reached in the discussion of the annual inequality in the diurnal variation of the declination (Part II. of the discussion). For convenience in the analytical treatment, a column headed "mean difference" has been added to Table V., obtained by changing the signs of the 2 P. M. differences (the annual variation being then opposite to the morning values), and taking the mean of the 9 A. M. and 2 P. M. differences. The values in this column are tolerably well represented by the following formula:—

$$\Delta_a = +0.00129 \; sin \; (\theta + 79°) + 0.00018 \; sin \; (2\theta + 191°),$$

the angle θ counting from January 1, at the rate of 30° a month. Accordingly, we find the transition to take place shortly before the middle of April and October, or about twenty-two days after the equinoxes. This is about twelve days later than the epoch found in Part II. for the declination.

Analysis of the Solar-Diurnal Variation of the Horizontal Force.—For convenience of investigation and proper comparison with similar results at other localities, the values given in Table I. have been put in an analytical form, and are represented by the following expressions. It will be seen that the difference between any monthly normal mean and the corresponding mean in Table V. of Part IV., which latter mean is affected with the disturbances, does not exceed $2\frac{1}{2}$ scale divisions. This small difference includes also a small effect due to the necessity of different methods of interpolation in the construction of the two tables. In the determination of the numerical quantities (by application of the method of least squares) in the monthly equations, due attention was paid to the relative weights of the values for the even and odd hours. The coefficients are expressed in scale divisions (increasing numbers denoting decrease of force), and the angle θ counts from midnight at the rate of 15° an hour.

For January, $\Delta_h = +\,793^d.3 + 3^d.77\,sin\,(\theta + 236°\,52') + 6^d.56\,sin\,(2\theta + 96°\,52') + 3^d.99\,sin\,(3\theta + 282°\,13') + 2^d.00\,sin\,(4\theta + 97°)$

For February, $\Delta_h = +\,800^d.6 + 5^d.50\,sin\,(\theta + 218°\,26') + 4^d.57\,sin\,(2\theta + 102°\,29') + 3^d.27\,sin\,(3\theta + 282°\,40') + 1^d.66\,sin\,(4\theta + 121°)$

For March, $\Delta_h = +\,805^d.7 + 6^d.56\,sin\,(\theta + 243°\,31') + 5^d.35\,sin\,(2\theta + 114°\,14') + 4^d.23\,sin\,(3\theta + 316°\,04') + 1^d.91\,sin\,(4\theta + 113°)$

For April, $\Delta_h = +\,828^d.3 + 7^d.65\,sin\,(\theta + 257°\,37') + 9^d.55\,sin\,(2\theta + 123°\,06') + 5^d.15\,sin\,(3\theta + 306°\,44') + 1^d.18\,sin\,(4\theta + 163°)$

For May, $\Delta_h = +\,832^d.2 + 2^d.24\,sin\,(\theta + 314°\,31') + 7^d.81\,sin\,(2\theta + 140°\,53') + 4^d.40\,sin\,(3\theta + 330°\,05') + 1^d.34\,sin\,(4\theta + 214°)$

For June, $\Delta_h = +\,856^d.8 + 2^d.12\,sin\,(\theta + 356°\,03') + 6^d.40\,sin\,(2\theta + 140°\,32') + 4^d.48\,sin\,(3\theta + 327°\,14') + 0^d.92\,sin\,(4\theta + 216°)$

For July, $\Delta_h = +\,676^d.3 + 3^d.42\,sin\,(\theta + 4°\,11') + 11^d.50\,sin\,(2\theta + 139°\,14') + 6^d.14\,sin\,(3\theta + 330°\,15') + 0^d.78\,sin\,(4\theta + 210°)$

For August, $\Delta_h = +\,702^d.2 + 5^d.32\,sin\,(\theta + 310°\,58') + 10^d.37\,sin\,(2\theta + 153°\,46') + 6^d.79\,sin\,(3\theta + 335°\,55') + 2^d.88\,sin\,(4\theta + 203°)$

For September, $\Delta_h = +\,724^d.6 + 8^d.02\,sin\,(\theta + 271°\,57') + 9^d.59\,sin\,(2\theta + 137°\,25') + 7^d.08\,sin\,(3\theta + 345°\,17') + 1^d.99\,sin\,(4\theta + 215°)$

For October, $\Delta_h = +\,738^d.2 + 8^d.06\,sin\,(\theta + 237°\,57') + 6^d.40\,sin\,(2\theta + 123°\,37') + 1^d.34\,sin\,(3\theta + 325°\,20') + 0^d.29\,sin\,(4\theta + 174°)$

For November, $\Delta_h = +\,738^d.5 + 4^d.13\,sin\,(\theta + 237°\,36') + 6^d.08\,sin\,(2\theta + 100°\,01') + 1^d.93\,sin\,(3\theta + 310°\,45') + 0^d.46\,sin\,(4\theta + 211°)$

For December, $\Delta_h = +\,768^d.4 + 5^d.03\,sin\,(\theta + 212°\,48') + 8^d.07\,sin\,(2\theta + 94°\,14') + 3^d.98\,sin\,(3\theta + 269°\,17') + 1^d.31\,sin\,(4\theta + 88°)$

We have also: For summer half year (April to September inclusive), for winter half year (October to March inclusive), and for the whole year, the following expressions for the regular solar diurnal variations:—

For summer, $\Delta_h = +\,770^d.1 + 3^d.79\,sin\,(\theta + 293°\,49') + 9^d.11\,sin\,(2\theta + 139°\,10') + 5^d.36\,sin\,(3\theta + 329°\,17') + 1^d.42\,sin\,(4\theta + 202°)$

For winter, $\Delta_h = +\,774^d.1 + 5^d.36\,sin\,(\theta + 231°\,36') + 6^d.04\,sin\,(2\theta + 104°\,46') + 2^d.88\,sin\,(3\theta + 293°\,54') + 1^d.11\,sin\,(4\theta + 108°)$

For year, $\Delta_h = +\,772^d.1 + 3^d.95\,sin\,(\theta + 256°\,19') + 7^d.25\,sin\,(2\theta + 125°\,05') + 3^d.96\,sin\,(3\theta + 317°\,31') + 0^d.86\,sin\,(4\theta + 165°)$

The following expressions for January may serve as specimens of the agreement of the result derived from the even and odd hours independently:—

From even hours, $\Delta_h = 793^d.3 + 3^d.81\,sin\,(\theta + 238°\,01') + 6^d.56\,sin\,(2\theta + 94°\,32') + 4^d.10\,sin\,(3\theta + 280°\,19') + 2^d.08\,sin\,(4\theta + 86°)$

From odd hours, $\Delta_h = 793^d.4 + 3^d.71\,sin\,(\theta + 234°\,35') + 6^d.56\,sin\,(2\theta + 101°\,32') + 3^d.76\,sin\,(3\theta + 286°\,00') + 1^d.85\,sin\,(4\theta + 119°)$

giving to the first equation the weight 2 and to the second the weight 1, we obtain the equation as given above.

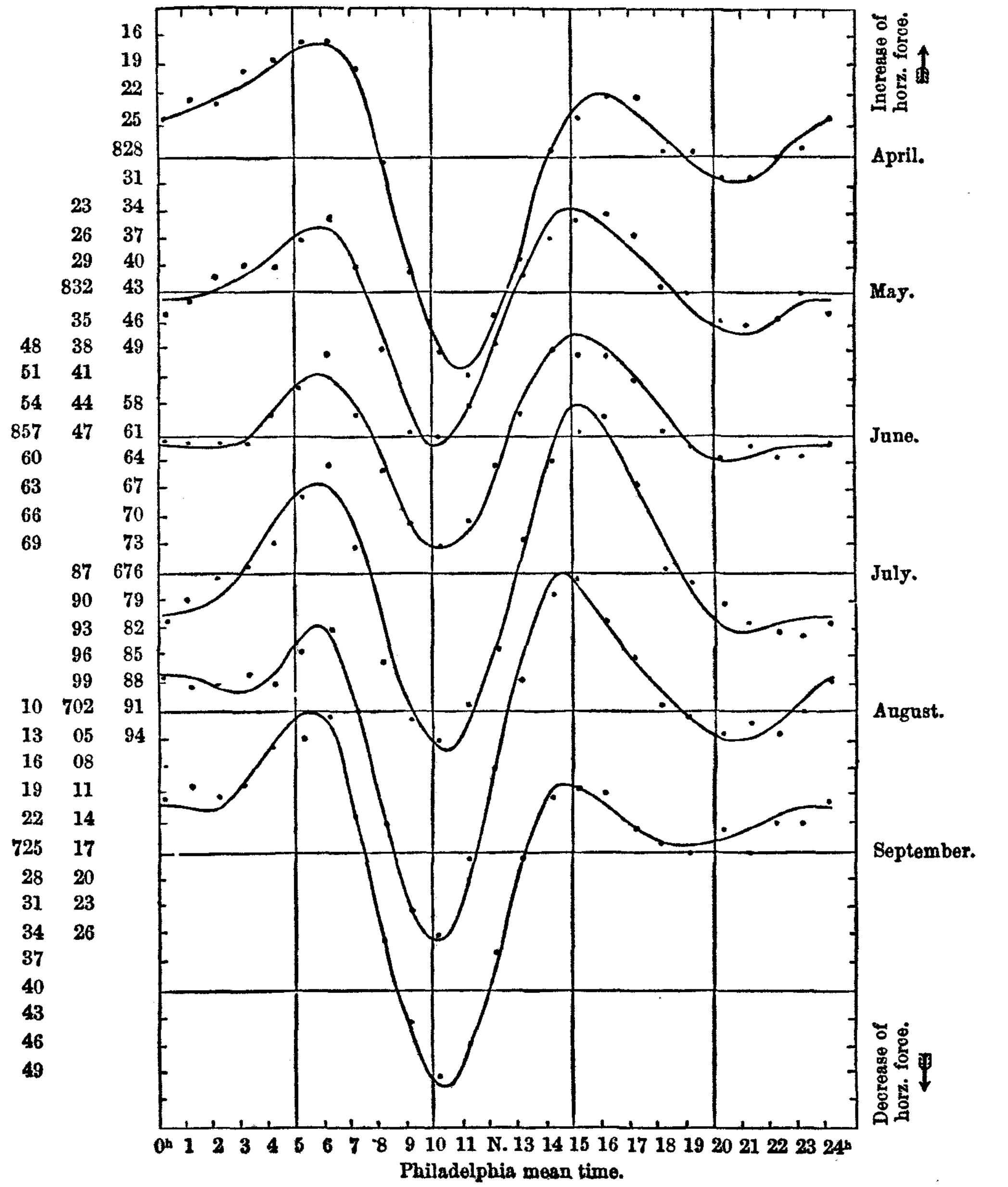
The following comparison will show the agreement of the observed and computed values we have for August:—

(A. M.)	Computed.	Observed.	Δ	(P. M.)	Computed.	Observed.	Δ
$0^h\ 21\frac{1}{2}^m$	698.3	698	0	$12^h\ 21\frac{1}{2}^m$	707.7	708	0
1 "	698.3	699	—1	13 "	695.1	698	—3
2 "	699.6	699	+1	14 "	688.4	689	—1
3 "	699.7	698	+2	15 "	888.7	688	+1
4 "	697.6	699	—1	16 "	692.5	692	0
5 "	694.3	695	—1	17 "	697.1	696	+1
6 "	694.5	693	+1	18 "	700.3	701	—1
7 "	701.2	702	—1	19 "	702.6	703	0
8 "	712.7	714	—1	20 "	704.5	704	0
9 "	723.6	724	0	21 "	704.8	703	+2
10 "	727 1	726	+1	22 "	703.3	704	—1
11 "	720.4	718	+2	23 "	700.6	702	—1

Diagrams C and D exhibit the regular solar-diurnal variation of the horizontal force; the dots represent the observations directly taken from Table 1; the curves give the computed values from the preceding equations. These diagrams also exhibit the general agreement between the observed and computed values. The summer months are represented on diagram C, the winter months on diagram D; their comparison shows plainly the much greater range of the diurnal variation when the sun is north of the equator than when south of it, as was also the case with the magnetic declination.

(C.)—Solar-diurnal Variation of the Horizontal Force; April to September, 1840 to 1845.

Scale divisions. $1^d = 0.0000365$ parts of the horizontal force.



(D.)—SOLAR-DIURNAL VARIATION OF THE HORIZONTAL FORCE; OCTOBER TO MARCH, 1840 TO 1845.

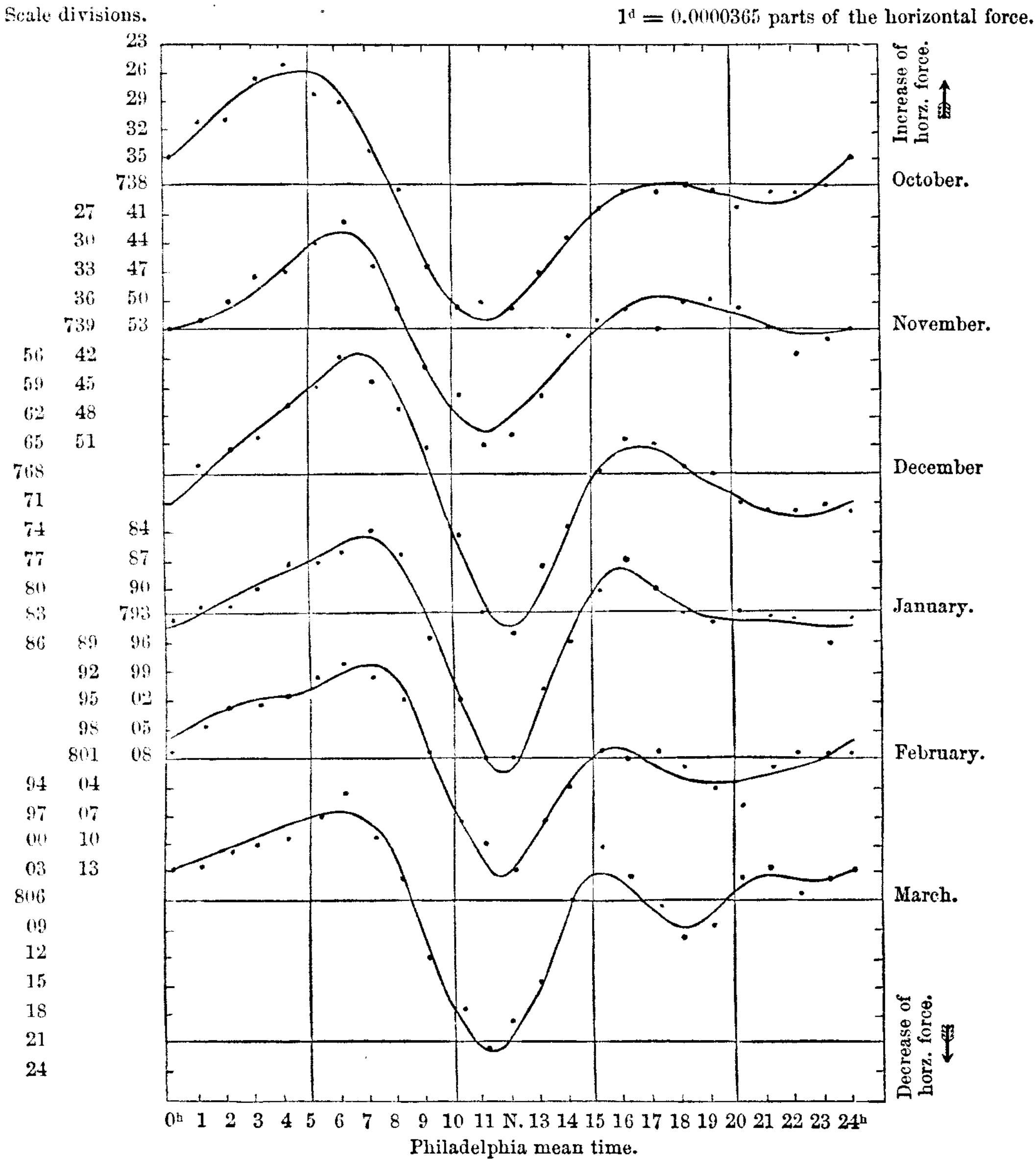


Table VI. contains the coefficients B_1 B_2 B_3 B_4 of the general equation:—

$$\Delta_h = A + B_1 \sin (\theta + C_1) + B_2 \sin (2\,\theta + C_2) + B_3 \sin (3\,\theta + C_3) + B_4 \sin (4\,\theta + C_4)$$

expressed in parts of the horizontal force, by multiplying the corresponding quantities in the preceding equations with the value of a scale division. The angles C_1 C_2 C_3 C_4 will be found in Table VII.; they are the same as given before, increased by 180°, so as to make a corresponding change in the direction of the scale readings; increasing numbers will now indicate increasing force.

The first three decimals (0.000) have been placed in front of the table.

Table VI.

Month.		B_1	B_2	B_3	B_4
January	0.000	138	239	146	073
February		202	167	119	060
March		239	195	154	070
April		279	349	188	043
May		082	285	161	049
June		077	234	164	034
July		125	420	224	029
August		194	379	248	105
September		295	350	258	073
October		294	234	048	011
November		151	222	071	017
December		184	295	145	048
Summer		138	333	196	052
Winter		196	220	105	040
Year		144	265	145	031

In Table VII. the same quantities are given in absolute measure; the first two places of decimals (0.00) are placed at the head of the columns. (Increasing numbers denote increase of force.) The numerical values of *A* will be found in connection with the discussion of the annual variation of the horizontal force.

Table VII.

Month.	B_1 0.00	C_1	B_2 0.00	C_2	B_3 0.00	C_3	B_4 0.00	C_4
January	057	56° 52′	100	276° 52′	061	102° 13′	030	277°
February	084	38 26	070	282 29	050	102 40	025	301
March	100	63 31	082	294 14	064	136 04	029	293
April	117	77 37	146	303 06	079	126 44	018	343
May	034	134 31	119	320 53	067	150 05	020	34
June	032	176 03	098	320 32	068	147 14	014	36
July	052	184 11	175	319 14	094	150 15	012	30
August	081	130 58	158	333 46	104	155 55	044	23
September . . .	122	91 57	146	317 25	108	165 17	030	35
October	123	57 57	098	303 37	020	145 20	005	354
November . . .	063	57 36	093	280 01	029	130 45	007	31
December . . .	077	32 48	123	274 14	061	89 17	020	268
Summer	058	113 49	139	319 10	082	149 17	022	22
Winter	082	51 36	092	284 46	044	113 54	017	288
Year	060	76 19	111	305 05	060	137 31	013	345

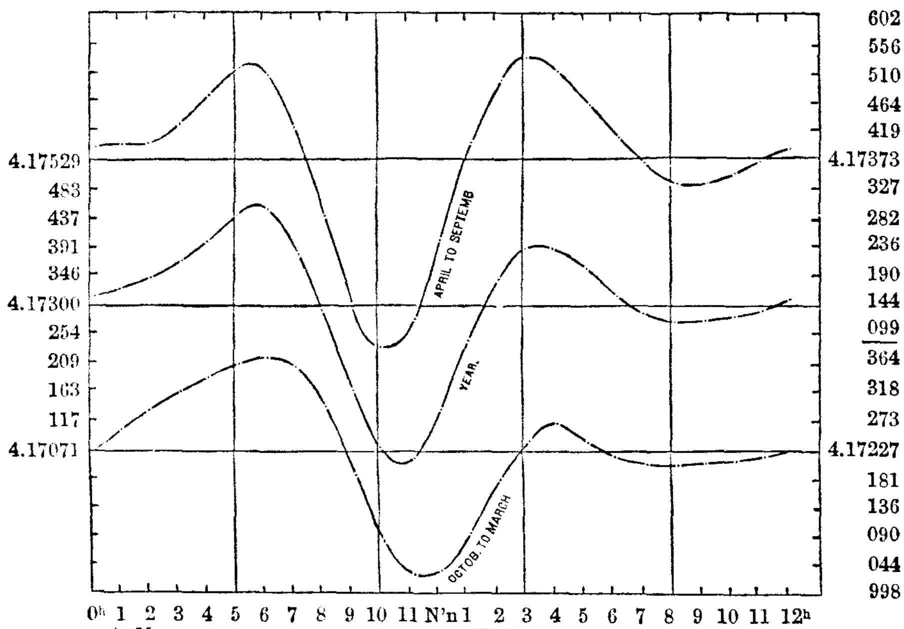
On diagram E the average value of the diurnal variation throughout the year, together with the summer and winter value, has been represented as resulting from the numerical quantities in the above table. It exhibits the noticeable feature in the annual curve of a greater morning maximum (about 6 A. M.) than afternoon maximum (about $3\frac{1}{2}$ P. M.), whereas in the summer curve it is the afternoon maximum which is the greater of the two.[1] In the winter season the contrast is more

---

[1] The same is the case at Prague; in May, June, and July, the afternoon maximum was the greater of the two. Karl Kreil, in vol. VIII. Proceedings of the Academy of Sciences of Vienna, 1855: "Resultate aus den magnetischen Beobachtungen zu Prag."

marked, the morning maximum being considerably greater. These curves also show the gradual shifting of the maxima and minimum to a later hour in winter than in summer, a phenomenon also well exhibited in the preceding diagrams C and D. The numerical values of this change of hours will be given in tabular form further on. The small afternoon minimum about 9 P. M. is less distinctly marked than any other feature of the diurnal curve.

(E.)—Regular Solar-Diurnal Variation of the Horizontal Force for Summer, Winter, and whole Year.
(In absolute measure.)



Philadelphia mean time.

Table VIII. contains the computed values of the time and amount of the morning maximum and minimum, and of the afternoon maximum. The values for the secondary afternoon minimum are taken from the diagrams. The time of the A. M. maximum and minimum is within the nearest eighth minute; that of the P. M. maximum within the nearest tenth minute. The time for the P. M. secondary minimum is within the nearest hour. The amount of change of horizontal force is expressed in scale divisions.

TABLE VIII.

MONTH.	Morning maximum.		Morning minimum.		Afternoon maximum.		Secondary afternoon minimum.		Interval A. M. min. to P. M. max.
January . . .	$7^h 10^m$	$-9^d.2$	$11^h 50^m$	$+15^d.7$	$4^h 10^m$	$-5^d.3$	11^h	$+2^d$	$4^h 20^m$
February . . .	7 15	−9.6	11 40	+12.7	4 00	−0.9	7	+2	4 20
March	6 15	−9.2	11 30	+16.4	3 20	−2.3	6	+3	3 50
April	6 00	−12.3	11 20	+22.5	3 55	−6.6	9	+3	4 35
May	5 50	−7.9	10 25	+15.5	3 10	−9.8	9	+4	4 45
June	5 50	−6.3	10 30	+12.5	3 20	−10.4	8	+3	4 50
July	5 35	−9.9	10 30	+19.3	3 25	−17.5	9	+6	4 55
August	5 55	−8.5	10 10	+24.8	2 45	−14.2	9	+3	4 35
September . . .	5 35	−14.9	10 20	+25.9	3 05	−6.7	7	−1	4 45
October . . .	5 00	−12.6	11 15	+13.7	5 10	−0.1	9	+2	5 55
November . . .	6 00	−9.8	11 25	+11.0	5 15	−3.0	11	+0	5 50
December . . .	7 05	−12.1	12 05	+16.1	4 35	−5.1	10	+4	4 30
Summer . . .	5 50	−9.8	10 30	+19.6	3 25	−10.5	20½	+3	4 55
Winter	6 15	−9.4	11 45	+13.9	4 10	−2.2	21	+2	4 25
Year	5 55	−9.6	11 00	+15.6	3 35	−6.0	20¾	+2.5	4 35

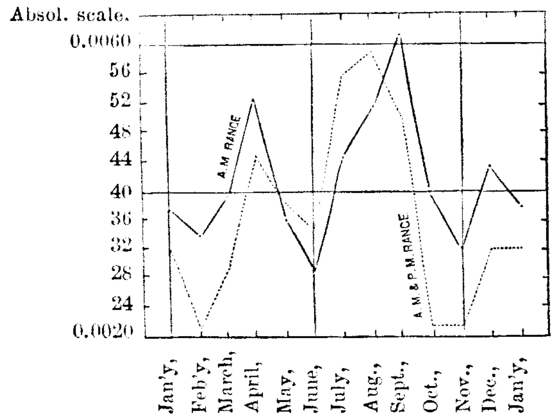
The extreme variation in the epoch of the A. M. maximum is therefore $2^h\ 15^m$; the variation for the A. M. minimum is $1^h\ 55^m$; for the P. M. maximum it is $2^h\ 30^m$, and for the secondary afternoon minimum between 3 and 4 hours. In all cases, the earlier hours occur in the summer season.

Table IX. shows the diurnal range, expressed in scale divisions, parts of the horizontal force and in absolute measure. In the second column the range between the A. M. maximum and minimum is given; in the third column that between the A. M. minimum and the P. M. maximum. These two amplitudes for A. M., and for A. M. and P. M., are further illustrated in diagram F, which shows the curve to be double crested, with maxima near the time of the equinoxes, and the greater of these near the autumnal equinox.

TABLE IX.—AMPLITUDE OF THE DIURNAL VARIATION OF THE HORIZONTAL FORCE.

MONTH.	For A. M.	For A. M. and P. M.	For A. M.	For A. M. and P. M.	For A. M.	For A. M. and P. M.
January	$24^d.9$	$21^d.0$	0.00091	0.00077	0.0038	0.0032
February	22.3	13.6	081	050	34	21
March	25.6	18.7	093	068	39	29
April	34.8	29.1	127	106	53	45
May	23.4	25.3	085	092	36	38
June	18.8	22.9	069	084	29	35
July	29.2	36.8	106	134	45	56
August	33.3	39.0	122	142	51	59
September	40.8	32.6	149	119	62	50
October	26.3	13.6	096	050	40	21
November.	20.8	14.0	076	051	32	21
December	28.2	21.2	0.00103	077	0.0043	0.0032
Summer	29.4	30.1	0.00107	0.00110	0.0045	0.0046
Winter	23.3	16.1	0.00085	0.00059	0.0036	0.0025
Year	25.2	21.6	0.00092	0.00079	0.0038	0.0033
	In scale divisions.		In parts of the horizontal force.		In absolute measure.	

(F.)—Solar-Diurnal Range of the Horizontal Force for each Month of the Year.



The next table contains the epochs when the mean horizontal force is reached in each day, as computed by the preceding formulæ. The diurnal curves intersect the axis of abscissæ four times, of which the table contains only the A. M. and first P. M. intersection: those later in the afternoon and near midnight occur in summer, winter, and whole year at 7 P. M., $5\frac{3}{4}$ P. M., and $6\frac{1}{2}$ P. M. respectively, and at $11\frac{1}{4}$ P. M., 12 P. M., and $11\frac{3}{4}$ P. M. respectively.

Table X.—Principal Epochs of Mean Horizontal Force.

Month.	A. M.	P. M.
January	$9^h 20^m$	$2^h 36^m$
February	9 23	2 58
March	8 42	2 28
April	8 14	2 19
May	7 44	0 59
June	7 47	0 48
July	7 57	0 53
August	7 28	0 44
September	7 42	1 29
October	8 08	5 00
November	8 40	3 28
December	9 34	3 03
Summer	$7^h 45^m$	$1^h 12^m$
Winter	9 00	3 07
Year	8 14	1 54

The above times are generally correct within two minutes (according to the formulæ). The morning hour of average daily horizontal force is less variable in the course of a year than the afternoon hour.

The following table contains the computed diurnal variation of the horizontal force. The values have been expressed in absolute measure. It compares directly with Table IV., which contains the observed values. It will be useful for the interpolation of observations, or for their reduction to the mean value of the day from observations taken at irregular hours. The table also forms the basis for the construction of diagram G.

TABLE XI.—COMPUTED SOLAR-DIURNAL VARIATION OF THE HORIZONTAL FORCE IN ABSOLUTE MEASURE.

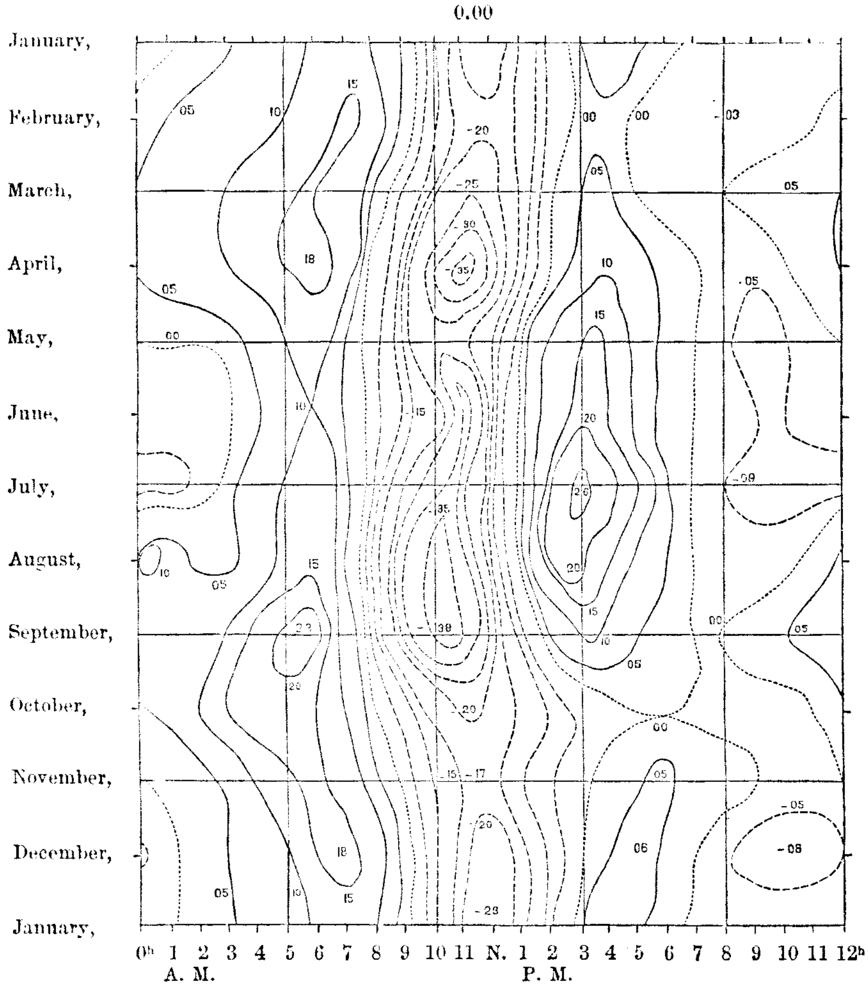
The first two places of decimals (0.00) are placed in front of the table.

	1840–1845.	0^h	1^h	2^h	3^h	4^h	5^h	6^h	7^h	8^h	9^h	10^h	11^h	$+21\frac{1}{2}^m$
0.00	July	—061	—061	—030	+015	+091	+137	+137	+046	—107	—244	—290	—244	
	August	+122	+061	+030	+030	+061	+122	+122	+015	—167	—335	—381	—274	
	September	+061	+061	+061	+107	+182	+229	+198	+061	—152	—320	—381	—320	
	October	+046	+091	+122	+167	+182	+182	+137	+076	—030	—122	—182	—213	
	November	000	+015	+030	+061	+107	+152	+152	+122	+030	—061	—137	—167	
	December	—046	000	+030	+061	+091	+137	+167	+182	+122	+015	—122	—229	
	January	—030	000	+030	+046	+061	+091	+107	+122	+091	—015	—137	—229	
	February	+030	+061	+076	+076	+091	+107	+137	+152	+137	+015	—107	—182	
	March	+046	+061	+076	+107	+107	+137	+137	+122	+030	—076	—198	—244	
	April	+061	+076	+091	+107	+137	+167	+182	+122	—015	—198	—320	—351	
	May	000	000	000	+030	+061	+107	+107	+046	—076	—198	—244	—182	
	June	—015	—030	—030	000	+046	+091	+091	+046	—061	—152	—182	—305	

	1840–1845.	12^h Noon.	13^h	14^h	15^h	16^h	17^h	18^h	19^h	20^h	21^h	22^h	23^h	$+21\frac{1}{2}^m$
0.00	July	—091	+076	+213	+259	+229	+152	+046	—015	—061	—091	—076	—076	
	August	—091	+107	+213	+198	+152	+076	+030	—015	—030	—046	—015	+015	
	September	—152	—015	+091	+107	+076	+046	+015	000	+030	+030	+046	+076	
	October	—182	—137	—076	—030	—015	000	000	—015	—015	—030	—015	+015	
	November	—137	—091	—046	000	+030	+046	+046	+030	+015	000	—015	—015	
	December	—244	—182	—076	+015	+061	+061	+015	—015	—046	—061	—076	—061	
	January	—229	—137	—030	+061	+076	+046	+015	—015	—030	—015	—015	—030	
	February	—182	—107	—030	+015	+015	—015	—030	—030	—030	—015	—015	000	
	March	—198	—107	000	+046	+015	—015	—046	—030	+015	+046	+046	+046	
	April	—274	—137	000	+076	+091	+061	+030	000	—030	—030	000	+030	
	May	—076	+046	+122	+152	+122	+076	+030	—015	—046	—061	—046	—015	
	June	—046	+061	+137	+167	+137	+076	+030	—015	—030	—046	—030	—015	

Diagram G exhibits the changes in the horizontal force (in absolute measure) from the monthly normal value for each hour of the day and for each month of the year. The three variables are: the hour of the day, the month of the year, and the difference of the horizontal force from the normal. The contour lines of the magnetic surface differ 0.0005 of horizontal force in absolute measure. Full lines indicate greater value, lines of dashes less value than the mean; dotted lines represent the normal value.

(G.)—CHANGES OF THE HORIZONTAL FORCE FROM ITS NORMAL VALUE, FOR EACH HOUR OF THE DAY AND MONTH OF THE YEAR. Expressed in absolute measure.



Philadelphia mean time.

Annual Variation of the Horizontal Force.—For the discussion of the annual variation we make use of the monthly normal readings of the horizontal force as given in Table II. If m equals the monthly effect of the total progressive change, we obtain from the twelve equations by the usual method the value $m = + 15.49$, and the correction for progressive change for July and June, for instance, becomes $+ 5.5\ m$ and $- 5.5\ m$ respectively. The following table contains the monthly normals uncorrected and corrected for progressive change; also the differences from the mean for each month, constituting the annual variation.

TABLE XII.

MONTH.	Normals.	Corrected for progressive change.	Corrected normals.	Differences, or annual variation.		
					0.000	0.00
July	676.3	+85.2	761.5	+10.6	+39	+16
August	702.2	+69.7	771.9	+ 0.2	+01	+00
September . . .	724.6	+54.2	778.8	— 6.7	—24	—10
October . . .	738.2	+38.7	776.9	— 4.8	—17	—07
November . . .	738.5	+23.2	761.7	(+10.4)	(+38)	(+16)
December . . .	768.4	+ 7.7	776.1	— 4.0	—15	—06
January . . .	793.3	— 7.7	785.6	—13.5	—49	—20
February . . .	800.6	—23.2	777.4	— 5.3	—19	—08
March	805.7	—38.7	767.0	+ 5.1	+19	+08
April	828.3	—54.2	774.1	— 2.0	— 7	—03
May	832.2	—69.7	762.5	+ 9.6	+35	+15
June	856.8	—85.2	771.6	+ 0.5	+02	+01
Mean	772.1	0.0	772.1	In scale divisions.	In parts of the horizontal force.	In absolute measure.

With the exception of the month of November, the values given above for the annual variation are tolerably regular in their progression, and considering the delicacy of the test applied to the observations in deducing the annual variation, this exceptional irregularity in the November value will not affect the general conclusion. We have as the general result: a greater horizontal force in summer (from April to August), and a smaller horizontal force in winter (from September to March) than the average annual value. The maximum occurs in July (at Toronto in June), and the minimum in January (at Toronto in December).

For Toronto we have the expression for the annual variation:—

$$3.531 + 0.002 \sin (\theta + 312^\circ).$$

For Philadelphia (omitting the November value):

$$4.176 + 0.001 \sin (\theta + 306^\circ);$$

the angle θ in both equations counting from January 15th.

The annual range is 0.0021 (in absolute measure). The transition appears to take place about the time of the equinoxes or a short time before.

Table XIII. contains the monthly normal values of the horizontal force in absolute measure, obtained by adding (algebraically) 4.1730 to the values in the last column of Table XII. These numbers, it will be observed, are corrected for secular change; if we apply the same we obtain the resulting monthly mean values of the horizontal force answering to the epoch January, 1843. The quantity A, mentioned in the explanatory remarks to Table VII., is given in the last column of Table XIII.

TABLE XIII.

MONTH	Normals corrected for secular change.	Monthly means (affected with secular change).
July	4.1746	4.1759
August	4.1730	4.1740
September	4.1720	4.1727
October	4.1723	4.1728
November	4.1746	4.1749
December	4.1724	4.1725
January	4.1710	4.1709
February	4.1722	4.1719
March	4.1738	4.1733
April	4.1727	4.1720
May	4.1745	4.1735
June	4.1731	4.1718
Mean	4.1730	4.1730

PART VI.

INVESTIGATION

OF THE

LUNAR INFLUENCE ON THE MAGNETIC HORIZONTAL FORCE.

INVESTIGATION

OF THE

INFLUENCE OF THE MOON ON THE MAGNETIC HORIZONTAL FORCE.

THE method pursued in the investigation of the lunar effect on the horizontal force is, in general, the same as that explained in Part III. of the discussions of the Girard College observations. The process may be briefly recapitulated as follows: Each horizontal force observation, after it had been corrected for the effect of difference from the standard temperature and for progressive change, the disturbed readings being omitted (as fully explained in Part IV.), was marked with its corresponding lunar hour; the observation nearest to the time of the moon's upper transit over the true meridian of the observatory was marked 0^h, that nearest to the lower transit was marked 12^h, and the observations between, for western and eastern hour angles of the moon, were marked with the proper lunar hour by interpolation. In the hourly series where thirteen observations are recorded in twelve lunar hours, that observation which is nearest midway between any two consecutive lunar hours was omitted. Each observation and reduced reading thus marked with its corresponding lunar hour was subtracted from the monthly normal belonging to its respective hour, and these differences were set down in tabular form, arranged according to lunar hours and keeping each monthly result separate for future combination. Let n = any normal belonging to any reduced reading r, the following tables contain the mean monthly values of the differences $n - r$; a positive sign, therefore, indicates greater force, a negative sign less force than the normal. It need hardly be repeated that in the original record of the horizontal force increasing numbers denote a decrease of the force. The greatest possible difference is 33, the number of scale divisions, which, according to the criterion, separates a disturbed from an undisturbed observation. For the formation of these differences which amount to more than 22,000, the manuscript tables of the reduced record were used: these tables have already been referred to in the preceding Part IV.

The units in which the differences $n - r$ are expressed are scale divisions, one division being equal 0.0000365 parts of the horizontal force, or equal to 0.000152 in absolute measure, the mean X being = 4.173 (in units of grains and feet).

The lunar effect on terrestrial magnetism being exceedingly minute, the process required for its elucidation is proportionally delicate; all the regular and irregular deviations arising from other sources must first be eliminated. In the method, as indicated above, the magnetic disturbances (as far as they could be recognized as such), the diurnal and annual solar variation, as well as the eleven (or ten) year inequality and secular change, are all eliminated, leaving numbers fitted for the lunar research.

The readings taken in the month of June, 1840, have not been used in this discussion (these had likewise been rejected in the two preceding parts), on account of the imperfect manner in which the allowance for the progressive change could only be made at that time. For the lunar hour 21 in July, 1840, the number of differences is so small that the mean had necessarily to be reduced; one-fourth of its amount was set down in the table. In January, February, and March, 1843, the observations were discontinued, excepting a single daily reading. These months, therefore, do not occur in the lunar discussion.

The number of observations used are distributed over the several months and years, as shown in the following table.

TABLE I.—NUMBER OF OBSERVATIONS FOR LUNAR DISCUSSION.

MONTH.	1840–1841.	1841–1842.	1842–1843.	1843–1844.	1844–1845.	Sum.
July	157	297	284	294	627	1659
August	235	295	318	313	622	1783
September	258	269	265	296	556	1644
October	255	281	257	*602	597	1992
November	245	279	297	603	564	1988
December	199	297	318	603	559	1976
January	179	298	- - - -	621	601	1699
February	238	250	- - - -	575	541	1604
March	260	297	- - - -	576	601	1734
April	262	271	286	586	575	1980
May	264	271	299	623	612	2069
June	212	295	309	579	522	1917
Sum	2764	3400	2633	6271	6977	22045

TABLE II.—DISTRIBUTION OF THE NUMBER OF OBSERVATIONS ACCORDING TO WESTERN AND EASTERN HOUR ANGLES OF THE MOON.

YEAR.	Western hour angles.	Eastern hour angles.
1840–41	1371	1393
1841–42	1688	1712
1842–43	1320	1313
1843–44	3138	3133
1844–45	3499	3478
Sum	11016	11029

Tables III., IV., V., VI. and VII. contain the monthly and annual means of the lunar diurnal variation for the years 1840 to 1845. The numbers are expressed in scale divisions.

* Commencement of the hourly series.

TABLE III.—DIFFERENCES FROM THE MONTHLY NORMALS, 1840–41, WESTERN HOUR ANGLES OF THE MOON.

1840–41.	0^h Up. cul.	1^h	2^h	3^h	4^h	5^h	6^h	7^h	8^h	9^h	10^h	11^h
July	+2	+1	+3	−9	+3	+7	−1	−17	+12	−4	−3	− 8
August	0	−4	+3	−5	+6	+2	−4	− 4	− 2	+2	+5	− 1
September	−2	0	+8	+1	0	−4	−5	+ 2	− 4	−7	+8	+ 4
October	−3	−1	−1	+1	0	−4	−4	+ 1	− 7	+7	+4	+10
November	−6	+3	0	−4	+1	−5	+4	− 4	− 4	+1	+6	− 1
December	−4	−3	+3	+4	+2	−7	−5	− 3	0	+3	−8	+ 9
January	+1	+3	−1	+8	−7	+3	−8	+ 1	− 9	−4	0	+ 2
February	+7	+5	0	+6	−3	+4	−2	0	− 4	0	−2	0
March	−4	+4	+1	0	+3	+3	−7	− 1	+ 8	+1	+3	− 2
April	0	+6	+1	−2	−2	−1	−1	+ 3	− 1	−2	+1	− 1
May	+3	−3	+1	−1	−8	+2	−1	− 1	− 6	0	+2	− 5
June	−1	−5	0	−3	+4	+4	+5	+ 8	− 1	+1	−1	− 8
Mean	−0.4	+0.5	+1.5	−0.3	−0.1	+0.3	−2.4	− 1.3	− 1.5	−0.2	+1.3	− 0.1

1840–41.	12^h Low. cul.	13^h	14^h	15^h	16^h	17^h	18^h	19^h	20^h	21^h	22^h	23^h
July	+11	− 9	−5	+ 2	+6	0	− 2	− 5	+6	− 5	−4	− 2
August	+ 7	+ 6	+9	+ 1	+5	+2	+ 5	− 3	+5	−11	0	− 2
September	− 2	− 1	+2	+ 6	+5	+4	− 4	+ 1	−2	− 3	−1	− 2
October	−16	+14	−9	+ 4	−7	+3	−10	− 2	−1	+ 6	−3	+ 5
November	− 2	+ 1	−1	+ 4	−6	0	+ 1	− 1	+4	+ 6	+1	+ 5
December	+ 6	+ 9	+2	+10	−3	+2	− 6	−12	−3	− 6	+3	+ 5
January	− 2	− 4	+3	− 1	+1	−1	+ 4	− 2	−2	+ 1	+3	+ 7
February	− 5	+ 4	−4	− 7	−6	+5	+ 1	+ 2	+1	− 5	+3	+ 4
March	− 4	0	−5	+ 2	−1	+4	−10	+ 2	−2	− 2	+2	+ 2
April	− 1	− 3	+3	− 8	−3	−4	0	+ 3	−2	+ 2	+4	+ 2
May	+ 8	− 3	0	− 3	0	0	− 2	+ 8	+3	− 2	−2	+ 2
June	+ 8	− 4	+6	− 5	+7	−8	− 5	− 7	0	− 7	+1	−11
Mean	+ 1.0	+0.8	+0.1	+ 0.4	−0.2	+0.6	− 2.3	− 1.3	+0.6	− 2.1	+0.6	+ 1.2

TABLE IV.—DIFFERENCES FROM THE MONTHLY NORMALS, 1841–42, WESTERN HOUR ANGLES OF THE MOON.

1841–42.	0h Up. cul.	1h	2h	3h	4h	5h	6h	7h	8h	9h	10h	11h
July	+4	+5	0	+8	−4	0	0	−2	0	−8	+2	0
August	−1	0	− 2	+2	+2	+3	+3	0	−5	0	+1	+ 5
September	−3	+8	+ 2	−1	0	−1	−5	−3	+1	0	−3	+10
October	+7	−1	+ 4	+4	0	+1	−1	+4	+1	−2	0	+ 1
November	0	+6	− 3	+1	−7	0	−3	−1	+6	+3	+1	+ 3
December	+8	−4	+12	−2	−1	−3	+2	−3	−2	−1	−3	0
January	−2	+8	+ 2	+2	−1	+7	0	+3	−2	+1	+5	0
February	−5	+1	− 1	−3	+4	+2	+4	−7	−5	+5	0	+ 2
March	+4	+3	+ 2	−1	+2	−1	+2	0	−1	−1	−2	− 3
April	0	0	+ 1	0	0	+4	+1	+3	+2	−1	0	+ 1
May	0	−2	+10	+1	+5	+4	+6	−4	+5	−7	−3	− 4
June	+1	0	+ 3	0	+4	−3	−1	−3	−5	−5	0	− 3
Mean	+1.1	+2.0	+ 2.5	+0.9	+0.3	+1.1	+0.7	−1.1	−0.4	−1.3	−0.2	+ 1.0

1841–42.	12h Low. cul.	13h	14h	15h	16h	17h	18h	19h	20h	21h	22h	23h
July	+3	−5	+3	+5	+1	−1	−8	−4	+1	−1	−1	+3
August	+1	+3	0	+2	+1	−5	−1	+3	−4	−3	−5	−1
September	+3	+2	+2	−6	+5	−1	−5	−2	+1	+4	−5	+6
October	+3	−1	−3	−5	−4	−3	+7	−3	−1	−3	−1	+1
November	−1	+4	+3	−6	−1	−5	+1	−2	0	−3	−4	−3
December	−1	0	−1	+2	−4	−3	−1	+1	+1	+1	+6	+1
January	+4	−2	−2	−4	−1	−5	−3	−3	+5	0	−3	+2
February	+7	+1	+1	−2	−3	0	−8	+6	−7	+1	+3	+2
March	−2	+3	−2	0	−6	−1	0	−2	−1	+2	−6	−1
April	+1	+1	+3	−3	+1	−1	−3	+2	−5	−3	−2	+1
May	0	−5	−3	−3	+4	−6	+6	−4	0	+3	+4	+2
June	−4	−2	−4	+2	0	+2	+2	+6	+6	−2	+2	+4
Mean	+1.2	−0.1	−0.2	−1.5	−0.6	−2.4	−1.1	−0.2	−0.3	−0.3	−1.0	+1.4

TABLE V.—DIFFERENCES FROM THE MONTHLY NORMALS, 1842–43, WESTERN HOUR ANGLES OF THE MOON.

1842–43.	0h Up. cul.	1h	2h	3h	4h	5h	6h	7h	8h	9h	10h	11h
July	+3	−3	+2	− 1	+1	0	+7	+2	0	−4	+1	−8
August	+3	+1	−3	0	−1	−4	−3	+4	+2	+1	+1	+3
September	+3	−6	−1	+ 9	+4	−1	+7	+1	0	+2	−3	0
October	+2	−7	0	+ 1	−6	+3	−1	+3	−3	+5	−3	+2
November	+1	+3	−1	+ 2	+1	+1	0	−2	−2	−1	−4	+1
December	−2	−3	−6	+ 1	−5	0	−2	0	+1	−1	+5	0
January	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -
February	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -
March	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -
April	−1	+2	0	+10	+4	+9	−1	+1	−3	−1	+1	−4
May	+3	−2	+2	+ 5	+3	+4	−1	+9	−1	+1	−6	+3
June	−6	+7	−4	− 1	0	+2	0	−1	−5	+4	−1	+3
Mean	+0.7	−0.9	−1.0	+ 2.9	+0.1	+1.6	+0.7	+1.9	−1.2	+0.7	−1.0	0.0

1842–43.	12h Low. cul.	13h	14h	15h	16h	17h	18h	19h	20h	21h	22h	23h
July	+1	+1	+4	−2	+4	−3	−4	−1	0	−2	+ 3	−1
August	−2	+1	+1	+2	+2	−4	+3	−5	−1	+2	+ 2	−2
September	+6	−1	−1	−8	−3	−1	−4	−1	−2	−7	+ 1	−6
October	−7	−3	+2	−1	+1	0	−1	+4	+4	−3	+11	+3
November	−2	−2	−1	−3	+1	+4	−1	+6	0	+1	+ 1	+2
December	+3	+3	+2	+1	+3	0	+1	+2	−2	+4	− 3	+3
January	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -
February	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -
March	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -
April	+2	−2	+3	+2	0	−2	−5	0	−3	−2	+1	−1
May	0	+1	+4	0	+1	−4	−3	−1	−1	−1	−1	−2
June	0	+3	−1	+4	−2	+2	−3	−3	+2	+1	−7	+4
Mean	+0.1	+0.1	+1.4	−0.6	+0.8	−0.9	−1.9	+0.1	−0.3	−0.8	+0.9	0.0

TABLE VI.—DIFFERENCES FROM THE MONTHLY NORMALS, 1843–44, WESTERN HOUR ANGLES OF THE MOON.

1843–44.	0^h Up. cul.	1^h	2^h	3^h	4^h	5^h	6^h	7^h	8^h	9^h	10^h	11^h
July	+6	+4	+2	+4	+5	—4	+3	+1	+1	—2	—5	—2
August	+2	+2	0	—1	+2	+1	—3	+4	0	—1	—2	—2
September	+1	—1	—3	+6	0	—2	—1	—4	—1	0	—2	0
October	—1	+4	+3	+5	+2	+3	+2	+1	0	—2	—1	—3
November	+1	+1	0	0	0	0	0	—2	0	0	+1	+1
December	+2	+1	+2	0	0	—2	—1	—1	—1	—2	+1	—1
January	+1	0	0	0	—1	—1	+1	+1	+1	0	—1	0
February	—1	—1	+1	+2	+1	—1	0	0	+3	0	—1	+2
March	—1	—3	+1	+1	+1	+1	+2	+2	0	+2	+1	+1
April	+2	+2	+3	+2	0	0	+1	+1	—1	—2	—3	—2
May	—2	—2	0	—1	—2	0	—1	—1	—2	—1	0	—1
June	0	—2	0	0	+2	+2	+2	+2	+1	—1	—2	0
Mean	+0.9	+0.4	+0.8	+1.5	+0.9	—0.3	+0.4	+0.3	+0.1	—0.8	—1.2	—0.6

1843–44.	12^h Low. cul.	13^h	14^h	15^h	16^h	17^h	18^h	19^h	20^h	21^h	22^h	23^h
July	—2	—7	—2	—3	+3	—1	+4	—2	+1	—1	+2	+2
August	+4	0	+2	—1	—2	0	+2	+1	+1	—2	+4	0
September	+3	0	+3	+3	+8	+2	—6	—1	—6	—3	—2	0
October	—3	—4	—2	—1	—2	0	0	—1	—1	+1	—2	—1
November	+1	+2	+2	+2	+2	0	—1	—2	—1	—1	0	—1
December	0	+1	+1	0	+1	—1	—3	—4	—4	—3	—2	0
January	+1	+2	—1	—1	—1	—2	0	—1	+1	+2	+1	+2
February	+2	+1	+1	+2	+2	—3	—2	+1	—2	—1	—1	—2
March	+1	0	—1	0	0	0	—1	—1	—1	—1	+1	—3
April	—4	0	0	0	+2	0	0	+1	0	0	—1	0
May	0	0	—2	0	0	—2	—3	+1	+1	+1	+2	+1
June	0	+2	+1	+3	+2	0	—1	0	0	—1	—2	0
Mean	+0.3	—0.2	+0.2	+0.3	+1.3	—0.6	—0.9	—0.7	—0.9	—0.8	0.0	—0.2

Equal weight has been given to each monthly result in the formation of the annual mean.

TABLE VII.—DIFFERENCES FROM THE MONTHLY NORMALS, 1844–45, WESTERN HOUR ANGLES OF THE MOON.

1844–45.	0^h Up. cul.	1^h	2^h	3^h	4^h	5^h	6^h	7^h	8^h	9^h	10^h	11^h
July	0	+1	+1	+1	0	+1	+2	+2	0	+1	0	—2
August	—3	+1	—1	0	+2	0	+1	+3	+1	—3	—2	+1
September	—2	0	—1	0	—2	+2	0	+3	+2	+2	+2	+4
October	0	+4	+5	+2	+3	+4	+2	0	0	+1	—3	0
November	—1	+3	+1	+2	+1	+3	+3	+3	+3	+3	+2	—1
December	—1	0	—1	0	—2	—3	—3	—2	—1	—1	+1	—1
January	+1	+2	+4	—2	—3	—4	—1	—3	0	—1	+4	+2
February	+1	+1	0	0	+1	+1	+1	+2	—1	—2	0	+4
March	+1	—3	—3	—3	0	0	+1	+1	0	+1	+1	0
April	—4	+2	+2	+2	0	+2	0	+2	—2	—2	—1	—1
May	+2	0	+2	+2	0	—2	0	—1	—2	—2	+1	0
June	—5	—4	—3	—1	0	+3	+1	+1	+1	0	—5	—4
Mean	—0.9	+0.6	+0.5	+0.3	0.0	+0.6	+0.6	+0.9	+0.1	—0.3	0.0	+0.2

1844–45.	12^h Low. cul.	13^h	14^h	15^h	16^h	17^h	18^h	19^h	20^h	21^h	22^h	23^h
July	0	0	+1	0	—1	0	0	0	—2	—2	—2	—3
August	+3	+2	+3	—1	0	—2	0	0	—3	—3	0	—2
September	+2	+3	+1	+1	—1	—2	—3	—3	—4	—3	—4	—4
October	+1	+2	+1	+2	0	—2	—2	—4	—4	—5	0	—2
November	—1	—4	0	—2	0	0	—3	—2	—1	+1	—4	—3
December	+1	0	—1	+2	0	+2	+2	+1	0	+3	+2	0
January	+4	0	+2	—1	—5	—4	—4	—4	0	+2	+1	+2
February	+1	+2	+1	—1	—3	—1	—2	—1	—5	—1	—1	—1
March	+1	+2	—1	0	+4	+3	+2	+1	—3	—2	—1	—3
April	—2	+1	0	0	+1	+1	+3	—1	—3	—4	—3	—4
May	+1	—2	—2	—2	—2	0	—1	0	+1	—3	0	+2
June	+1	—1	+1	+2	—1	+4	+4	+3	+2	+2	0	—1
Mean	+1.0	+0.4	+0.5	0.0	—0.7	—0.1	—0.3	—0.8	—1.8	—1.2	—1.0	—1.6

TABLE VIII.—RECAPITULATION OF THE ANNUAL MEANS EXHIBITING THE LUNAR-DIURNAL VARIATION, FROM 22,045 OBSERVATIONS BETWEEN 1840 AND 1845, EXPRESSED IN SCALE DIVISIONS.

July to July.	0^h Up. cul.	1^h	2^h	3^h	4^h	5^h	6^h	7^h	8^h	9^h	10^h	11^h
1840–41	—0.4	+0.5	+1.5	—0.3	—0.1	+0.3	—2.4	—1.3	—1.5	—0.2	+1.3	—0.1
1841–42	+1.1	+2.0	+2.5	+0.9	+0.3	+1.1	+0.7	—1.1	—0.4	—1.3	—0.2	+1.0
1842–43	+0.7	—0.9	—1.0	+2.9	+0.1	+1.6	+0.7	+1.9	—1.2	+0.7	—1.0	0.0
1843–44	+0.9	+0.4	+0.8	+1.5	+0.9	—0.3	+0.4	+0.3	+0.1	—0.8	—1.2	—0.6
1844–45	—0.9	+0.6	+0.5	+0.3	0.0	+0.6	+0.6	+0.9	+0.1	—0.3	0.0	+0.2
Mean	+0.3	+0.5	+0.9	+1.1	+0.2	+0.7	0.0	+0.1	—0.6	—0.4	—0.2	+0.1

July to July.	12^h Low. cul.	13^h	14^h	15^h	16^h	17^h	18^h	19^h	20^h	21^h	22^h	23^h
1840–41	+1.0	+0.8	+0.1	+0.4	—0.2	+0.6	—2.3	—1.3	+0.6	—2.1	+0.6	+1.2
1841–42	+1.2	—0.1	—0.2	—1.5	—0.6	—2.4	—1.1	—0.2	—0.3	—0.3	—1.0	+1.4
1842–43	+0.1	+0.1	+1.4	—0.6	+0.8	—0.9	—1.9	+0.1	—0.3	—0.8	+0.9	0.0
1843–44	+0.3	—0.2	+0.2	+0.3	+1.3	—0.6	—0.9	—0.7	—0.9	—0.8	0.0	—0.2
1844–45	+1.0	+0.4	+0.5	0.0	—0.7	—0.1	—0.3	—0.8	—1.8	—1.2	—1.0	—1.6
Mean	+0.7	+0.2	+0.4	—0.3	+0.1	—0.7	—1.3	—0.6	—0.5	—1.0	—0.1	+0.2

If we give weight to the annual means according to the number of observations, they would be; one for the first and second year, three-fourths for the third year, one and three-fourths for the next year, and two for the last year: a general examination, however, shows that, owing to the disturbing effect of the progressive change, the monthly means are very nearly of equal value, derived either from the bi-hourly or the hourly series. It will also be shown in the sequel that the lunar diurnal variation is nearly the same in the summer and winter seasons; the means of Table V. and the final means of Table VIII. have therefore been adopted without reference to combinations or weights.

A comparison of the values of Table VIII. among themselves shows them to be very irregular, although derived from many thousand observations; a five year series of observations seems barely sufficient to exhibit a tolerably regular progression. In the following table two groups have been formed, one of results from three years, 1840 to 1843, comprising 8,797 observations, the other from the remaining two years comprising 13,248 observations. From these it appears that the lunar diurnal variation during these two periods exhibits the same general character.

LUNAR-DIURNAL VARIATION DURING THE PERIODS 1840–43 AND 1843–45.

Groups.	0^h	1^h	2^h	3^h	4^h	5^h	6^h	7^h	8^h	9^h	10^h	11^h
1840–43	+0.5	+0.5	+1.0	+1.2	+0.1	+1.0	—0.3	—0.2	—1.0	—0.3	0.0	+0.3
1843–45	0.0	+0.5	+0.7	+0.9	+0.4	+0.3	+0.5	+0.6	+0.1	—0.6	—0.6	—0.2
Groups.	12^h	13^h	14^h	15^h	16^h	17^h	18^h	19^h	20^h	21^h	22^h	23^h
1840–43	+0.8	+0.3	+0.4	—0.6	0.0	—0.9	—1.8	—0.5	0.0	—1.2	+0.2	+0.9
1843–45	+0.7	+0.1	+0.4	+0.2	+0.3	—0.4	—0.6	—0.7	—1.3	—1.0	—0.5	—0.9

Before proceeding to the analysis of the final result of Table VIII. the separate results have been combined into summer and winter groups; the first group comprising the months from April to September, the second group the months from October to March.

Table IX. exhibits the lunar diurnal variation of the horizontal force during the summer and winter seasons.

TABLE IX.—LUNAR-DIURNAL VARIATION IN SUMMER.

(In scale divisions.)

Apr. to Sept.	0h Up. cul.	1h	2h	3h	4h	5h	6h	7h	8h	9h	10h	11h
1840–41	+0.7	—0.9	+2.7	—3.2	+0.5	+1.7	—1.1	—1.5	—0.3	—1.7	+2.0	—3.2
1841–42	+0.2	+1.8	+2.3	+1.7	+1.2	+1.2	+0.7	—1.5	—0.3	—3.5	—0.5	+1.5
1842–43	+0.8	—0.2	—0.7	+3.6	+1.8	+1.7	+1.5	+2.7	—1.2	+0.5	—1.2	—0.5
1843–44	+1.5	+0.5	+0.3	+1.7	+1.2	—0.5	+0.2	+0.5	—0.3	—1.2	—2.3	—1.2
1844–45	—2.0	0.0	0.0	+0.7	0.0	+1.0	+0.7	+1.7	0.0	—0.7	—0.8	—0.3
Mean	+0.2	+0.2	+0.9	+0.9	+0.9	+1.0	+0.4	+0.4	—0.4	—1.3	—0.6	—0.7

	12h	13h	14h	15h	16h	17h	18h	19h	20h	21h	22h	23h
1840–41	+5.8	—2.3	+2.5	—1.2	+3.2	—1.0	—1.3	—0.5	+1.7	—4.3	—0.3	—2.2
1841–42	+0.7	—1.0	+0.2	—0.5	+2.0	—2.0	—1.5	+0.2	—0.2	—0.3	—1.2	+2.5
1842–43	+1.2	+0.5	+1.7	—0.3	+0.3	—2.0	—2.7	—1.8	—0.8	—1.5	—0.2	—1.3
1843–44	+0.2	—0.8	+0.3	+0.3	+2.2	—0.2	—0.7	0.0	—0.5	—1.0	+0.5	+0.5
1844–45	+0.8	+0.5	+0.7	0.0	—0.7	+0.2	+0.5	—0.2	—1.5	—2.2	—1.5	—2.0
Mean	+1.7	—0.6	+1.1	—0.3	+1.4	—1.0	—1.1	—0.5	—0.3	—1.9	—0.5	—0.5

LUNAR-DIURNAL VARIATION IN WINTER.

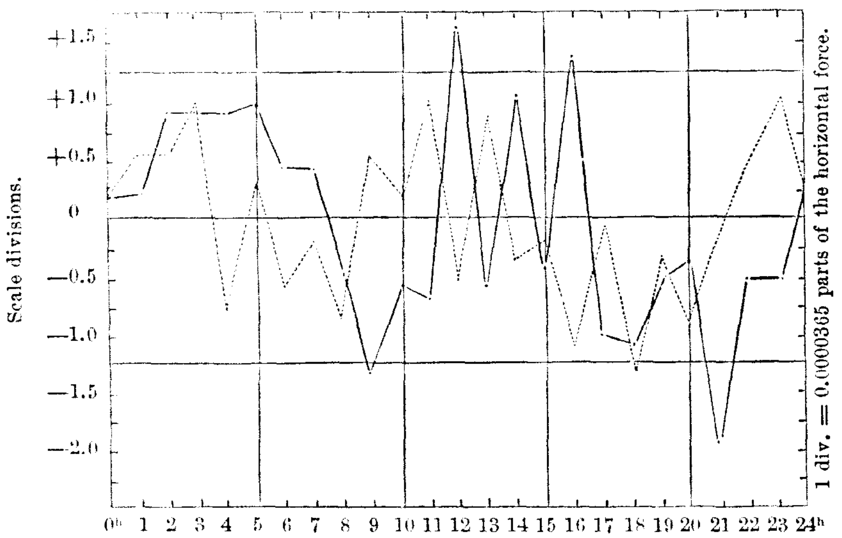
(In scale divisions.)

Oct. to Mar.	0h Up. cul.	1h	2h	3h	4h	5h	6h	7h	8h	9h	10h	11h
1840–41	—1.5	+1.8	+0.3	+2.5	—0.7	—1.0	—3.7	—1.0	—2.7	+1.3	+0.5	+3.0
1841–42	+2.0	+2.2	+2.7	+0.2	—0.5	+1.0	+0.7	—0.7	—0.5	+0.8	+0.2	+0.5
1842–43	+0.3	—2.3	—2.3	+1.3	—3.3	1.3	—1.0	+0.3	—1.3	+1.0	—0.7	+1.0
1843–44	+0.2	+0.3	+1.2	+1.3	+0.5	+0.0	+0.7	+0.2	+0.5	—0.3	0.0	0.0
1844–45	+0.2	+1.2	+1.0	—0.2	0.0	+0.2	+0.5	+0.2	+0.2	+0.2	+0.8	+0.7
Mean	+0.2	+0.6	+0.6	+1.0	—0.8	+0.3	—0.6	—0.2	—0.8	+0.6	+0.2	+1.0

	12h	13h	14h	15h	16h	17h	18h	19h	20h	21h	22h	23h
1840–41	—3.8	+4.0	—2.3	+2.0	—3.7	+2.2	—3.3	—2.2	—0.5	0.0	+1.5	+4.7
1841–42	+1.7	+0.8	—0.7	—2.5	—3.2	—2.8	—0.7	—0.5	—0.5	—0.3	—0.8	+0.3
1842–43	—2.0	—0.7	+1.0	—1.0	+1.7	+1.3	—0.3	+4.0	+0.7	+0.7	+3.0	+2.3
1843–44	+0.3	+0.3	0.0	+0.3	+0.3	—1.0	—1.2	—1.3	—1.3	—0.5	—0.5	—0.8
1844–45	+1.2	+0.3	+0.3	0.0	—0.7	—0.3	—1.2	—1.5	—2.2	—0.3	—0.7	—1.2
Mean	—0.5	+0.9	—0.3	—0.2	—1.1	—0.1	—1.3	—0.3	—0.8	—0.1	+0.5	+1.1

The results are exhibited in the annexed diagram. The number of observations (about 11,000 for each group) is evidently too small to eliminate the greater irregularities.

(A.)—Lunar-Diurnal Variation of the Horizontal Force in Summer and Winter.



If there is any marked difference in the lunar diurnal variation in the summer and winter season, the summer range is slightly greater than the winter range; as to the epoch, there is no doubt that in winter the lunar maxima and minima are earlier than in summer. It is a remarkable fact that we have found the same features in the lunar effect on the declination, viz., a greater amplitude in summer and an earlier occurrence of the maxima and minima in winter; the amount of the shifting of the two curves appears to be nearly the same. From the ten year series of observations at Prague (1840–49) Mr. Karl Kreil found a larger lunar effect in the summer months than in the winter months.

Recurring to the final values of the lunar-diurnal variation of the horizontal force, as given in Table VIII., they can be represented by the usual Besselian form of periodic functions.

The angle θ counts from the moon's upper culmination westward at the rate of 15° to an hour; a + sign indicated greater, a — sign, less force than the average normal. The observed values are represented by the following expression:—

$$H_{☾} = -0.01 + 0.40 \sin(\theta + 13^\circ 29') + 0.60 \sin(2\theta + 38^\circ 43') + 0.155 \sin(3\theta + 244^\circ 31').$$

The three coefficients are expressed in scale divisions; if expressed in parts of the horizontal force the equation may be written as follows: (M signifies millionth parts of the force.)

$$H_{☾} = -\overset{M}{0.36} + \overset{M}{14.60} \sin(\theta + 13^\circ.5) + \overset{M}{21.90} \sin(2\theta + 38^\circ.7) + \overset{M}{5.64} \sin(3\theta + 244^\circ.5.)$$

If expressed in absolute measure and if n = number of hours after the upper culmination, it may be written

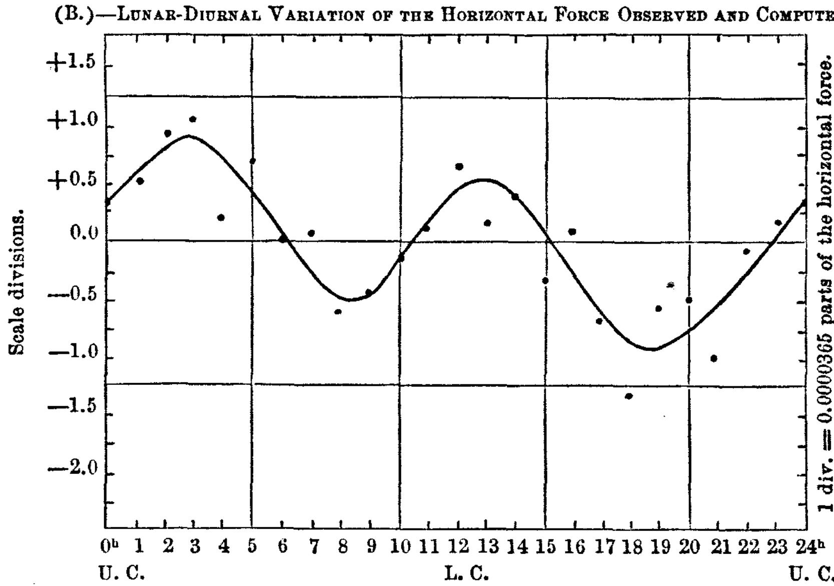
$$H_{☾} = -\overset{M}{1.5} + \overset{M}{61.0} \sin(15n + 13^\circ.5) + \overset{M}{91.5} \sin(30n + 39^\circ) + \overset{M}{23.6} \sin(45n + 244^\circ.5.)$$

The curve is double-crested and is exhibited, together with the observed values, in the annexed diagram. It presents two maxima and two minima, which are found from the equation

$$\frac{dH}{d\theta} = 0 = +\,0.40\,\cos\,(\theta + 13^\circ) + 1.20\,\cos\,(2\,\theta + 39^\circ) + 0.45\,\cos\,(3\,\theta + 245^\circ).$$

The lunar effect on the declination we have found likewise to present two maxima and two minima. (See Part III. of the discussion.)

(B.)—Lunar-Diurnal Variation of the Horizontal Force Observed and Computed.



We find:	Principal maximum	$2^h\ 52^m$ after Upper Culmination;	+ 0.87	scale divisions.
	Secondary "	1 7 " Lower "	+ 0.51	" "
	Principal minimum	6 41 " " "	— 0.87	" "
	Secondary "	8 19 " Upper "	— 0.45	" "

The epoch of the horizontal force tide for the high values is nearly 2 hours after the culminations, and for the low values it is $7\frac{1}{2}$ hours after the same phases.

For Makerstoun, in Scotland, at General Sir Thomas M. Brisbane's observatory, in 1843–46, Mr. J. A. Broun found (Trans. Royal Society of Edinburgh, Vol. XIX. p. 11, 1849) the smaller maximum of the horizontal force 2 hours after upper culmination, the greater maximum $1\frac{1}{4}$ hours after the lower culmination, the smaller minimum 8 hours after the upper culmination, and the greater minimum 9 hours after the lower culmination.

At Prague all extremes appear from 2 to 3 hours later. Mr. Karl Kreil (Denkschriften of the Imperial Academy of Sciences, at Vienna, Vol. V. 1853), found from the ten year series at Prague (1840—49) maxima of horizontal force between four and five hours after the upper and lower culminations, the latter being the greater of the two; and minima between ten and eleven hours after the same epoch, that after the upper culmination being the greater of the two.

From the Toronto observations, continued for five years, Major-General Sabine deduced the formula (see Vol. III. of the Toronto Magnetical and Meteorological Observations, London, 1857).

$$\Delta_x = +\,0.05 + 0.215\,\sin\,(a + 353^\circ.6) + 0.3324\,\sin\,(2\,a + 13^\circ.5).$$

The coefficients are in decimals of scale divisions (1 div. = 0.000087) parts of the horizontal force); the angle a counts from the superior culmination, giving a curve of which the general features are in exact accordance with those deduced from the Philadelphia observations, viz: a principal maximum after Upper Culmination, followed by the secondary minimum; the secondary maximum after the Lower Culmination, followed by a principal minimum. The times and amount of these values are compared in the following Table X.

TABLE X.—COMPARISON OF THE LUNAR-DIURNAL VARIATION OF THE HORIZONTAL COMPONENT OF THE MAGNETIC FORCE AS DEDUCED FROM 22,045 OBSERVATIONS BETWEEN 1840 AND 1845 AT PHILADELPHIA, AND AS DEDUCED FROM 34,303 OBSERVATIONS BETWEEN 1844 AND 1848 (A FIVE YEAR SERIES) AT TORONTO, CANADA.

	Philadelphia.	Toronto.
Time of principal maximum	$2^h.9$ after up. cul.	3^h after up. cul.
" " secondary minimum	8.3 " "	9 " "
" " secondary maximum	1.1 " low. cul.	2 " low. cul.
" " principal minimum	6.7 " "	8 " "
	In parts of horizontal force.	
Amount of principal maximum	+0.000032	+0.000046
" " secondary minimum	—0.000016	—0.000010
" " secondary maximum	+0.000019	+0.000024
" " principal minimum	—0.000032	—0.000041
	In absolute measure.	
Amount of principal maximum	+0.000133	
" " secondary minimum	—0.000068	
" " secondary maximum	+0.000078	
" " principal minimum	—0.000133	

Probable error of any single representation of the Philadelphia values = $\pm$ $0^d.25$ = $\pm$ 0.000009 parts of the horizontal force = $\pm$ 0.000038 in absolute measure.

Investigation of the Horizontal Force in Reference to the Lunar Phases.—The following process of reduction has been adopted: After marking the days of the full and new moon, and also the days preceding and following, the daily means of the horizontal force readings were taken (already corrected for difference of temperature and progressive change.) In the place of any disturbed observation, the monthly normal, belonging to the respective hour, was substituted before taking the daily mean. All accidental omissions in the record of the hourly or bi-hourly series were supplied by the hourly normal of the month. The means thus obtained are independent of the solar diurnal variation. The monthly normal was next compared with each daily mean and the differences (normal minus mean) were tabulated.

A positive sign signifies a greater; a negative sign, a less force than the normal value. As the results deduced from a single year are yet too much affected by the incidental irregularities of the observations, the collective results from the five year series (1840–45) are herewith presented.

TABLE XI.—INFLUENCE OF THE LUNAR PHASES ON THE HORIZONTAL FORCE.

	Scale divisions.	Parts of the hor. force.	In absolute measure.
One day before full moon	—1.0	—0.000036	—0.00015
On the day of full moon	—1.5	—0.000055	—0.00023
One day after full moon	—0.2	—0.000007	—0.00003
One day before new moon	+0.0	+0.000000	+0.00000
On the day of new moon	+2.4	+0.000091	+0.00038
One day after new moon	+0.9	+0.000033	+0.00014
Difference for new-full moon	3.9	0.000146	0.00061

The average number of observations from which any one of the above six means were deduced, is over 800, and the probable error, in scale divisions, of any one of the results is $\pm$ 0.7 (nearly).

From the Makerstoun observations, Broun found for the years 1843–46, a minimum at the time of the full moon, and a maximum at the time of the new moon; Kreil, from the Prague observations, between 1843–46, found the same result, all in accordance with the Philadelphia results, as given above. It must be remarked, however, that after the year 1848, Kreil found that the signs were reversed and consequently it appears that the lunar influence on the horizontal force is subject to a cycle of short period. This last remark does not apply to the effect of the moon's declination and variations in distance.

Influence of the Moon's Changes of Declination on the Horizontal Force.—The method of investigation is precisely the same as that adopted for the phases. We find:—

TABLE XII.

	Scale divisions.	
One day before the greatest north declination	+0.8	Mean +1.1.
On the day of " " " "	+0.6	
One day after " " " "	+2.2	
Two days after " " " "	+0.9	
On the day of the moon's crossing the equator	—1.2	Probable error of any one result $\pm$0.9.
One day before the greatest south declination	—3.4	Mean —0.6.
On the day of " " " "	—0.9	
One day after " " " "	+0.9	
Two days after " " " "	+1.0	

It seems probable that the greatest effect takes place rather a day after than on the day of the moon's greatest declination. Taking means, as indicated in the above table, we find about the time of the maximum north declination an increase of horizontal force of 1.1 scale divisions (or 0.000040 parts of the horizontal force); at the time of the moon's crossing the equator the force is decreased 1.2 scale divisions (or 0.000044 parts of the horizontal force); the horizontal force also appears decreased about the time of the moon's greatest south declination; the amount is about half that of the other two cases, and is somewhat doubtful, from an apparently excessive value on the preceding day.

According to Broun, there is at Makerstoun a maximum horizontal force at the time of the moon's greatest north and south declination, with a minimum force at the time of her crossing the equator; in two cases, therefore, viz: for north declination and no declination, the Makerstoun and Philadelphia results agree; while in the third case they disagree or remain doubtful. Kreil's results, from the Prague observations, do not appear to me sufficiently decisive and regular to admit of comparison.

Influence of the Moon's Variation in Distance on the Horizontal Force.—By a process of reduction similar to that followed in the preceding investigation we find:—

TABLE XIII.

	Scale divisions.	
One day before perigee	—1.5	s. d.
On the day of "	—1.9	Mean —1.8.
One day after "	—2.0	
One day before apogee	+2.3	
On the day of "	+2.3	Mean —2.4.
One day after "	+2.7	

The probable error of any one result is about the same as in the preceding results (Tables XI. and XII.). The results for variation in the moon's distance are more consistent and satisfactory than those depending on the phases and declination changes. The lunar effect is to diminish the horizontal force by its 0.000066 part in perigee, and to increase it by its 0.000088 part when she is in apogee.

The Prague results are the same, viz: a greater horizontal force at and after the moon's apogee than at and after her perigee; a three years' series of observations at Milan, however, do not agree therewith.

In no branch of magnetic research would additional results from independent observations, particularly at stations widely apart, be more acceptable and valuable than in the study of the lunar effect in its various manifestations.

PUBLISHED BY THE SMITHSONIAN INSTITUTION,

WASHINGTON CITY,

NOVEMBER, 1862.

SMITHSONIAN CONTRIBUTIONS TO KNOWLEDGE.
175

DISCUSSION

OF THE

MAGNETIC AND METEOROLOGICAL OBSERVATIONS

MADE AT THE GIRARD COLLEGE OBSERVATORY, PHILADELPHIA,
IN 1840, 1841, 1842, 1843, 1844, AND 1845.

THIRD SECTION,

COMPRISING PARTS VII, VIII, AND IX. VERTICAL FORCE.

INVESTIGATION OF THE ELEVEN (OR TEN) YEAR PERIOD AND OF THE DISTURBANCES OF THE VERTICAL COMPONENT OF THE MAGNETIC FORCE, AND APPENDIX ON THE MAGNETIC EFFECT OF THE AURORA BOREALIS; WITH AN INVESTIGATION OF THE SOLAR DIURNAL VARIATION, AND OF THE ANNUAL INEQUALITY OF THE VERTICAL FORCE; AND OF THE LUNAR EFFECT ON THE VERTICAL FORCE, THE INCLINATION, AND TOTAL FORCE.

BY

A. D. BACHE, LL.D., F.R.S.,

MEM. CORR. ACAD. SC. PARIS; PREST. NAT. ACAD. SCIENCES; SUPERINTENDENT U. S. COAST SURVEY.

[ACCEPTED FOR PUBLICATION, AUGUST, 1863.]

PART VII.

INVESTIGATION

OF THE

ELEVEN (OR TEN) YEAR PERIOD, AND OF THE DISTURBANCES OF THE VERTICAL FORCE.

INVESTIGATION

OF THE

ELEVEN (OR TEN) YEAR PERIOD, AND OF THE DISTURBANCES OF THE VERTICAL FORCE.

THE observations of the vertical component of the magnetic force were commenced in June, 1840, and continued, with an exception in January, 1841, without interruption to the last of June, 1845. To keep up the continuity of the series, a daily reading was taken at 2^h 17^m P. M. during the months of January, February, and March, 1843. Up to October, 1843, the observations were bi-hourly, afterwards hourly.

Instruments.—From June, 1840, to the end of the year, the observations were made with a balance vertical force magnetometer of Lloyd's pattern. It was at first mounted in the eastern building of the College, but was removed to the observatory in the latter part of July. While in the College an increase of the readings corresponds to a decrease of vertical force; at the observatory increasing readings denote increasing force. The instrument was made by Robinson, of London; the magnet, the axis of which was mounted as nearly as possible transversely to the magnetic meridian, was 12 inches in length, having at its ends cross wires set in copper rings. For a full description see Dr. Lloyd's account of the Magnetical Observatory of Dublin, and the preface in volume I, of the record of the Philadelphia observations.

In January, 1841, the Lloyd instrument was replaced by a reflecting vertical force magnetometer, made at my suggestion by Mr. J. Saxton. The bar of this instrument was two feet and one inch in length, two inches wide in the middle, one and a half near the ends, tapering to nothing at the ends, and a quarter of an inch thick. The magnet was of steel and hardened as perfectly as the maker could effect. By means of a ball moving on a fine screw, its equilibrium could be changed. The mirror projected outside the box, and the motion of the bar was observed by means of a telescope. At the top of the box was a piece of plate glass through which a thermometer (of Francis' make) could be read. For further particulars see p. vii, of the preface to volume I of the record. For some time (between three and four months) after being put up, the bar lost considerably of its magnetic force, and after being in use four months, a movement of the adjusting ball upon the screw was required for placing the readings again near the middle of the scale. By this adjustment, the sensibility of the apparatus was not interfered with.

The value of a scale division of the Lloyd instrument, expressed in parts of the vertical force, was carefully determined and found to be $=0.0000165$, both in the

College building and at the observatory. This value being known, I considered that the value of the scale of the new reflecting magnetometer could best be ascertained by comparison with the former. The result of this, continued at intervals, was, that two divisions of the new scale were equivalent to one of the old, or that a change of one division of the reflecting instrument corresponded to a change of vertical force of 0.000033 parts. This was after the instrument had been finally adjusted.

The only disadvantage in the new instrument was the large effect of changes of temperature upon it; by direct observations it was found that a change of 1° (F.) of temperature produced a corresponding change of 13.5 ± 0.25 scale readings, whereas in the Lloyd instrument the corresponding change was but 3.12 scale divisions. We have accordingly for the Lloyd instrument $q=0.0000515$, and for the reflecting instrument $q=0.000446$. The values actually used in the reduction of the observed reading to a standard temperature will be seen further on.

The importance of ascertaining the most correct and suitable coefficients of temperature for the two series of observations, demands a more detailed statement and elaborate discussion of the observations themselves independently of the special trials. Experience has shown that the value for q deduced from the differential intensity observations themselves, with the magnet subject generally to gradual and small changes of temperature, is smaller by a considerable fraction than the value found by direct and special observation during which the temperature changes are necessarily more violent. There is no doubt that in the reduction to a standard temperature that value of q should be used which was obtained while the magnet was under its ordinary influences and condition. The same view is taken by General Sabine, and was also carried out in the discussion of the horizontal component of the magnetic force; for which see the preceding paper (Part IV).

Determination of the Effect of a Change of Temperature on the Readings of the Vertical Force.

(A.) Results of special observations made for determining the temperature coefficient. The correction for temperature of the Lloyd vertical force magnetometer was ascertained by the usual method of vibrating the bar when suspended horizontally, and when alternately heated and cooled artificially. The thermometer was placed with its ball near the axis of the magnet. The changes of the horizontal force magnetometer, while these experiments were going on, were noted and allowed for.

Date. Feb'y, 1841.	Time of 10 oscillations.	Temp. (F.)	Readings of Horz'l force.	Temp. (F.)
9th	87ˢ.950	37°.2	1128.8	25°.6
"	87.900	41.0	1079.3	36.5
"	88.117	94.6	1139.5	36.1
Result	87.990	39.1		
"	88.117	94.6		

hence $q=\frac{2}{t'-t}\cdot\frac{\tau'^2-\tau^2}{\tau'^2+\tau^2}=0.0000520$

which is equivalent to 3.15 scale divisions; in the first reduction of the record 3.12 was used.

Before putting the reflecting vertical force magnetometer in its place in January, 1841, observations were made for its correction for temperature by means of deflections; the result, however, was not satisfactory, owing to the small difference in the deflections at high and low temperatures, and the necessity of keeping the bar at a proper distance from the declinometer to prevent the possibility of a permanent change of magnetism. The weight of the mirror and other fixtures of the bar rendered the method of horizontal oscillations impracticable without their removal, and it was finally decided to determine the value of q by means of a subsidiary instrument kept at a uniform temperature in a separate building, while the vertical force instrument at the observatory was subject to considerable fluctuations of temperature. The subsidiary instrument consisted of a small dipping needle mounted on a knife edge, and rendered horizontal by weighting it. The indications, however, did not prove very satisfactory; 14 scale divisions were indicated as the correction for 1° change in temperature. Subsequently an inclinometer, according to Prof. Lloyd's plan, was mounted as a subsidiary instrument, and observed twice a day with the vertical force instrument at the observatory. The mean values, expressed in scale divisions, thus found between February, 1843, and January, 1844, are as follows:—

13.3 14.3 14.4 12.3 12.2 13.1 and 15.4.

Average value 13.56 ± 0.25. In the first reduction the value 13.5 was used.

(B.) Investigation of the temperature coefficient from the regular series of observations. We will first examine the principal series observed between 1841 (February), and 1845 (June), with the reflecting magnetometer. In February, March, April, and May, 1841, the readings gradually increased and approached the end of the scale, requiring a readjustment of the instrument after May 22. It was supposed that —529 scale divisions would be an approximate correction for referring the observations to the indications of the scale subsequent to May 22, the uninterrupted series of observations commencing with June 1, 1841. The following table contains the *uncorrected* monthly means of the vertical force magnetometer together with the observed mean monthly temperature taken directly from the record. The tabular means for January, February, and March, 1843, when the instrument was read only once a day (at 2^h 17^m P. M.), were obtained as follows: The difference between the daily mean and the mean at 2^h 17^m P. M. was ascertained for each month, from the records of the preceding year (1842) and the following year (1844). The mean correction to the average reading at 2^h 17^m P. M. to refer the same to the mean of the day and month is +18.6, +14.4, and +11.2 scale divisions for the months of January, February, and March, respectively. These corrections have been applied.

TABLE I.—UNCORRECTED MONTHLY MEANS OF VERTICAL FORCE MAGNETOMETER AND CORRESPONDING MEAN MONTHLY TEMPERATURES.

1841.			July . . .	$792^{d}.9$	$77^{\circ}.5$	1844.		
February . .	$1059^{d}.0$	$35^{\circ}.58$	August . .	775.9	75.4	January . .	$636^{d}.1$	$57^{\circ}.0$
March . . .	1236.2	43.96	September .	775.0	74.3	February . .	657.4	59.2
April . . .	1357.4	50.8	October . .	763.8	71.4	March . . .	769.5	67.2
May . . .	1445.6	56.2	November .	725.0	66.8	April . . .	750.1	65.4
June . . .	722.6	74.5	December .	672.8	62.5	May . . .	798.4	69.9
July . . .	810.3	77.6	1843.			June . . .	845.1	73.6
August . .	776.1	75.85	January . .	681.9	64.0	July . . .	913.2	77.2
September .	702.5	71.1	February . .	615.8	57.6	August . .	877.2	75.4
October . .	448.7	54.5	March . . .	555.5	53.8	September .	860.7	71.7
November .	339.8	47.8	April . . .	762.4	71.3	October . .	834.7	71.2
December .	534.1	59.6	May . . .	734.1	70.4	November .	748.2	63.9
1842.			June . . .	784.6	75.9	December .	695.6	60.8
January . .	679.4	67.7	July . . .	800.5	77.4	1845.		
February . .	695.5	65.0	August . .	817.1	77.8	January . .	712.6	62.2
March . . .	668.9	66.8	September .	797.9	74.3	February . .	650.1	56.3
April . . .	671.2	67.4	October . .	749.0	69.6	March . . .	775.7	60.2
May . . .	693.9	69.7	November .	728.0	64.7	April . . .	721.1	65.7
June . . .	718.0	71.9	December .	705.9	62.5	May . . .	688.8	64.1
						June . . .	811.1	74.2

Combining the preceding values by months, we obtain a complete series extending over four years. The first five months are necessarily omitted; the break in the series occurred between the 4th and 5th month.

Uncorrected readings of the vertical force magnetometer. One division of scale =0.000033 parts of the vertical force.

Month.	1841–42.	1842–43.	1843–44.	1844–45.	1841–45.
July	810.3	792.9	800.5	913.2	829.2
August	776.1	775.9	817.1	877.2	811.6
September	702.5	775.0	797.9	860.7	784.0
October	448.7	763.8	749.0	834.7	699.1
November	339.8	725.0	728.0	748.2	635.2
December	534.1	672.8	705.9	695.6	652.1
January	679.4	681.9	636.1	712.6	677.5
February	695.5	615.8	657.4	650.1	654.7
March	668.9	555.5	769.5	675.7	667.4
April	671.2	762.4	750.1	721.1	726.2
May	693.9	734.1	798.4	688.8	728.8
June	718.0	784.6	845.1	811.1	789.6
				Mean	721.3

Corresponding readings of the thermometer (F.).

Month.	1841–42.	1842–43.	1843–44.	1844–45.	1841–45.
July	$77^{\circ}.6$	$77^{\circ}.5$	$77^{\circ}.4$	$77^{\circ}.2$	$77^{\circ}.42$
August	75.85	75.4	77.8	75.4	76.11
September	71.1	74.3	74.3	71.7	72.85
October	54.5	71.4	69.6	71.2	66.68
November	47.8	66.8	64.7	63.9	60.80
December	59.6	62.5	62.5	60.8	61.35
January	67.7	64.0	57.0	62.2	62.72
February	65.0	57.6	59.2	56.3	59.53
March	66.8	53.8	67.2	60.2	62.00
April	67.4	71.3	65.4	65.7	67.45
May	69.7	70.4	69.9	64.1	68.52
June	71.9	75.9	73.6	74.2	73.90
				Mean	67.45

The last column contains the mean readings. They may be represented by the equation:—

$$V = V_m + \Delta e x + \Delta t y$$

where x= monthly amount of loss of magnetism and effect of secular change.
y= change in magnetometer reading for a change of temperature of 1° F.
Δe= epoch—middle epoch. The middle epoch is January 1st.
Δt= temperature—mean temperature.
V_m= mean reading of the vertical force magnetometer.
V= any of the monthly means to be represented.

From the 12 conditional equations, we form the normal equations

$$-828.90 = +143.000\,x - 85.335\,y$$
$$+4685.73 = -85.335\,x + 443.120\,y$$

whence $x = +0.577$, the monthly change, equal to nearly 7 scale divisions for each year.

And $y = +10.68$ scale divisions, the correction for temperature for 1° F. This is not quite three-fourths of the value found by direct measure.

Second determination of the temperature coefficient by means of alternate combinations by seasons.

The mean values for each season have been directly formed from table No. 1. The value in June, 1845, is necessarily omitted.

				Alternate means.		Differences.		Temp. coefficient.
1841	June to November	633d.3	66°.89					
1841–2	December to May	657.2	66.03	695d.8	69°.88	+38d.6	+3°.85	10d.0
1842	June to November	758.4	72.88	663.8	64.65	—94.6	—8.23	11.5
1842–3	December to May	670.4	63.27	769.0	73.08	+98.6	+9.81	10.1
1843	June to November	779.5	73.28	695.0	63.40	—84.5	—9.88	8.6
1843–4	December to May	719.6	63.53	813.0	72.72	+93.4	+9.19	10.2
1844	June to November	846.5	72.17	705.1	62.54	—141.4	—9.63	14.7
1844–5	December to May	690.6	61.55					
					Mean			10.85
					By preceding method . .			10.68
					Mean, adopted			10.77

We have for the reflecting magnetometer $k = 0.000033$ $\frac{q}{k} = 10.77$, hence $q = 0.000355$. For comparison we have the corresponding values at Toronto $\frac{q}{k} = 1.80$ and $q = 0.000113$.

The scale value k at Toronto is 0.0000628, nearly twice as large as at Philadelphia. The comparatively large value for q at Philadelphia is most probably due to the large size of the bar which prevents a thorough hardening, a circumstance which undoubtedly also contributes to the difference exhibited by the resulting value of q as found by the direct and indirect methods.

The magnitude of the temperature coefficient requires that the standard temperature should be the mean temperature at all the readings. The average temperature between February, 1841, and June, 1845, is 66°.0, which has been adapted as the standard temperatare to which all the vertical force readings, taken with the reflecting magnetometer, have been referred.

A close examination of the record of the Lloyd balance magnetometer, which was used in June and July, 1840, in the College, and afterwards at the observatory during five months, proved that in point of accuracy it would not compete with the reflecting magnetometer mounted in January, 1841, and continued in use for four years and a half. Owing to some imperfection in the first named instrument, its indications were very unsteady, and at times fitfully changeable; thus in September, October, and December, there are differences in the daily means (deduced from twelve readings and referred to 32° Fahrenheit) of adjacent days of more than 200 scale divisions, and in one instance (October 19–20) amounting even to 256 divisions. In August there is a change of 389 scale divisions in three consecutive days, and in October (17th to the 20th) one of 477 divisions in the means during the same interval. There is besides a large progressive change, showing that the instrument was in a very unstable equilibrium; this change amounted in the first month to over 300 scale divisions. An attempt was also made to deduce a temperature coefficient by comparing mean daily readings of short and specially selected periods of a few days each, with average high and low temperatures, but it failed for want of sufficient uniformity in the indications of the instrument. In such a series the disturbed indications could not be recognized and separated from the regular readings. It was finally concluded to make no use of the observations prior to January, 1841.

Reduction of the Observations, between February, 1841, *and June,* 1845, *to a uniform Temperature.*—A table has been constructed, with the observed temperature as the argument, giving the reduction for difference of temperature from the normal temperature (66° Fahr.); by means of this table each observation has been referred to its corresponding value as the standard temperature. Table No. 2 contains the monthly mean readings for each observing hour; the time is local time, and reckoned from midnight to midnight to 24 hours. The tenths in the record have been omitted, as of no special value, since an error in the recorded temperature of only 0°.1 affects the magnetometer reading by more than a scale division. An increase of scale readings corresponds to a decrease of vertical force, and one division equals 0.000033 parts of the force. Accidental irregularities in the record are especially referred to in foot notes.

The tabular values are directly taken from the manuscript tables containing the single reduced readings and their monthly means.

In the present state of our knowledge regarding the occurrence of the disturbances it is not safe to make any interpolations in the magnetometer record in case of an accidental omission; a rule which has been strictly adhered to.

TABLE II.—RECORD OF THE MONTHLY MEANS OF THE VERTICAL FORCE MAGNETOMETER READINGS FOR EACH OBSERVING HOUR, AND REDUCED TO UNIFORM TEMPERATURE OF 66° FAH.

1841.	0^h	2	4	6	8	10	Noon.	14	16	18	20	22^h	$+17^m$
February	861	861	860	857	851	846	845	849	847	878	872	867	
March	956	954	949	943	936	933	931	928	941	953	957	957	
April	1004	997	994	988	982	982	981	982	989	997	1006	1009	
May	1033	1031	1030	1021	1012	1009	1008	1008	1012	1022	1037	1043	
June	641	634	631	621	618	618	616	620	627	641	652	653	
July	704	698	686	675	669	666	667	700	684	695	708	707	
August	684	680	673	665	664	655	653	659	663	672	686	690	
September	665	660	657	651	642	633	632	632	637	646	656	662	
October	583	582	578	572	562	559	561	561	569	577	581	582	
November	540	543	540	544	531	528	526	527	532	535	538	540	
December	602	595	595	599	593	598	605	600	606	613	613	613	

Notes to the above table:—February 25th, 6^h 17^m, temperature interpolated, 28°. March 2d, 0^h 17^m, reading 32 minutes late, t=46°.8. March 11th, 22^h 17^m, reading 49^m late, t=41°.7. March 27th, 20^h 17^m, reading 40^m late, t=64°.8. March 29th, 22^h 17^m, reading 43^m late, t=50°.5. April 9th, 16^h 17^m, reading 25^m late, r=920, t=57°.6. April 30th, 0^h 17^m, reading 59^m late, r=805, t=49°.3. May 22d, 14^h 17^m, observations discontinued. Between this date and June 1st the instrument was readjusted, the corrections required to make the readings of the first four months comparable with the continuous series following will be investigated further on. June 29th, 22^h 17^m, reading 38^m late, temperature 81°.8, interpolated. July 22d, 16^h 17^m, temperature 84°.5 interpolated. August 23d, 24th, seven observations were omitted between 20^h 17^m and 8^h 17^m on account of the magnet being fixed by a spider's line which was found attached to the mirror. August 24th, 14^h 17^m, observation rejected, the sun shining on the box. October 4th, 16^h 17^m, sun shining on the needle; October 13th, 22^h 17^m, observation 7^m late; October 28th, 22^h 17^m, observation 67^m late. In December the variations of temperature are unusually large; they seem to demand a greater value of the temperature coefficient. December 8th, 16^h 17^m, observation 8^m late; December 20th, 4^h 17^m, observation 9^m late; December 30th, 18^h 17^m, temperature 69°.0, interpolated.

TABLE II.—*Continued.* VERTICAL FORCE READINGS AT 66° FAH.

1842.	0^h	2	4	6	8	10	Noon.	14	16	18	20	22^h	$+17\frac{1}{2}^m$
January	651	651	663	659	657	651	661	648	660	679	674	674	
February	705	704	714	714	707	698	704	696	699	714	707	701	
March	661	656	662	664	663	662	655	649	655	667	673	666	
April	666	655	659	655	653	654	649	643	645	655	667	670	
May	662	665	657	649	646	647	652	644	647	654	664	665	
June	674	669	663	658	648	643	639	640	636	652	665	663	
July	690	685	675	667	663	656	652	651	656	669	681	684	
August	689	687	685	682	675	668	655	655	665	677	684	686	
September	698	692	696	689	686	677	671	671	673	679	690	698	
October	707	699	703	712	696	709	708	707	707	709	711	706	
November	718	710	723	725	713	715	713	713	716	711	718	718	
December	708	708	714	709	716	711	709	707	705	710	713	713	

Notes to above table:—February 3d, 14^h $17\frac{1}{2}^m$, the temperature 73°.5 is interpolated. May 9th, 10^h $17\frac{1}{2}^m$, the temperature 59°.6 is interpolated. June 6th, 0^h $17\frac{1}{2}^m$, 2^h $17\frac{1}{2}^m$, and 18^h $17\frac{1}{2}^m$, the temperatures 70°.4, 71°.0, and 74°.4 respectively, were interpolated. August 3d, 12^h $17\frac{1}{2}^m$; 5th, 22^h $17\frac{1}{2}^m$; 6th, 10^h $17\frac{1}{2}^m$, and 31st, 14^h $17\frac{1}{2}^m$, the temperatures 69°.0, 73°.7, 76°.0, and 67°.6 respectively, were interpolated. September 1st, 22^h $17\frac{1}{2}^m$, the temperature 77°.0 is interpolated. October 8th, 2^h $17\frac{1}{2}^m$; 21st, 10^h $17\frac{1}{2}^m$, and 28th, 6^h $17\frac{1}{2}^m$, the temperatures 66°.1, 68°.1, and 70°.8 respectively, were interpolated. November 3d, 14^h $17\frac{1}{2}^m$; and 16th, 6^h $17\frac{1}{2}^m$, the observations are 6^m and 7^m late.

2

TABLE II.—*Continued.* VERTICAL FORCE READINGS AT 66° FAH.

1843.	0^h	2	4	6	8	10	Noon.	14 $+17^m$	16	18	20	22^h	$+23\frac{1}{2}^m$
January								685					
February								692					
March								676					
April	715	713	713	712	708	702	705	687	697	696	706	709	
May	698	697	695	690	683	680	677	666	678	679	697	690	
June	696	691	693	690	677	669	664	660	659	671	681	690	
July	692	693	693	689	681	673	664	660	660	667	678	686	
August	702	703	707	705	695	682	670	672	672	682	694	698	
September	722	720	721	716	707	705	694	693	694	701	710	708	
October	714	708	714	717	704	703	702	703	704	714	719	714	
November	740	737	744	744	743	740	734	735	746	750	748	745	
December	749	737	740	740	739	728	724	738	755	759	762	754	

Additional odd hours observed.

1843.	1^h	3	5	7	9	11	13	15	17	19	21	23^h	$+23\frac{1}{2}^m$
October	710	709	718	712	704	700	701	702	709	717	715	717	
November	737	740	745	747	742	735	731	741	750	751	746	747	
December	744	738	742	743	738	720	725	747	755	763	756	757	

Notes to the above table:—January 4th, February 1st, and March 24th, observations 7^m, $7\frac{1}{2}^m$, and 20^m late, respectively. In April seven readings were supplied by the observers, also one in May and one in June. July 14th, 0^h $23\frac{1}{2}^m$, observation 6^m late. August 10th, 16^h $23\frac{1}{2}^m$, observation supplied by observer. August 29th, 0^h $23\frac{1}{2}^m$, observation 12^m late. September 20th, 0^h $23\frac{1}{2}^m$, temperature supplied by observer. November, six readings supplied by observers. December 1st, 4^h $23\frac{1}{2}^m$; December 9th, 1^h $23\frac{1}{2}^m$; December 12th, 21^h $23\frac{1}{2}^m$, and December 22d, 5^h $23\frac{1}{2}^m$, observations 5^m, 6^m, 15^m, and 8^m late, respectively. December 19th, 2^h $23\frac{1}{2}^m$, a printing error of 200 scale divisions was corrected. December 30th, 9^h $23\frac{1}{2}^m$, reading supplied by observer.

TABLE II.—*Continued.* VERTICAL FORCE READINGS AT 66° FAH.

1844.	0^h	1	2	3	4	5	6	7	8	9	10	11^h	$+23\frac{1}{2}^m$
January	735	733	731	731	733	736	733	733	731	730	726	717	
February	736	731	729	730	733	732	733	734	727	725	727	720	
March	763	758	758	759	760	762	763	760	755	759	761	762	
April	7[illegible]6	765	763	765	766	765	763	759	752	751	746	740	
May	772	769	766	758	767	764	760	754	749	747	747	744	
June	778	776	772	772	771	768	765	7[illegible]0	752	750	749	747	
July	809	807	803	802	801	798	794	789	780	779	778	777	
August	794	792	788	787	785	783	780	774	768	765	761	759	
September	815	813	811	808	809	807	805	802	796	793	788	783	
October	779	776	773	776	778	780	781	782	775	774	775	771	
November	775	771	768	769	772	772	771	773	768	772	772	767	
December	756	752	753	754	756	757	756	7[illegible]0	752	752	754	749	

1844.	Noon.	13	14	15	16	17	18	19	20	21	22	23^h	$+23\frac{1}{2}^m$
January	717	713	717	724	732	740	743	745	747	746	744	745	
February	716	719	720	723	731	735	742	743	743	739	738	737	
March	758	751	752	755	752	751	751	749	750	751	759	762	
April	739	735	738	744	750	754	758	764	764	762	765	766	
May	744	740	740	744	746	748	753	762	765	766	768	772	
June	746	745	748	752	754	757	765	772	775	774	775	778	
July	776	775	772	778	780	787	795	801	804	804	806	809	
August	756	756	752	765	767	769	777	787	788	788	790	793	
September	779	777	766	791	790	793	804	810	809	810	812	815	
October	769	767	771	778	787	789	792	790	786	786	782	782	
November	766	763	761	758	774	779	777	778	774	770	769	772	
December	745	739	740	735	750	756	754	753	753	750	751	754	

Notes to preceding table:—January 2d, $10^h\ 23\frac{1}{2}^m$, temperature observation 30^m late. January 8th, $10^h\ 23\frac{1}{2}^m$, instrument disturbed. January 15th, $3^h\ 23\frac{1}{2}^m$, temperature 56°.3 interpolated. February 6th, $4^h\ 23\frac{1}{2}^m$, and 13th, $9^h\ 23\frac{1}{2}^m$, temperature observation 15^m and 20^m late, respectively. April 11th, $0^h\ 23\frac{1}{2}^m$ and $1^h\ 23\frac{1}{2}^m$, readings supplied by observer. July 13th, $12^h\ 23\frac{1}{2}^m$, observation 36^m late. August 26th and 27th, thirteen readings supplied by observers. October 1st, $22^h\ 23\frac{1}{2}^m$, observation 8^m late.

TABLE II.—*Continued.* VERTICAL FORCE READINGS AT 66° FAH.

1845.	0^h	1	2	3	4	5	6	7	8	9	10	11^h	$+23\frac{1}{2}^m$
January	754	748	749	751	752	756	762	767	760	758	753	751	
February	761	756	753	757	759	760	763	764	756	756	752	749	
March	749	743	739	742	745	744	745	743	735	733	732	729	
April	732	727	726	728	729	727	725	724	719	718	718	717	
May	722	720	718	721	718	717	715	711	707	704	702	701	
June	733	731	729	729	729	727	726	722	717	715	715	711	
1845.	**Noon.**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23^h**	**$+23\frac{1}{2}^m$**
January	752	748	749	739	754	759	756	753	748	746	746	753	
February	747	741	740	739	753	759	761	761	758	753	752	756	
March	730	729	713	731	737	741	745	746	740	740	739	742	
April	716	713	715	724	726	723	732	737	735	729	726	728	
May	699	698	680	700	704	704	712	719	719	717	719	720	
June	711	711	711	714	713	715	722	730	733	731	731	732	

Notes to above table:—April 27th, $4^h\ 23\frac{1}{2}^m$, reading supplied by observer. April 14th, $2^h\ 23\frac{1}{2}^m$, observation 12^m late. April 22d, 23d, and 28th, $14^h\ 23\frac{1}{2}^m$; also April 22d, $15^h\ 23\frac{1}{2}^m$, readings supplied by observer. April 22d, $16^h\ 23\frac{1}{2}^m$, temperature supplied by observer. May 2d, $14^h\ 23\frac{1}{2}^m$, reading supplied by observer. May 12th, $4^h\ 23\frac{1}{2}^m$, 9^m late. June 6th, $5^h\ 23\frac{1}{2}^m$, and 28th, $1^h\ 23\frac{1}{2}^m$, observations 13^m and 9^m late, respectively. June 12th, $0^h\ 23\frac{1}{2}^m$ and $1^h\ 23\frac{1}{2}^m$, readings supplied by observer.

TABLE III.—MEAN MONTHLY READINGS OF THE VERTICAL FORCE REDUCED TO THE TEMPERATURE OF 66° FAH.

	1841.	1842.	1843.	1844.	1845.
January		661	699	733	752
February	858	705	702	731	754
March	945	661	684	757	738
April	993	656	705	756	725
May	1022	654	686	756	710
June	631	654	678	762	722
July	688	669	678	792	
August	670	676	690	776	
September	648	685	708	799	
October	572	706	710	779	
November	535	716	742	770	
December	603	710	744	751	
Mean		679	702	763	

The monthly mean for January, February, and March, 1843, was obtained by adding 14, 10, and 8 divisions to the readings at $14^h\ 7^m$ respectively; these corrections were found by comparisons in 1842 and 1844.

Corrections for progressive and irregular changes.—The difficulty of fully eliminating all effects of changes of temperature, and adjustment, particularly during the first year (1841), demanded the application of a secondary process analogous to that used in the reduction of the horizontal force for progressive change. The progressive change in the readings of the vertical force is less decided and more fluctuating than in the horizontal force. Half monthly means, and in special cases, means of even less periods of time, have been taken and were compared with the monthly mean, the differences were applied either progressively (increasing or diminishing) or as constants, as the case seemed to demand.

Seventeen months required no such correction, and in many months it was applied very sparingly.

The process leaves the diurnal variation, relatively, undisturbed, and prepares the series for the application of Peirce's Criterion for the recognition of the disturbances. The individual figures thus corrected were inserted in blue ink in the manuscript tables.

Recognition and separation of the larger disturbances.—Peirce's Criterion for the recognition of the disturbances was applied to the observations extending over four years, and commencing with July, 1841, in the following order: July 0^h, August 2^h, September 4^h, October 6^h, November 8^h, December 10^h, January (1842) 12^h, etc. The odd hours were selected from July, 1844, to the close of the series, thus July 1^h, August 3^h, September 5^h, etc. The following limits of separation, in scale divisions, have been found for each year:—

July,	1841 —	June,	1842,	limit,	52
"	1842 —	"	1843,	"	46
"	1843 —	"	1844,	"	40
"	1844 —	"	1845,	"	33
				Average limit,	43

As this limit would only separate 1 in every 34 observations, and would not furnish a sufficient number of disturbances to investigate their laws to advantage, it was necessary to contract the above limit, and 30 scale divisions were finally selected. There can be no doubt that the limiting number as found by the use of the criterion is too high, owing to the unavoidable presence of irregularities ascribable to imperfection in the corrections for temperature in some cases, and in others due to apparently fitful changes in the instrument. 30 scale divisions =0.00099 parts of the vertical force =0.0127[1] in absolute measure, adopted as limit of deviation of any observation from its corresponding mean monthly value for the same hour, will furnish an average value for the ratio of the number of disturbances to the whole number of observations. The ratio of a disturbance to the whole force is also nearly the same for the horizontal and vertical component.

All deviations over 30 divisions from the mean were marked, and a new mean was taken, the hourly observations were again compared with this new mean, and

[1] The vertical force, in absolute measure, is on the average, between 1841 and 1845, equal to 12.84 (English units), as stated in a subsequent number of this discussion.

the process was repeated, if necessary, until all deviations above 30 had been separated; the final hourly means for each month, thus found and known as the "normals," are given in the following tables.

TABLE IV.—BI-HOURLY NORMALS OF THE VERTICAL COMPONENT OF THE MAGNETIC FORCE IN 1841.

One division of the scale =0.000033 parts of the vertical force. Increasing numbers denote decrease of force. The observations are made 17^m after the full hours.

1841.	0^h	2	4	6	8	10	Noon.	14	16	18	20	22^h
February	664	6 4	662	656	654	650	644	648	647	673	668	665
March	670	661	661	655	651	645	643	646	656	665	666	673
April	671	662	658	653	645	646	646	649	656	666	670	676
May	669	665	660	654	647	649	644	646	650	660	674	679
June	646	631	622	617	624	616	603	624	635	650	653	657
July	703	697	687	671	667	664	665	676	680	698	708	706
August	686	680	676	664	662	652	653	662	666	676	689	691
September	6 3	655	646	647	637	631	627	628	634	645	653	660
October	579	578	573	568	558	556	561	562	571	577	581	583
November	532	537	538	533	526	523	520	521	526	529	531	538
December	597	591	590	606	592	598	605	597	607	610	605	604

The normals for February, March, April, and May, have been diminished by 198, 278, 333, and 361 scale divisions respectively; the uncorrected monthly means are 856, 936, 991, and 1019, which can be exactly represented by the expression $r=966+54.4\ \Delta t-12.8\ \Delta t^2$, where $r=$ monthly reading and t, expressed in units of a month, counts from April 1 as the epoch. It shows that the monthly increase is uniformly retarded. The mean reading from the four succeeding months is 658, the corrections to February, March, April, and May, as applied, will produce the same mean.

The rapid change in the monthly means for some adjacent months makes a small correction necessary to the monthly means, viz: of plus one scale division to the February, March, and December means of the hours 0, 2, and 4, and to the September and October means at the hours 18, 20, and 22; of minus one scale division to the February, March, and December means at the hours 18, 20, and 22, and to the September and October means at the hours 0, 2, and 4. This small correction is included in the above normals.

TABLE IV.—*Continued.* BI-HOURLY NORMALS OF THE VERTICAL COMPONENT IN 1842.

The observations are made $17\frac{1}{2}^m$ after the full hours.

1842.	0^h	2	4	6	8	10	Noon.	14	16	18	20	22^h
January	(658)	642	656	649	656	653	664	650	663	675	670	663
February	706	701	713	721	699	698	709	692	698	712	723	710
March	655	·43	654	663	657	661	651	650	657	668	673	665
April	668	658	655	655	656	654	657	651	650	660	672	672
May	673	670	661	644	646	647	651	644	645	656	668	670
June	674	669	6[illegible]4	658	647	642	635	639	635	653	671	668
July	683	672	664	659	657	650	643	643	647	663	677	682
August	689	689	683	682	679	672	652	659	669	679	688	688
September	692	686	689	690	681	671	671	673	672	679	687	693
October	706	698	702	714	695	708	707	706	706	708	710	709
November	717	713	723	725	712	715	711	713	716	712	718	718
December	713	707	709	706	715	711	709	707	706	711	713	713

In January at 0^h the final mean is 637 which differs so much from the standard value at this hour that it was preferred to substitute the mean of the month (658) as a close approximation.

TABLE IV.—*Continued.* BI-HOURLY NORMALS OF THE VERTICAL COMPONENT IN 1843.

The observations are made $23\frac{1}{2}^m$ after the full hours.

1843.	0^h	2	4	6	8	10	Noon.	14	16	18	20	22^h
January								690				
February								697				
March								683				
April	715	712	717	716	708	702	709	700	696	696	710	709
May	698	699	695	690	682	680	677	668	677	685	691	695
June	698	691	693	687	677	668	663	658	659	669	681	689
July	691	692	692	686	679	672	662	658	659	666	677	685
August	703	703	708	706	698	683	669	671	672	682	695	699
September	721	719	721	716	707	706	693	692	692	703	714	710
October	714	707	712	717	706	703	702	703	704	714	719	714
November	742	745	745	744	742	737	735	731	746	749	749	746
December	752	733	740	740	740	729	727	743	758	767	764	754

Normals at additional odd hours.

1843.	1^h	3	5	7	9	11	13	15	17	19	21	23^h
October	710	713	714	714	704	701	700	701	709	717	715	717
November	740	743	744	748	739	729	731	738	749	751	747	748
December	744	742	742	743	740	720	729	749	760	763	757	757

TABLE IV.—*Continued.* HOURLY NORMALS OF THE VERTICAL COMPONENT IN 1844.

The observations were made $23\frac{1}{2}^m$ after the full hours.

1844.	0^h	1	2	3	4	5	6	7	8	9	10	11^h
January	733	739	730	728	732	733	732	732	730	725	720	713
February	734	729	725	726	729	728	729	730	723	724	722	717
March	768	761	763	760	764	762	765	762	758	761	762	764
April	776	773	771	765	766	765	759	755	749	749	744	740
May	772	769	766	768	767	764	760	754	749	747	747	744
June	776	772	767	768	767	764	760	755	747	745	744	744
July	816	811	804	806	805	802	798	793	784	783	782	780
August	794	790	786	781	783	777	776	769	763	760	756	754
September	816	815	813	812	811	810	809	805	798	795	790	785
October	775	771	769	773	776	779	780	780	774	773	773	770
November	775	772	768	769	772	771	770	773	768	772	772	767
December	754	753	754	755	757	756	755	758	749	751	750	746

1844.	Noon.	13	14	15	16	17	18	19	20	21	22	23^h
January	715	708	715	724	737	740	741	743	744	745	744	744
February	718	716	716	720	727	731	738	739	740	737	735	733
March	762	753	758	760	754	757	753	754	752	753	763	764
April	737	733	737	741	745	744	756	767	768	765	767	775
May	744	740	747	744	746	748	754	763	766	766	768	772
June	742	739	744	747	749	754	760	769	771	770	771	775
July	780	776	773	781	783	791	799	805	808	809	811	816
August	750	750	746	759	764	771	778	789	790	789	792	794
September	780	779	772	794	793	795	806	813	812	812	814	817
October	769	766	773	777	786	788	791	789	785	785	781	781
November	756	763	762	765	774	779	777	778	774	770	769	772
December	743	733	736	724	748	757	755	751	751	748	750	750

TABLE IV.—*Continued.* HOURLY NORMALS OF THE VERTICAL COMPONENT IN 1845.

The observations are made $23\frac{1}{2}^m$ after the full hours.

1845.	0^h	1	2	3	4	5	6	7	8	9	10	11^h
January	754	747	747	7[illegible]2	752	757	763	767	760	758	753	752
February	760	756	752	757	759	760	763	764	756	756	752	749
March	749	741	736	742	746	748	749	746	736	733	732	729
April	732	727	728	729	728	727	725	721	718	715	717	717
May	720	718	717	719	716	715	713	709	705	702	701	699
June	733	731	729	730	729	728	726	722	717	715	715	711

1845.	Noon.	13	14	15	16	17	18	19	20	21	22	23^h
January	751	748	749	742	753	760	756	753	749	746	747	754
February	747	741	740	744	753	759	761	761	758	753	752	756
March	729	729	713	734	740	747	751	746	741	740	739	743
April	716	713	717	720	724	724	732	737	736	729	726	727
May	697	696	701	698	702	705	710	720	718	717	718	719
June	711	711	713	714	713	714	722	730	735	731	731	733

TABLE V.—NUMBER OF OBSERVATIONS AND LARGER DISTURBANCES IN EACH MONTH.

	1841.		1842.		1843.		1844.		1845.	
	Obs.	Dis.	Obs	Dis.	Obs.	Dis.	Obs.	Dis.	Obs.	Dis.
January . . .			300	76	26	5	646	81	648	17
February . . .	288	49	284	86	24	3	600	33	576	5
March	321	64	322	36	27	12	624	106	624	68
April	304	46	306	51	300	50	624	83	624	24
May	223	16	293	47	324	36	648	8	648	28
June	310	91	310	37	312	16	600	52	600	9
July	323	64	305	24	312	4	648	45		
August . . .	304	21	318	34	324	10	648	94		
September . .	307	40	303	57	312	20	600	14		
October . . .	308	28	310	12	624	25	648	20		
November . .	312	37	312	13	624	79	624	10		
December . .	323	84	319	15	624	65	624	47		
Sum . . .	3323	540	3682	488	3833	325	7534	593	3720	151
Ratio . . .	1 dis. in 6.2 obs.		1 dis. in 7.5 obs.		1 dis. in 11.8 obs.		1 dis. in 12.7 obs.		1 dis. in 24.6 obs.	

Total number of observations used,	22092
" " larger disturbances,	2097
Ratio of disturbances to observations,	1 to 10.5

Investigation of the eleven year (also called ten year) period in the inequality of the amplitude of the Diurnal Variation of the Vertical Force.—The preceding monthly means of the bi-hourly and hourly normals were rearranged in four groups of one year each, necessarily omitting the first five months; the annual means have for their mean epoch, January, as the monthly means were arranged from July to July.

The means for the year 1842–43 depend on nine months only, to refer them to the mean of twelve months, the differences for every observing hour, between the same nine months and twelve months for the preceding and following year, were made out and the mean correction, giving the weight two to the following year, as indicated by the readings taken at the hour 14, was applied to the values of 1842–43.

	From 9 months.	Correction.	Annual means		From 9 months.	Correction.	Annual means
0h	701	+3	704	Noon	682	+8	690
2	696	+3	699	14	681	+6	687
4	697	+5	702	16	683	+7	690
6	697	+6	703	18	689	+7	696
8	690	+6	696	20	697	+5	702
10	686	+7	693	22	700	+5	705

The normals for 1843–44 at the even hours are complete, at the odd hours they extend only over nine months. To refer the latter to twelve months, the difference between the means of the same nine months and the annual mean at the even hours was made out and applied as a correction to the means of the odd hours; the correction thus applied is the mean difference as deduced from the preceding and following even hour.

	Means of 9 months.	Correction.	Annual means.		Means of 9 months.	Correction.	Annual means.
1h	749	—11	738	13h	728	—14	714
3	746	—10	736	15	726	—16	720
5	746	—11	735	17	744	—16	728
7	744	—11	733	19	752	—15	737
9	737	—12	725	21	751	—14	737
11	730	—13	717	23	754	—13	741

TABLE VI.—ANNUAL MEANS OF THE BI-HOURLY AND HOURLY NORMAL VALUES OF THE REGULAR SOLAR-DIURNAL VARIATION OF THE VERTICAL FORCE.

The numbers are expressed in scale divisions, increasing values indicate decrease of force. The minutes at the head of each column are to be added to the hour given in the first column. Each year commences with the month of July. The time is local Philadelphia time counted from midnight to midnight.

Hour.	1841–42. $+17\frac{1}{4}^{m}$	1842–43. $+20\frac{1}{2}^{m}$	1843–44. $+23\frac{1}{2}^{m}$	1844–45. $+23\frac{1}{2}^{m}$
0 A. M.	650	704	740	765
1 "			738	761
2 "	643	699	735	759
3 "			736	760
4 "	643	702	737	761
5 "			735	761
6 "	640	703	734	761
7 "			733	759
8 "	634	696	727	752
9 "			725	751
10 "	632	693	722	749
11 "			717	747
12 P. M.	633	690	717	745
13 "			714	742
14 "	631	687	718	741
15 "			720	746
16 "	636	690	724	753
17 "			728	758
18 "	647	696	732	762
19 "			737	764
20 "	654	702	738	763
21 "			737	761
22 "	652	705	738	761
23 "			741	764
Means	641	697	730	756

The following formulæ of the mean diurnal variation of the vertical force were deduced from the above tabular values. The angle θ counts from midnight at the rate of 15° an hour.

$$1841\text{–}42\quad V = 641^d + 10^d.4 \sin(\theta + 106^\circ\, 40') + 3^d.1 \sin(2\theta + 198^\circ\, 25') + 1^d.7 \sin(3\theta + 250^\circ)$$
$$1842\text{–}43\quad V = 697 + 7.6 \sin(\theta + 69\quad 17) + 2.9 \sin(2\theta + 196\quad 48) + 1.3 \sin(3\theta + 195\ )$$
$$1843\text{–}44\quad V = 730 + 11.0 \sin(\theta + 79\quad 54) + 3.4 \sin(2\theta + 226\quad 29) + 0.6 \sin(3\theta + 45\ )$$
$$1844\text{–}45\quad V = 756 + 9.2 \sin(\theta + 83\quad 40) + 4.3 \sin(2\theta + 233\quad 41) + 1.1 \sin(3\theta + 1\ )$$

In the construction of the equation for 1843-44 weighted normals were used, those of the even hours have the weight 4, of the odd hours the weight 3.

To show the degree of accordance in the expressions when deduced from the even and odd hours separately, the resulting equations for the last year are added:—

$$\text{Even hours:}\quad V = 756 + 9.32 \sin(\theta + 84^\circ\, 45') + 4.07 \sin(2\theta + 235^\circ\, 17') + 1.2 \sin(3\theta + 353^\circ)$$
$$\text{Odd}\quad \text{“}\quad V = 756 + 8.99 \sin(\theta + 82\quad 36) + 4.52 \sin(2\theta + 232\quad 05) + 1.0 \sin(3\theta + 10\ )$$

The observed and computed values compare as follows. The differences, observed less computed, are expressed in scale divisions:—

Hour.	1841–42.	1842–43.	1843–44.	1844–45.	
0	+2	+2	+1	+3	
2	–1	–1	–2	–2	
4	0	+1	+1	0	$17\frac{1}{4}$, $20\frac{1}{2}$, $23\frac{1}{2}$, and $23\frac{1}{2}$ minutes are to be added to the full hours for the four years respectively.
6	+1	+1	+1	+2	
8	0	–1	–2	–3	
10	0	+1	+1	0	
Noon	+1	+1	+1	+2	
14	–1	–1	+1	–2	
16	0	+1	0	+1	
18	+1	+1	0	+1	
20	0	0	+1	–1	
22	–1	0	–1	–1	

The graphical representation of the observed and computed values exhibits a maximum of the vertical force between 1 and 2 P. M., and a minimum of force between $8\frac{1}{2}$ and $10\frac{1}{2}$ A. M.; the diagrams also show a tendency of a secondary maximum about two hours after midnight followed by a secondary minimum about two hours later, with a range probably less than two scale divisions (0.000066 parts of the force, or 0.00085 in absolute measure). This small nocturnal inequality is only exhibited by one of the formulæ, in 1842–43, when it has its greatest value; in the preceding year there is but a faint trace of it, in the two succeeding years it is indicated in the diagram by dashes. The average diurnal range is nearly 22 scale divisions (0.00073 parts of the vertical force, or 0.00932 in absolute measure).

If we take the monthly aggregate amount of the disturbances, all referred to a uniform series of bi-hourly observations, and form a table of these values for each year (Table VIII), the mean aggregate amount for each year is as follows:—

	Mean amount of disturbances.
In 1841–42	2306 div.
" 1842–43	1521
" 1843–44	959
" 1844–45	636

This again points to the end of the year 1844 for the epoch of the minimum amount of disturbances, and considering the three elements, declination, horizontal and vertical force, the spring of 1844 might be assumed as the time of the minimum magnitude of the magnetic disturbances.

Altogether, the inequalities in the diurnal amplitude and in the number and magnitude of the disturbances of the magnetic elements, as observed at Philadelphia, fix the end of the year 1843, or the beginning of 1844, as the epoch of the minimum of the eleven (or ten) year inequality.

We now proceed with the analysis of the disturbances, their diurnal and annual inequality in number and amount, and for increasing and decreasing values.

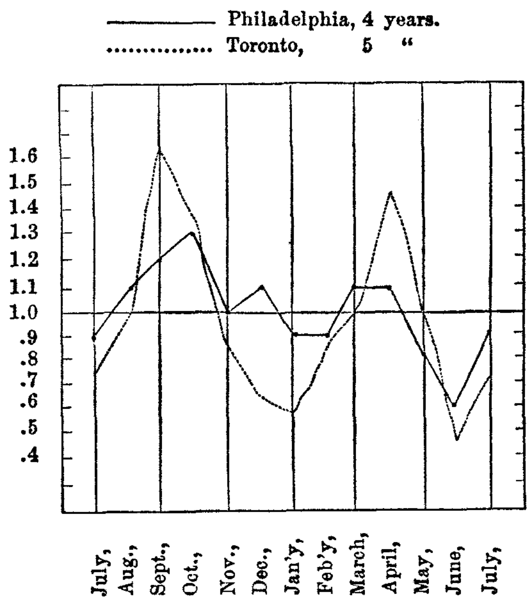
Annual Inequality in the number of Disturbances.—The numbers for each month have been referred to a uniform series of bi-hourly observations as explained above. The ratios of the monthly means to the annual means is given, and also, for comparison, similar ratios found for the horizontal force and declination.

TABLE VII.—ANNUAL INEQUALITY IN THE NUMBER OF DISTURBANCES.

	1841–1842.	1842–1843.	1843–1844.	1844–1845.	Means from four years.	Vert. force ratio.	Hor. force ratio.	Declination ratio.	Mean ratio Vert. force hor. force and declination.	Mean ratio Hor. force and declination.
July	64	24	4	22	28	0.9	1.1	0.9	1.0	0.9
August	21	34	10	47	28	0.9	0.9	1.6	1.3	1.1
September	40	57	20	7	31	1.0	1.4	1.4	1.4	1.2
October	28	12	12	10	15	0.5	1.4	2.1	1.7	1.3
November	37	13	40	5	24	0.8	1.0	1.1	1.1	1.0
December	84	15	33	23	39	1.3	1.0	1.0	1.0	1.1
January	76	58	40	9	46	1.5	0.6	0.8	0.7	0.9
February	86	51	16	3	39	1.3	1.0	0.5	0.7	0.9
March	36	44	53	34	42	1.4	1.1	0.7	0.9	1.1
April	51	50	42	12	39	1.3	1.1	0.9	1.0	1.1
May	47	35	4	14	25	0.8	1.0	0.6	0.8	0.8
June	37	16	26	4	21	0.7	0.6	0.5	0.6	0.6
Mean	51	34	25	16	31					

The months of maximum disturbance are March and September (the high value in January and the low one in October appear anomalous, and would no doubt disappear in a longer series of observations). The minimum occurs in June; there is no well expressed second minimum. The horizontal force and declination ratios, as well as the ratios of the three elements at Toronto, give the maximum number of disturbances at the equinoxes, and the minimum number at the solstices, and as the winter solstice minimum only is wanting in the Philadelphia vertical force ratios, it is probably due to the small number of observations, and the difficulty in keeping the instrument in adjustment and allowing for its irregularities. I have, therefore, given the mean ratio of the Philadelphia disturbances in the last column of Table VII, and compared the result, graphically, with those deduced by General Sabine for Toronto.[1]

ANNUAL INEQUALITY OF DISTURBANCES.



If we separate the disturbances into two parts, those increasing and those decreasing the force, we obtain the numbers of Table VIII. A positive sign indicates disturbances increasing, a negative sign those decreasing the vertical force. The law of the annual variation seems to be the same as shown by the ratios in the last two columns; this accords with the result at Toronto.

TABLE VIII.—ANNUAL INEQUALITY OF DISTURBANCES INCREASING AND DECREASING THE FORCE.

	1841-42.		1842-43.		1843-44.		1844-45.		Ratios.	
	+	−	+	−	+	−	+	−	+	−
July	31	33	8	16	0	4	12	10	0.7	1.1
August	14	7	9	25	8	2	36	11	1.0	0.8
September	22	18	16	41	9	11	5	2	0.7	1.2
October	22	6	8	4	6	6	3	7	0.6	0.4
November	17	20	5	8	14	26	4	1	0.6	1.0
December	51	33	8	7	23	10	13	10	1.4	1.1
January	38	38	32	26	26	14	6	3	1.5	1.5
February	53	33	32	19	11	5	2	1	1.4	1.0
March	16	20	23	21	30	23	23	11	1.3	1.3
April	32	19	30	20	23	19	5	7	1.3	1.1
May	28	19	18	18	3	1	11	3	0.9	0.7
June	23	14	5	11	7	19	3	1	0.6	0.8

[1] Page lxx., Vol. III.

TABLE IX.—AGGREGATE AND MEAN AMOUNT OF DISTURBANCES IN EACH MONTH OF THE YEAR.

The numbers are expressed in scale divisions and referred to a uniform series of bi-hourly observations. The mean amount or average magnitude is found by dividing the number in the preceding column by 4 and by the number of disturbances found in Table VII.

	Aggregate amount.				Sum of 4 years.	Mean amount.
	1841–42.	1842–43.	1843–44.	1844–45.		
July . . .	2593	1255	149	784	4781	42
August . .	791	1323	622	2017	4753	43
September .	1612	2798	770	301	5481	44
October . .	1216	432	488	4[illegible]8	2574	42
November .	1564	503	1504	206	3777	40
December .	4187	560	831	872	6450	42
January . .	3899	2745	1592	314	8550	47
February . .	3900	2279	659	109	6947	45
March . . .	1672	1898	2125	1245	6940	42
April . . .	2324	2111	1642	468	6545	42
May . . .	2445	1625	186	704	4960	49
June . . .	1472	723	934	168	3297	40

The last column shows that the magnitude of the disturbances is rather irregularly distributed over the several months without following any apparent law.

TABLE X.—AGGREGATE AND MEAN AMOUNT OF DISTURBANCES IN EACH MONTH OF THE YEAR, SEPARATED INTO TWO GROUPS OF INCREASING (+) AND DECREASING FORCE (—).

The mean amount is obtained by means of the numbers of Table VIII.

	1841–42.		1842–43.		1843–44.		1844–45.		Sum of 4 years.		Mean amount.	
	+	—	+	—	+	—	+	—	+	—	+	—
July	1130	1463	340	915	0	149	402	382	1872	2909	37	46
August	555	236	359	964	279	343	1568	449	2761	1992	41	45
September	835	777	775	2023	397	373	251	50	2258	3223	43	45
October	999	217	300	132	250	238	128	310	1677	897	43	39
November	653	911	223	280	504	1000	154	52	1534	2243	38	41
December	2745	1442	276	284	508	323	489	383	4018	2432	42	41
January	2128	1771	1589	1156	1050	542	208	106	4975	3575	48	44
February	2615	12[illegible]5	1538	741	462	197	76	33	4691	2256	48	39
March	671	1001	910	988	1149	976	875	370	3605	3335	39	44
April	1471	853	1361	750	853	790	172	296	3856	2689	43	41
May	1535	910	928	697	170	16	598	106	3231	1729	54	42
June	999	473	246	477	246	688	133	35	1624	1673	43	37
Sums	16336	11339	8845	9407	5867	5635	5054	2572	36102	28953		
									Mean . . .		43	42

The magnitudes of the disturbances, as before, do not appear to follow any law. The disturbances which increase the force preponderate over those which decrease it; the ratio of the annual means is 1.3 to 1.0. At Toronto the reverse was found; the disturbances which decrease the force preponderate over those which increase in the ratio of 1.4 to 1.0.

Diurnal Inequality of the Disturbances.—In the bi-hourly combination of the disturbances we make use of the series of observations extending from February, 1841, to June, 1845, omitting only the single daily observation in January, February, and March, 1843. Strictly speaking the time is 21 minutes later than indicated in the table.

TABLE XI.—DIURNAL INEQUALITY IN THE NUMBER OF DISTURBANCES.
The ratios of the three elements have been collected to facilitate comparison.

	Number vertical force.	Ratios.		
		Vertical force.	Horizontal force.	Declination.
0h	168	1.3	1.1	1.0
2	15[illegible]	1.2	0.9	1.2
4	156	1.2	0.7	1.0
6	133	1.0	0.7	1.1
8	117	0.9	0.8	1.0
10	115	0.8	1.1	1.1
Noon	131	1.0	1.3	0.9
14	163	1.2	1.0	0.8
16	127	0.9	1.1	0.9
18	116	0.8	1.1	0.9
20	110	0.8	1.1	1.0
22	123	0.9	1.1	1.1
Mean	135			

The greatest number of disturbances occur about A. M. (at Toronto at 3 A. M.), with the least number at 10 A. M. (at Toronto at 11 A. M.); the secondary maximum and minimum occur about 2 P. M. and 7 P. M. (at Toronto the hours are 5 P. M. and 9 P. M.). On the average, therefore, the maxima and minima occur 1^h 40^m earlier at Philadelphia than at Toronto. At neither station do the three elements show the same law; they agree only in so far as to exhibit a systematic increase and decrease with the solar hours, and in having two maxima and two minima.

The diagram shows the law of the disturbances of the vertical force for Philadelphia and Toronto.

DIURNAL VARIATION OF DISTURBANCES.

———— Philadelphia, 4½ years.
.... Toronto, 5 "

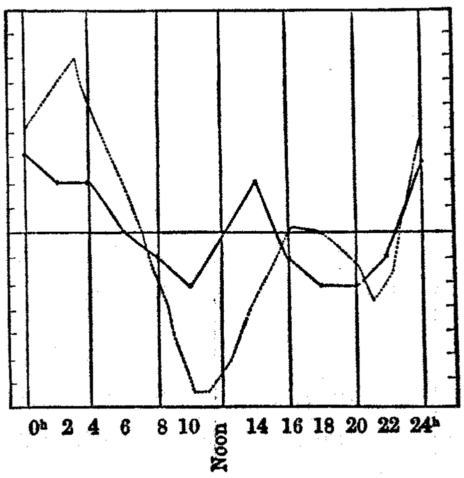


TABLE XII.—CONTAINS THE NUMBER OF DISTURBANCES DISTRIBUTED OVER THE HOURS OF THE DAY, SEPARATED INTO THOSE WHICH INCREASE (+) AND THOSE WHICH DECREASE (—) THE VERTICAL FORCE.

	Number of disturbances.		Ratios.	
	+	—	+	—
0^h	93	75	1.3	1.2
2	73	86	1.0	1.3
4	69	87	1.0	1.4
6	62	71	0.9	1.1
8	59	58	0.8	0.9
10	53	62	0.8	1.0
Noon	61	70	0.9	1.1
14	96	67	1.3	1.0
16	69	58	1.0	0.9
18	72	44	1.0	0.7
20	67	43	0.9	0.7
22	76	47	1.1	0.7
Mean	71	64		

The laws which regulate the diurnal occurrence of the number of disturbances, increasing and decreasing the vertical force, are evidently not the same, yet they are by no means the converse of one another as has been found to be the case in the disturbances of the declination and the horizontal force. At Toronto also, where the horizontal force and declination curves were exactly opposed, that of the vertical force is not so, and at Philadelphia rather favors an agreement between the increasing and decreasing disturbances than an opposition.

Principal maximum of increasing disturbances 2 P.M., principal minimum 9 A.M. (at Toronto 5 P. M. and 5 A. M. respectively). Secondary maximum at midnight; this may possibly be the principal maximum; secondary minimum at 8 P. M.

Principal maximum of decreasing disturbances 4 A. M., principal minimum 8 P. M. (at Toronto 3 A. M. and 6 P. M. respectively). The secondary maximum at noon is less distinctly marked; secondary minimum at 8 A. M.

Thus the epochs of the curves for increasing and decreasing force seem to be 12 hours apart.

Diurnal Inequality in the Magnitude of the Disturbances.

TABLE XIII.—CONTAINS THE AGGREGATE AMOUNT OF DISTURBANCES AND THEIR AVERAGE MAGNITUDE, THE LATTER FOUND BY MEANS OF TABLE XI. ALL EXPRESSED IN SCALE DIVISIONS.

	Aggregate amount.	n	Mean amount.
0^h	7049	168	42
2	6876	159	43
4	6480	156	42
6	5418	133	41
8	5022	117	43
10	5096	115	44
Noon	5526	131	42
14	7101	163	44
16	5591	127	44
18	5571	116	48
20	4773	110	43
22	5246	123	43

Average magnitude 43 scale divisions, the disturbances appear to be nearly of the same size at all hours, there is a slight preponderance in magnitude between 10 A. M. and 10 P. M. over the other half of the day.

TABLE XIV.—AGGREGATE AMOUNT AND MEAN AMOUNT OF DISTURBANCES, SEPARATED INTO THOSE WHICH INCREASE AND THOSE WHICH DECREASE THE VERTICAL FORCE.

	Aggregate amount.		Mean amount.		Difference of aggregate amount.
	+	—	+	—	
0^h	3737	3312	40	44	+ 425
2	3221	3655	44	43	— 434
4	2866	3614	42	42	— 748
6	2577	2841	42	40	— 264
8	2524	2498	43	43	+ 26
10	2507	2589	47	42	— 82
Noon	2731	2795	45	40	— 64
14	4456	2645	46	40	+1811
16	2969	2622	43	45	+ 347
18	3456	2115	48	48	+1341
20	3003	1770	45	41	+1233
22	3258	1988	43	42	+1270
Mean			44.0	42.5	+4861

The magnitude of the disturbances, either increasing or decreasing the force, apparently does not vary with the hours of the day. The disturbances which increase the force preponderate between the hours 2 P. M. and 2 A. M.; those which decrease the same occur in the other half of the day, the average ratio of the preponderance of increase over decrease is as 4 to 1.

Dividing the numbers in the last column of the preceding table, or the excess of the sum of disturbances increasing the force over the sum of those decreasing the same, by the total number of days (1297) of observation, we find the diurnal disturbance variation as follows:—

	Scale divisions.	In parts of vertical force.	In absolute measure.
0^h	+0.3	+0.00001	+0.00013
2	—0.3	—0.00001	—0.00013
4	—0.6	—0.00002	—0.00025
6	—0.2	—0.00001	—0.00008
8	0.0	0.00000	0.00000
10	—0.1	0.00000	—0.00004
Noon	0.0	0.00000	0.00000
14	+1.4	+0.00004	+0.00059
16	+0.3	+0.00001	+0.00013
18	+1.0	+0.00003	+0.00042
20	+1.0	+0.00003	+0.00042
22	+1.0	+0.00003	+0.00042
Mean	+0.3	+0.00001	+0.00013

The value for the hour 14 is evidently anomalous, the mean of the hours 12 and 16 or $+ 0^d.2$ (0.00001 in parts of force, 0.00008 in absolute measure) should be substituted. The average daily effect of the larger disturbances is therefore to increase the vertical force between 1 P. M. and midnight, and to decrease it between 1 A. M. and noon, with an amplitude of about 1.6 scale division (0.00005 parts of the force, 0.00067 in absolute measure). The maximum value takes place at 8 P. M., the same hour at which the horizontal force disturbance is greatest (decreasing that force).

The disturbance law at Toronto is nearly the same as at Philadelphia, the disturbances increase the force between noon and 9 P. M., and decrease it in the remaining hours of the day; the range at Toronto appears to be larger.

If we classify the disturbances according to their magnitude in eight groups, each differing 25 scale divisions from the preceding, we find the following scale numbers:—

Disturbances. In scale divisions.	Between limits in parts of the force.	Number.
30 and 55	0.00099 and 0.00181	1840
55 " 80	0.00181 " 0.00263	211
80 " 105	0.00263 " 0.00346	28
105 " 130	0.00346 " 0.00428	15
130 " 155	0.00428 " 0.00511	0
155 " 180	0.00511 " 0.00593	2
180 " 205	0.00593 " 0.00676	0
205 " 230	0.00676 " 0.00759	1
Beyond		None.

APPENDIX TO PART VII.

EFFECT OF THE AURORA BOREALIS ON THE MAGNETIC DECLINATION, AND THE HORIZONTAL AND VERTICAL FORCE AS OBSERVED AT THE GIRARD COLLEGE OBSERVATORY.

There were in all 22 auroras recorded; these, however, comprise only the brighter displays. Of those observed, 7 occurred between May 30, 1840, and July 1, 1841; 1 occurred between July, 1841, and July, 1842; 6 occurred between July, 1842, and July, 1843; and 7 between July, 1843, and July, 1844. One is recorded in the last year, ending June 30, 1845. They are distributed over the several months as follows:—

January	2	July	6
February	0	August	3
March	1	September	2
April	2	October	0
May	3	November	1
June	2	December	0

In the summer months there were 18, in the winter months 4. In reference to the hours of the night, the phenomenon was visible on the average between 9½ P. M. and 11¼ P. M.

Individual examination of the magnetic record during auroral displays. The time is local time, counted for convenience' sake from midnight to midnight to 24 hours.

I. 1840, May 29th—30th. As the twilight faded an aurora became visible. In the course of the display there were moving pillars, flashes from a low segment of light in the north, and a beautiful arch nearly or quite at right angles to the magnetic meridian. Pillars of aurora from $21^h\ 18^m$ to $22^h\ 2^m$, varying in brightness and position; low segment of light to the north, continued throughout the appearances; at $22^h\ 5^m$ an arch forms from east to west; streams of light, varying in brightness, fading and reappearing from $22^h\ 20^m$ to about $23^h\ 10^m$; the brightest flash at $23^h\ 6^m$. From $18^h\ 54^m$ the declination magnet commenced to move eastward (declination decreasing), reaching an extreme position at $20^h\ 34^m$, difference from average position about 56 divisions or 19′; the movement then became westerly with smaller fluctuations till $22^h\ 39^m$, when it reached its westerly extreme of about 71 divisions or 24′ from the normal place; the magnet reached a second easterly extreme at $23^h\ 44^m$ of about 48 divisions or 17′, at $1^h\ 24^m$ (30th) again a westerly extreme of about 7′, and at $2^h\ 49^m$ an easterly deflection of about 14′; after this the needle returned gradually to its ordinary position. About the time of the brightest flash the change (easterly motion) was very rapid, no extreme value, however, was reached. When the arch formed, the position was nearly normal. The horizontal force decreased steadily until $22^h\ 42^m$, when the readings fell beyond the scale; a minimum was reached between that time and $22^h\ 52^m$ of at least 0.016 (parts of the force) below the normal force. At the time of the brightest flash the retrograde movement was in progress. The disturbance of the vertical force commenced before $17^h\ 52^m$, at which time the force was a maximum; it then decreased very rapidly, and finally moved off the scale after $22^h\ 2^m$. (The value of a division of the scale was not ascertained.)

II. 1840, July 4th. At 20^h auroral light in the N. N. W. about 10° above the horizon, at 22^h very faint aurora still visible in N W. The declination was not at all affected. The horizontal force at these hours was 85 divisions (0.003 parts of the force) less than the normal amount. The vertical force is apparently undisturbed; it is slightly above the normal value.

III. 1840, July 6th. An aurora was noticed at $0^h\ 25^m$ and $2^h\ 25^m$. The declination was disturbed at $0^h\ 19\frac{1}{2}^m$ and $2^h\ 19\frac{1}{2}^m$; it indicated 50 divisions and 34 divisions, or 17′ and 12′ of easterly deflection. It is likely that there were disturbances two hours preceding and two hours following the above times, as the scale could not be read. The horizontal force was disturbed from midnight till 2 P. M.; the force was less during this time, and reached its minimum value at $2^h\ 22^m$ of 130 divisions or 0.005 parts of the force; between 2 and 8 A. M. the diminution was about 0.004 parts. The vertical force was also less from midnight till after $2^h\ 17^m$, the greatest diminution probably took place later as the observations failed at $4^h\ 17^m$. Minimum value at $2^h\ 17^m$, 0.004 parts of the force.

IV. 1840, July 29th. At $22^h\ 25^m$ a faint aurora. The declination was not disturbed. The horizontal force was very slightly affected. At $20^h\ 22^m$ it was 0.001 parts less than the normal, at $22^h\ 22^m$ it was nearly normal, and at $0^h\ 22^m$ (30th) it was greater by 0.002. There may have been ordinary disturbances not immediately connected with the aurora. At $22^h\ 17^m$ the vertical force was slightly affected, the force decreased 0.002 parts below the normal.

V. 1840, August 19th. At $20^h\ 25^m$ auroral light in N.; $22^h\ 25^m$ aurora continues in N. and N. W. The declination disturbance commenced at $22^h\ 20^m$ and continued to $2^h\ 20^m$ (20th), west deflection 48 divisions (22′), 10 divisions, and 10 divisions. The horizontal force was disturbed from $16^h\ 22^m$ to $22^h\ 22^m$, force less 43 divisions, 49, 102, and 85 divisions (in minimo 0.004 parts of the force). The vertical force seems lower than usual, but hardly reached the limit of a recognized disturbance.

VI. 1840, August 28th and 29th. An aurora appeared at $20^h\ 39\frac{1}{2}^m$ in N. N. E., disappeared at $21^h\ 19\frac{1}{2}^m$, reviving at $21^h\ 59\frac{1}{2}^m$; at $22^h\ 9\frac{1}{2}^m$ streamers moving from E. to W.; light continued in N.; streamers again in N. E. at $22^h\ 59\frac{1}{2}^m$ and $1^h\ 14\frac{1}{2}^m$, after which time the aurora was not observed. An easterly movement of the needle commenced about $20^h\ 19^m$ with a maximum eastern deflection of 125 divisions (or 57′) at $21^h\ 0^m$, the westerly motion continued till $21^h\ 55^m$ when the needle was yet 5′ east of its normal position; smaller fluctuations were observed till midnight, the deflection was then 19′ east; half an hour later it was 20′ east; the morning extreme was reached at $1^h\ 35^m$, when the deflection was 25′ east; after this the disturbance gradually subsided. There was a disturbance of the horizontal force about $18^h\ 20^m$; from about $21^h\ 52^m$ till 10 the next morning the horizontal force remained below its normal value. At $23^h\ 32^m$ it was 0.009 (parts) below, at $0^h\ 22^m$ it was 0.007, and at $1^h\ 22^m$ its minimum value of 0.010 (parts of the force) was reached. The disturbance in the vertical force appears to have commenced about $21^h\ 7^m$, when the force gradually decreased till $21^h\ 57^m$ when it reached a minimum of about 0.003 (parts); after this it gradually increased.

VII. 1840, September 21st. At $20^h\ 25^m$ faint aurora, $22^h\ 25^m$ aurora disappeared. Disturbance of the declination commenced at $20^h\ 20^m$ and continued to $4^h\ 20^m$ (22d), deflections 40 divisions (18′) W., 10 divisions E., 14 divisions W., and 23 divisions E. The horizontal force disturbance commenced at $16^h\ 22^m$ and ceased at $4^h\ 22^m$ next day; force less 69 divisions, 47 divisions, 71 divisions, 42 divisions, 94 divisions, 124 divisions (0.005 parts of the force), and 93 divisions. The vertical orce between 16^h and 23^h was slightly above the average, but suddenly became much smaller than the normal between midnight and 3 A. M. Minimum about 0.002 parts of the force.

VIII. 1842, April 14th. At $22^h\ 40^m$ appearance of aurora, a bright light in the N.; at $0^h\ 20^m$ (15th) an arc of light was visible extending to about 15° above the north horizon. Declination disturbed from $22^h\ 20^m$ to $8^h\ 20^m$ (15th), deflections at the regular observing hours 23 divisions W., 39 E., 11, 37, 10, and 14 divisions W. Maximum west deflection at $22^h\ 56^m$, 58 divisions (26′), maximum east deflection 39 divisions (18′), derived from the series of extra observations. The horizontal force disturbances commenced at $22^h\ 22^m$ and ceased at $4^h\ 22^m$, force less 39 divisions, 149 divisions, 37 divisions, and 50 divisions, minimum 279 divisions (0.010 parts of the force) at $1^h\ 16^m$ (15th). But one of the 69 extra readings during this aurora shows an increase of force. The vertical force disturbances commenced at $0^h\ 17\frac{1}{2}^m$ (15th) and continued to $6^h\ 17\frac{1}{2}^m$; force less 68 divisions, 69 divisions, 59 divisions, and 38 divisions. Minimum value 111 divisions or 0.0037 parts of the force at $1^h\ 24^m$ (15th).

IX. 1842, September 2d. At $2^h\ 22^m$ a bright light extending on each side of N. point about 15°, and to about 6° above the horizon; at $2^h\ 49^m$ light spreading and becoming more faint; at $3^h\ 12^m$ light faint and gradually subsiding. The declination was very slightly affected, maximum west deflection at $3^h\ 26^m$, 19 divisions (9′). The horizontal force was not disturbed. The vertical force was likewise undisturbed

X. 1842, November 21st and 22d. A well developed aurora and the best observed of the series. At $22^h\ 23^m$ a very luminous arc extending to about 15° above the horizon, and about 90° along it in the north; $22^h\ 38^m$ light slightly increasing; $22^h\ 53^m$ a slight decrease of light; $23^h\ 18^m$ light alternately appearing and disappearing; $23^h\ 33^m$ four streamers of unusual brightness reaching 30° above horizon; $23^h\ 36^m$ light particularly bright in N. W., whence a large streamer of 20° is shooting, also one due north of 15°; $23^h\ 40^m$ light subsided, no streamers; $23^h\ 43^m$ small streamers appearing; $23^h\ 46^m$ large streamers attended with great light in N. W.; $23^h\ 48^m$ the arc still remains about 15° above horizon, but has shortened its chord to 30°, no streamers; $23^h\ 51^m$ arc scarcely visible; $0^h\ 23^m$ (22d) two arcs visible; $0^h\ 28^m$ a large streamer of 20° in length; $0^h\ 36^m$ considerable light without the arc; $1^h\ 08^m$ light very faint; $1^h\ 23^m$ slight appearances of arc; $1^h\ 33^m$ faint streamer of 10°; $1^h\ 58^m$ faint streamer of 20°; $2^h\ 48^m$ a large but faint streamer due N. about 20° in length; light has nearly disappeared; $3^h\ 3^m$ light scarcely visible; $3^h\ 33^m$ no light visible, and readings of instrument ordinary. The declination disturbances commenced at $22^h\ 20^m$, and ceased at $10^h\ 20^m$ (22d); deflections 49 divisions W., 20 divisions W., 25, 20, 8, 25, and 16 divisions E. The maximum W. deflection (22′) occurred at the commencement, with the appearance of the luminous arc, the needle remained deflected to the westward until towards the end, when there was a smaller easterly deflection. No special effect of the streamers is noticed. The horizontal force disturbances commenced at $16^h\ 22^m$, and continued to $2^h\ 22^m$ (22d); horizontal force less 33 divisions, 68, 82, 183 divisions (0.007 parts of the force); this diminution was about the time of the appearance of the arc, 125 divisions and 73 divisions at the last two regular observing times. The streamers did not appear to have any special effect. The horizontal force always remained smaller than the normal value at the respective hours. The vertical component was not affected.

XI. 1843, May 6th and 7th. At $19^h\ 48^m$ a bright light; at $2^h\ 18^m$ (7th) light to N. about 23° high, but faint. The declination disturbances commenced at $16^h\ 20^m$, and continued to about 3^h (7th). The déflections at the regular hours were 28, 30, 16 divisions E., 15 divisions W. (20^m after midnight), maximum east deflection 18′, succeeding maximum west deflection 9′, next following maximum east deflection 33′, following maximum west deflection 15′. The horizontal force disturbances commenced at the same hour with the declination disturbances, and continued to the end of the series of observations. The change commenced with a violent increase of 113 divisions above the normal value, and increased to 330 divisions (0.012 parts of the force) at $18^h\ 04^m$, corresponding in time to the first maximum east deflection. The force then decreased, reaching 132 divisions below the normal value, and attaining shortly (16^m) after midnight the extraordinary low value 348 divisions (0.013 parts of the force); up to the end of the disturbance the force remained below the standard amount. The vertical force was suddenly disturbed, at $18^h\ 23\frac{1}{2}^m$ it was 161 divisions greater than the mean, and at $22^h\ 23\frac{1}{2}^m$ but 41 divisions above the normal. Maximum value 164 divisions (equal to 0.0054 parts of the force) at $18^h\ 12^m$.

XII. 1843, May 8th. At 0^h an aurora visible to north. The declination was but slightly affected; at $19^h\ 32^m$ there was an easterly deflection of 15 divisions or 7′; at $20^h\ 26^m$ it was west 4′; after this there was an easterly motion, changing again to west, which reached an extreme value at $23^h\ 44^m$ of 17 divisions or 8′ W. From the commencement of the horizontal force observations ($17^h\ 38^m$) the force was less than the normal; at $20^h\ 4^m$ the greatest depression was 0.003 (parts of the force). The disturbance continued till 6 A. M., the force being less than the standard value. From $17^h\ 38^m$, when the vertical force was observed, it was found less than the normal, at $20^h\ 36^m$ depression 20 divisions, at $0^h\ 23^m$ it was 31 divisions or 0.001 parts of the force below the standard value.

XIII. 1843, June 27th. At 22^h a bright diffused light to north, particularly bright to N. W., whence streamers are shooting up; general light weakens as it rises at $20^h\ 45^m$; at $21^h\ 15^m$ a brilliant light, dark cumulus spots in the bright light, and long streaks of dark clouds to N. Fades at $22^h\ 13^m$, light to N. faint; dark, fuzzy, low cumuli form and disappear to N. Neither the declination nor the horizontal force was disturbed by this aurora. The vertical force was slightly affected, force less 38 divisions or 0.0012 parts of the force.

XIV. 1843, June 30th. At 23^h aurora visible to the N. N. E., flaming to about 10°. The declination disturbances commenced at $22^h\ 20^m$ (9 divisions W.), they reach a maximum at $0^h\ 02^m$ (July 1st) of 20 divisions (9′), and gradually disappear, the deflections having been west throughout. The horizontal force is smaller than the normal value, a first minimum is reached about $20^h\ 44^m$ (about 45 divisions), and the principal minimum about $0^h\ 10^m$ (July 1st) of nearly 50 divisions (0.0018 parts of the force). The vertical force remained undisturbed.

XV. 1843, July 7th. At $20^h\ 52^m$ very light in the N. N. E. and N. W. The declination at $18^h\ 20^m$ is deflected 15 divisions E., the motion then became westerly and reached 29 divisions (13′) W. at $23^h\ 33^m$; at $2^h\ 20^m$ (8th) the deflection is again 16 divisions W. The horizontal force is less than usual, with a minimum value about $20^h\ 52^m$ of 55 divisions (0.002 parts). The force then increases, and about midnight reaches slightly above the normal. The vertical force was not disturbed.

XVI. 1843, July 24th and 25th. According to a letter (dated July 25th) from one of the observers, auroral disturbances commenced about 16^h (July 24th) and quieted down about 21^h. At $16^h\ 20^m$ the declinometer was deflected 15 divisions E., about 4^h (25th) the disturbances reappeared deflecting 10 divisions W., and changed to east deflection at $6^h\ 20^m$, reaching a maximum east of 34 divisions (15′) at $13^h\ 20^m$. At $16^h\ 22^m$ the horizontal force was about 46 divisions less, with disturbances reappearing about $8^h\ 22^m$, reaching at $8^h\ 46^m$ 96 divisions (0.0015 parts of the force) below the normal, and quieting down about one hour after noon. The vertical force was not sensibly disturbed.

XVII. 1843, July 25th. At $21^h\ 30^m$ (25th) streamers to N., flaming to about 30°; at 22^h streamers very bright, reaching about 40°; At $22^h\ 15^m$ light very faint and gradually disappearing. The declination was disturbed (deflections west) between $20^h\ 20^m$ and $22^h\ 20^m$ reaching a maximum at $21^h\ 58^m$ of 36 divisions (16′). The horizontal force decreased between $18^h\ 22^m$ and $22^h\ 22^m$, reaching at $21^h\ 34^m$ 91 divisions (0.0033 parts) below the normal value. The vertical force apparently undisturbed.

XVIII. 1843, August 22d. At $20^h\ 22^m$ there were streamers of 35° in length, bright light in N. Between $14^h\ 20^m$ and $18^h\ 20^m$ there was a small east deflection of the magnet reaching 28 divisions at the latter hour; at $19^h\ 56^m$ it changed to a west deflection of the same amount (13′). At the time of the appearance of the streamers the declination was normal. During the aurora the horizontal force diminished, reaching at $20^h\ 28^m$ 91 divisions (0.0033 parts) below the normal. The low value continued for about two hours after this time. The vertical force was not sensibly affected.

XIX. 1844, January 24th and 25th. Aurora visible to N. and N. N. E. at $0^h\ 22^m$ (25th), streamers running up 30°; $0^h\ 33^m$ streamers running up 15° and 20°. During this aurora the horizontal needle was deflected to the westward about 10 divisions, reaching a maximum at $6^h\ 58^m$ of 15 divisions (7′); at the time of the appearance of the shorter streamers the deflection was near 7′, the horizontal force was below the normal value, viz: decrease 36 divisions, 41 and 35 divisions at $22^h\ 22^m$, $23^h\ 22^m$, and $0^h\ 22^m$ (25th), minimum 47 divisions (0.0017 parts). At the time of the longer streamers there was an average decrease, and during the continuance of the shorter streamers the horizontal force was normal. At $0^h\ 23\frac{1}{2}^m$ and $2^h\ 23\frac{1}{2}^m$ the vertical force was 36 and 32 divisions smaller than the normal. Difference 0.0011 parts of the force.

XX. 1844, March 29th. At $16^h\ 51^m$ cloudy, aurora visible. The declination magnet is deflected to the east and west several times in succession; between $16^h\ 20^m$ and $18^h\ 20^m$ about 14 divisions E., and 16 divisions E.; the following greatest west deflection of 61 divisions (27′) occurred at $20^h\ 10^m$; the next east deflection reached a maximum at $0^h\ 22^m$ (30th) of 41 divisions; a maximum west deflection was again reached at $1^h\ 14^m$ of 50 divisions (23′). The horizontal force is throughout smaller than the normal value, with differences varying on the average from 50 to 70 divisions. The greatest difference was reached at $20^h\ 2^m$ of nearly 100 divisions (0.0036 parts of the force); at $23^h\ 47^m$ another small value of 90 divisions was observed. The vertical force was disturbed from $21^h\ 23\frac{1}{2}^m$ to $4^h\ 23\frac{1}{2}^m$ (30th). Force less 49 divisions, 55, 44, 73, 49, 52, 55, and 31 divisions. Minimum value 0.0024 parts of the force.

XXI. 1844, April 17th. At $2^h\ 20^m$, although cloudy, it was very bright at the north; same remark at $22^h\ 20^m$. The declination disturbances extend nearly over the whole day. The deflection was at first west (between $0^h\ 20^m$ and $4^h\ 20^m$) with a maximum value of 48 divisions (22′) at $3^h\ 10^m$; it then changed to the east, at $6^h\ 04^m$ it reached 52 divisions (23′); up to $20^h\ 20^m$ the deflection was slightly to the east. The horizontal force was diminished early in the morning, attaining a first minimum at $2^h\ 40^m$ of 47 divisions; it increased for a short time, reaching at $4^h\ 14^m$ 52 divisions above the normal, the force again decreased and reached at $5^h\ 47^m$ the lowest value of 151 divisions (0.0055 parts); it remained below the normal value for several hours. At $19^h\ 53^m$ the diminution was 41 divisions. Vertical force disturbed from 3^h to 8^h ($+23\frac{1}{2}^m$), force less 52 divisions, 58, 61, 66, 53, and 35 divisions. Minimum value 0.0022 parts of the force.

XXII. 1845, January 9th. At $17^h\ 20^m$ an aurora visible. The declination magnet is deflected east and west alternately; first maximum east deflection at $16^h\ 32^m$ of 20 divisions; following maximum west at $17^h\ 02^m$ of 11 divisions; following east deflection about 20 divisions 12^m later; next west deflection at $17^h\ 22^m$ 21 divisions; at $19^h\ 56^m$ the deflection again east 32 divisions; at $21^h\ 38^m$ it is west 40 divisions (18′), at $22^h\ 20^m$ it is east 33 divisions. The horizontal force between $15^h\ 52^m$ and midnight is considerably smaller than the normal value, a minimum is reached at $17^h\ 16^m$ of 155 divisions (0.0056 parts of the force). The disturbances ceased between 2^h and 3^h on the morning of the 10th. The vertical force was disturbed at 17^h, 20, 22, and 23^h ($+23\frac{1}{2}^m$), force greater 44 divisions, 31, 35, and 33 divisions. Average increase 0.0012 parts of the force.

From the preceding detailed account of the condition of the declination and of the horizontal and vertical components of the magnetic force during auroral displays, we obtain the following general results: Each of the 22 auroras recorded was accompanied by a corresponding disturbance of the earth's magnetism, at least in one of the three elements; in one case the declination alone was affected, in another case only the horizontal force, and in a third only the vertical force. The latter force was less subject to disturbances than the other two elements.

In the following table, showing the condition of the magnetic components during auroras, the first column contains the number of the aurora, the second the amount of declination deflection, the third its direction or the successive large excursions of the north end eastward or westward, the fourth the amount of the horizontal force disturbance expressed in parts of that force (a minus sign indicates less force than the normal belonging to that time, a plus sign indicates the reverse), the last column contains the amount of disturbance in the vertical force expressed in parts of that force; the signs have the same signification as for the horizontal force.

Number.	Amount of deflection.	Direction of deflection and excursions.	Excess (+) or defect (—) of horizontal force.	Excess (+) or defect (—) of vertical force.
1	24′	E. W. E. W. E.	—0.016	— (?)
2			—0.003	
3	17	E.	—0.005	—0.004
4			∓0.002	—0.002
5	22	W.	—0.004	
6	57	E.	—0.010	—0.003
7	18	W. E. W. E.	—0.005	—0.002
8	26	W. E. W.	—0.010	—0.004
9	9	W.		
10	22	W. E.	—0.007	
11	33	E. W. E. W.	±0.013	+0.005
12	8	E. W. E. W.	—0.003	—0.001
13				∸0.001
14	9	W.	—0.002	
15	13	E. W.	—0.002	
16	15	E. W. E.	—0.002	
17	16	W.	—0.003	
18	13	E. W.	—0.003	
19	7	W.	—0.002	—0.001
20	27	E. W. E. W.	—0.004	—0.002
21	23	W. E.	∓0.006	—0.002
22	18	E. W. E. W. E. W. E.	—0.006	+0.001

The action on the declination magnet appears to be that of alternate deflections either way from the normal position; in 5 cases the deflection was west, in 2 cases east, more frequently there were one or two successions of west and east deflections (or the reverse) in one instance even three; these alternate excursions appear to be a characteristic sign. In 5 cases the tendency of the deflection was easterly, in 6 cases westerly, and in the remaining 8 cases in both directions. The average amount of deflection is 17′. With but one exception the uniform effect upon the horizontal force was to decrease it. In the exceptional case a decrease followed the increase; in another case the reverse took place; during one aurora there was at first a fall in the force, then a rise, and again a fall. The average depression of the horizontal force below the normal value was 0.005 parts of the force (0.021 in absolute measure). The effect upon the vertical force is small; in 9 cases no disturbance occurred; in general the force is less than the normal; there are two exceptions to this in the 13 cases. The average depression of the vertical force below its normal was 0.0007 parts of the force (0.009 in absolute measure), or irrespective of sign 0.0013 parts of the force. If we wish to compare the tabular differences in declination and horizontal and vertical force with the magnitude of the recognized disturbances, the latter are 4′ and 0.001 (parts of the force) for either the horizontal or vertical component.

Of the auroras noted, that of May 29, 1840, was in many respects the most remarkable, and the best observed both as to its appearance and as to its magnetic effect; its study can be recommended to those who have occasion to test their theoretical views in reference to this phenomenon, and to terrestrial magnetism. Its appearance at New Haven, Conn., has been described by E. C. Herrick (Sill. Jour., 1840, Vol. XXXIX, p. 194). It was seen over a great portion of the United States, in Canada, and England. See, also, description in the volume of the Toronto Observations; also an extract from the proceedings of the British Association (Sill. Jour. Vol. XL, p. 337); also (p. 338 ibid.) note on the same by the Astronomer Royal.

The total number of auroras which occurred at Philadelphia was much greater than the number given above, as has already been stated. At Toronto the annual distribution of the phenomenon over the same period of time, and for three years beyond the close of the record at Philadelphia, is as follows:—

In 1840 (from March)	. . . .	on 23 nights.
" 1841	. . . .	36 "
" 1842	. . . .	14 "
" 1843	. . . .	16 "
" 1844	. . . .	20 "
" 1845	. . . .	19 "
" 1846	. . . .	27 "
" 1847	. . . .	29 "
" 1848	. . . .	66 "

These figures seem to indicate the existence of a period of frequency, probably of eleven years as conjectured by Prof. Wolf, the least number probably occurred in 1843, if we make an allowance for invisibility of the phenomenon either by daylight or by cloudy weather.

Between June, 1840, and July, 1845 (incl.), there were seen, according to the Toronto record, 109 auroras. The disturbances at Philadelphia on the dates of their appearance have been classified as follows: The numbers give the relative proportion to the total number, which latter is expressed by 100; the average numbers are given resulting from the examination of the disturbances of the declination, the horizontal and the vertical force.

	Number of cases.
No record at Philadelphia	19
None of the elements disturbed	30
But very few disturbances	20
An ordinary number of disturbances	14
An unusual number of disturbances	17

The number of unusual disturbances is therefore less than one-fifth of the total amount, and in fully one-half of the cases the magnetic elements were either not at all or but very slightly affected.

PART VIII.

INVESTIGATION

OF THE

SOLAR-DIURNAL VARIATION AND OF THE ANNUAL INEQUALITY OF THE VERTICAL COMPONENT OF THE MAGNETIC FORCE.

DISCUSSION

OF THE

SOLAR DIURNAL VARIATION, AND OF THE ANNUAL INEQUALITY OF THE VERTICAL COMPONENT OF THE MAGNETIC FORCE AT PHILADELPHIA.

THE necessary data for this investigation are given in the preceding Part (VII), which contains the normals resulting from the reduction of the observations to the same temperature (66° Fah.), from the allowance for irregularity in the progressive change and the exclusion of all recognized disturbances.

Owing to the greater irregularity in the indications of the vertical force instrument, and the comparatively small number of observations at odd hours, the normals are given for the even hours only; the observations at odd hours, however, are used to improve those taken at the intermediate even hours by means of a suitable process of interpolation.

The tabular numbers are expressed in scale divisions, one division being equal to 0.000033 parts of the vertical force, or equal to 0.000423 in absolute measure. Increasing numbers denote decrease of force. The hours count from midnight to midnight to 24 hours; the number of minutes the observations are made later than the full hour are given in the last column for each month.

NORMALS OF THE VERTICAL FORCE FOR JULY.

Year.	0^h	1	2	3	4	5	6	7	8	9	10	11^h
1841	703		697		687		671		667		664	
1842	683		672		664		659		657		650	
1843	691		692		692		686		679		672	
1844	816	811	804	806	805	802	798	793	784	783	782	780
Means[1]	722		717		712		703		698		692	

Noon	13	14	15	16	17	18	19	20	21	22	23^h	Min.
665		676		680		698		708		706		+17
643		643		647		663		677		682		+17½
662		658		659		666		677		685		+23½
780	776	773	781	783	791	799	805	808	809	811	816	+23½
687		688		693		706		717		721		+20.4

[1] Let reading for any even hour $= n$ for the year 1844, for the odd hours preceding and following $n\,p$ & $n\,f$, mean $\frac{n\,p + n\,f}{2}$; hence mean for the even hour $\frac{1}{2}\left(n + \frac{n\,p + n\,f}{2}\right)$ which was substituted before the general mean for the four years was taken.

Normals of the Vertical Force for August.

Year.	0h	1	2	3	4	5	6	7	8	9	10	11h
1841	686		680		676		664		662		652	
1842	689		689		683		682		679		672	
1843	703		703		708		706		698		683	
1844	794	790	786	781	783	777	776	769	763	760	756	754
Means	718		715		712		706		700		691	

Noon	13	14	15	16	17	18	19	20	21	22	23	Min.
653		662		666		676		689		691		+17
652		659		669		679		688		688		$+17\frac{1}{2}$
669		671		672		682		695		699		$+23\frac{1}{2}$
750	750	746	759	764	771	778	789	790	789	792	794	$+23\frac{1}{2}$
681		686		693		704		715		718		+20.4

Normals of the Vertical Force for September.

Year.	0h	1	2	3	4	5	6	7	8	9	10	11h
1841	663		655		646		647		637		631	
1842	692		686		689		690		681		671	
1843	721		719		721		716		707		706	
1844	816	815	813	812	811	810	809	805	798	795	790	785
Means	723		718		717		715		706		699	

Noon	13	14	15	16	17	18	19	20	21	22	23h	Min.
627		628		634		645		653		660		+17
671		673		672		679		687		693		$+17\frac{1}{2}$
693		692		692		703		714		710		$+23\frac{1}{2}$
780	779	772	794	793	795	806	813	812	812	814	817	$+23\frac{1}{2}$
693		693		698		708		717		719		+20.4

Normals of the Vertical Force for October.

Year.	0h	1	2	3	4	5	6	7	8	9	10	11h
1841	579		578		573		568		558		556	
1842	706		698		702		714		695		708	
1843	714	710	707	713	712	714	717	714	706	704	703	701
1844	775	771	769	773	776	779	780	780	774	773	773	770
[1]Means.	694		689		691		694		684		685	

Noon	13	14	15	16	17	18	19	20	21	22	23h	Min.
561		562		571		577		581		583		+17
707		706		706		708		710		709		+17½
702	700	703	701	704	709	714	717	719	715	714	717	+23½
769	766	773	777	786	788	791	789	785	785	781	781	+23½
685		685		692		697		699		697		+20.4

Normals of the Vertical Force for November.

Year.	0h	1	2	3	4	5	6	7	8	9	10	11h
1841	532		537		538		533		526		523	
1842	717		713		723		725		712		715	
1843	742	740	745	743	745	744	744	748	742	739	737	729
1844	775	772	768	769	772	771	770	773	768	772	772	767
Means	691		691		694		693		688		686	

Noon.	13	14	15	16	17	18	19	20	21	22	23h	Min.
520		521		526		529		531		538		+17
711		713		716		712		718		718		+17½
735	731	731	738	746	749	749	751	749	747	746	748	+23½
766	763	762	765	774	779	777	778	774	770	769	772	+23½
682		683		690		692		693		693		+20.4

[1] Let m equal any reading at an even hour in 1843 or 1844, mp and mf the same for the odd hours preceding and following, then mean for the even hour $\frac{1}{2}\left(m + \frac{mp + mf}{2}\right)$ which was substituted for the above value at the even hour in 1843 and 1844 before the general mean of the four years was taken.

NORMALS OF THE VERTICAL FORCE FOR DECEMBER.

Year.	0h	1	2	3	4	5	6	7	8	9	10	11h
1841	597		591		590		606		592		598	
1842	713		707		709		706		715		711	
1843	752	744	733	742	740	742	740	743	740	740	729	720
1844	754	753	754	755	757	756	755	758	749	751	750	746
Means	703		697		699		703		700		697	

Noon	13	14	15	16	17	18	19	20	21	22	23h	Min.
605		597		607		610		605		604		+17
709		707		706		711		713		713		+17½
727	729	743	749	758	760	767	763	764	757	754	757	+23½
743	733	736	724	748	757	755	751	751	748	750	750	+23½
695		695		704		710		707		705		+20.4

NORMALS OF THE VERTICAL FORCE FOR JANUARY.

Year.	0h	1	2	3	4	5	6	7	8	9	10	11h
1841												
1842	658		642		656		649		656		653	
1843												
1844	733	739	730	728	732	733	732	732	730	725	720	713
1845	754	747	747	752	752	757	763	767	760	758	753	752
Means	716		707		713		715		715		709	

Noon	13	14	15	16	17	18	19	20	21	22	23h	Min.
664		650		663		675		670		663		+17½
		[1]690										+17
715	708	715	724	737	740	741	743	744	745	744	744	+23½
751	748	749	742	753	760	756	753	749	746	747	754	+23½
709		705		717		724		721		719		+21.5

[1] No use is made of this reading, nor of the analogous readings in the following two months.

Normals of the Vertical Force for February.

Year.	0h	1	2	3	4	5	6	7	8	9	10	11h
1841	664		664		662		656		654		650	
1842	706		701		713		721		699		698	
1843												
1844	734	729	725	726	729	728	729	730	723	724	722	717
1845	760	756	752	757	759	760	763	764	756	756	752	749
Means	718		711		716		717		709		705	

Noon	13	14	15	16	17	18	19	20	21	22	23h	Min.
644		648		647		673		668		665		+17
709		692		698		712		723		710		+17½
		(697)										+17
718	716	716	720	727	731	738	739	740	737	735	733	+23½
747	741	740	744	753	759	761	761	758	753	752	756	+23½
704		700		706		720		722		716		+20.4

Normals of the Vertical Force for March.

Year.	0h	1	2	3	4	5	6	7	8	9	10	11h
1841	670		661		661		655		651		645	
1842	655		643		654		663		657		661	
1843												
1844	768	761	763	760	764	762	765	762	758	761	762	764
1845	749	741	736	742	746	748	749	746	736	733	732	729
Means	709		701		705		707		701		700	

Noon	13	14	15	16	17	18	19	20	21	22	23h	Min.
643		646		656		665		666		673		+17
651		650		657		668		673		665		+17½
		(686)										+17
762	753	758	760	754	757	753	754	752	753	763	764	+23½
729	729	713	734	740	747	751	746	741	740	739	743	+23½
696		694		703		709		708		710		+20.4

NORMALS OF THE VERTICAL FORCE FOR APRIL.

Year.	0^h	1	2	3	4	5	6	7	8	9	10	11^h
1841	671		662		658		653		645		646	
1842	668		658		655		655		656		654	
1843	715		712		717		716		708		702	
1844	776	773	771	765	766	765	759	755	749	749	744	740
1845	732	727	728	729	728	727	725	721	718	715	717	717
Means	712		706		705		702		696		693	

Noon	13	14	15	16	17	18	19	20	21	22	23^h	Min.
646		649		656		666		670		676		+17
657		651		650		660		672		672		+17½
709		700		696		696		710		709		+23½
737	733	737	741	745	744	756	767	768	765	767	775	+23½
716	713	717	720	724	724	732	737	736	729	726	727	+23½
693		691		694		702		711		711		+21.0

NORMALS OF THE VERTICAL FORCE FOR MAY.

Year.	0^h	1	2	3	4	5	6	7	8	9	10	11^h
1841	669		665		660		654		647		649	
1842	673		670		661		644		646		647	
1843	698		699		695		690		682		680	
1844	772	769	766	768	767	764	760	754	749	747	747	744
1845	720	718	717	719	716	715	713	709	705	702	701	699
Means	706		704		700		692		686		685	

Noon	13	14	15	16	17	18	19	20	21	22	23^h	Min.
644		646		650		660		674		679		+17
651		644		645		656		668		670		+17½
677		668		677		685		691		695		+23½
744	740	747	744	746	748	754	763	766	766	768	772	+23½
697	696	701	698	702	705	710	720	718	717	718	719	+23½
683		680		684		693		703		706		+21.0

Normals of the Vertical Force for June.												
Year.	0h	1	2	3	4	5	6	7	8	9	10	11h
1841	646		631		622		617		624		616	
1842	674		669		664		658		647		642	
1843	698		691		693		687		677		668	
1844	776	772	767	768	767	764	760	755	747	745	744	744
1845	733	731	729	730	729	728	726	722	717	715	715	711
Means	705		698		695		690		683		677	

Noon	13	14	15	16	17	18	19	20	21	22	23h	Min.
603		624		635		650		653		657		+17
635		639		635		653		671		668		+17½
663		658		659		669		681		689		+23½
742	739	744	747	749	754	760	769	771	770	771	775	+23½
711	711	713	714	713	714	722	730	735	731	731	733	+23½
671		675		678		691		702		703		+21.0

Table I.—Recapitulation of the Bi-hourly Normals of the Vertical Force (expressed in Scale Divisions) for each Month of the Year.

Increase of scale readings denotes decrease of force.

1841–5.	0h	2	4	6	8	10	Noon	14	16	18	20	22h	(+20m.6)
January	716	707	713	715	715	709	709	705	717	724	721	719	
February	718	711	716	717	709	705	704	700	706	720	722	716	
March	709	701	705	707	701	700	696	694	703	709	708	710	
April	712	706	705	702	696	693	693	691	694	702	711	711	
May	706	704	700	692	686	685	683	680	684	693	703	706	
June	705	698	695	690	683	677	671	675	678	691	702	703	
July	722	717	712	703	698	692	687	688	693	706	717	721	
August	718	715	712	706	700	691	681	686	693	704	715	718	
September	723	718	717	715	706	699	693	693	698	708	717	719	
October	694	689	691	694	684	685	685	685	692	697	699	697	
November	691	691	694	693	688	686	682	683	690	692	693	693	
December	703	697	699	703	700	697	695	695	704	710	707	705	
Year . .	709.7	704.5	704.9	703.1	697.2	693.2	689.9	689.6	696.0	704.7	709.6	709.8	
Summer .	714.3	709.7	706.8	701.3	694.8	689.5	684.7	685.5	690.0	700.7	710.8	713.0	
Winter .	705.2	699.3	703.0	704.8	699.5	697.0	695.2	693.7	702.0	708.7	708.3	706.7	

The months from April to September inclusive comprise the summer half year, those from October to March inclusive the winter half year.

Table II.—Mean Values of the Normals for each Month and Season.

1841–45.		1841–44.		1841–45.	
January	714.2	July	704.7	Year	701.0
February	712.0	August	703.3	Summer	700.0
March	703.6	September	708.8	Winter	702.0
April	701.3	October	691.0		
May	693.5	November	689.7		
June	689.0	December	701.2		

Subtracting each value of Table I from its respective monthly mean as given in Table II, and converting the remainder into parts of the force, we find the regular solar-diurnal variation presented in the following table:—

TABLE III.—REGULAR SOLAR-DIURNAL VARIATION OF THE VERTICAL FORCE EXPRESSED IN PARTS OF THE FORCE.

A plus sign indicates a greater, a minus sign a less value than the mean. The first three places of decimals 0.000 have been placed on the side.

1841–5.	0^h	2	4	6	8	10	Noon.	14	16	18	20	22^h	$+20^m.6$
January	—059	+238	+040	—026	—026	+172	+172	+304	—092	—323	—224	—158	
February	—198	+033	—132	—165	+099	+231	+264	+396	+198	—264	—330	—132	
March	—178	+086	—046	—112	+086	+119	+251	+317	+020	—178	—145	—211	
April	—353	—155	—122	—023	+175	+274	+274	+340	+241	—023	—320	—320	
May	—412	—346	—214	+049	+247	+280	+346	+445	+313	+016	—313	—412	
June	—528	—297	—198	—033	+198	+396	+594	+462	+363	—066	—429	—462	
July	—571	—406	—241	+056	+221	+419	+584	+551	+386	—043	—406	—538	
August	—485	—386	—287	—089	+109	+406	+736	+571	+340	—023	—386	—485	
September	—469	—304	—271	—205	+092	+323	+521	+521	+356	+026	—271	—337	
October	—099	+066	—000	—099	+231	+198	+198	+198	—033	—198	—264	—198	
November	—043	—043	—142	—109	+086	+122	+254	+221	—010	—076	—109	—109	
December	—059	+139	+040	—059	+040	+139	+205	+205	—092	—290	—191	—125	
Year	—287	—115	—131	—068	+129	+257	+366	+377	+165	—121	—282	—290	
Summer	—469	—317	—221	—040	+175	+350	+508	+482	+333	—020	—353	—426	
Winter	—106	+086	—040	—096	+082	+162	+224	+274	—000	—221	—211	—155	

Multiplying the above numbers by Y=12.83, we obtain the solar-diurnal variation in absolute value.

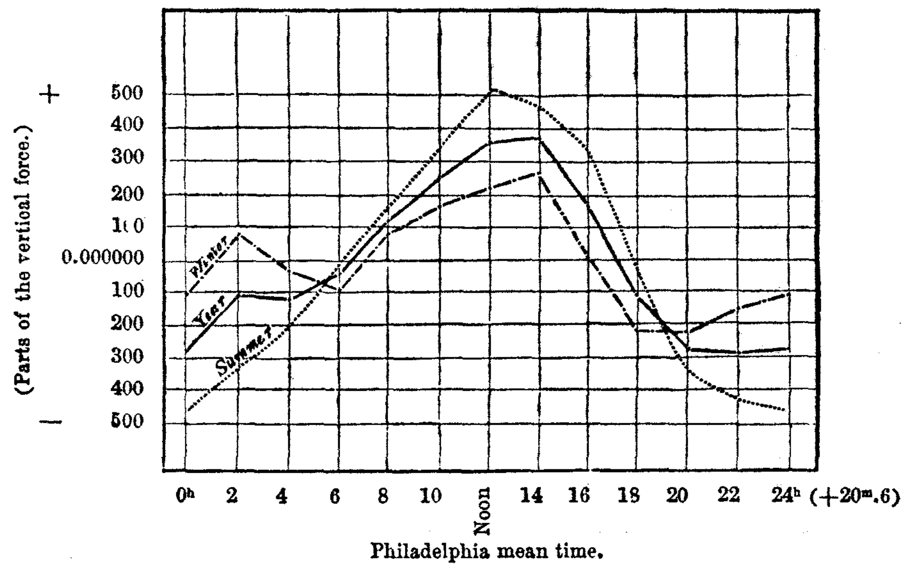
TABLE IV.—REGULAR SOLAR-DIURNAL VARIATION OF THE VERTICAL FORCE IN ABSOLUTE MEASURE.

A greater force than the mean is indicated by a plus sign, a less force by a minus sign. The first two places of decimals 0.00 are placed on the side.

1841–5.	0^h	2	4	6	8	10	Noon.	14	16	18	20	22^h	$+20^m.6$
January	—076	+305	+051	—034	—034	+220	+220	+389	—118	—415	—289	—203	
February	—254	+042	—170	—212	+127	+296	+338	+508	+254	—338	—423	—169	
March	—229	+110	—059	—144	+110	+152	+321	+406	+025	—228	—186	—271	
April	—452	—198	—157	—030	+224	+351	+351	+436	+309	—030	—410	—410	
May	—529	—444	—275	+064	+317	+360	+444	+571	+402	+021	—402	—529	
June	—677	—380	—254	—042	+254	+508	+761	+592	+465	—085	—550	—592	
July	—732	—520	—308	+072	+283	+537	+749	+706	+495	—055	—520	—690	
August	—622	—495	—368	—114	+140	+520	+943	+732	+436	—030	—495	—622	
September	—601	—389	—346	—263	+118	+415	+668	+668	+457	+034	—347	—431	
October	—127	+085	—000	—127	+296	+254	+254	+254	—042	—254	—338	—254	
November	—055	—055	—182	—140	+072	+157	+326	+283	—013	—098	—140	—140	
December	—076	+178	+051	—076	+051	+178	+263	+263	—118	—372	—245	—161	
Year	—368	—148	—168	—087	+165	+329	+470	+483	+212	—156	—364	—372	
Summer	—601	—406	—284	—051	+224	+448	+651	+618	+426	—025	—453	—546	
Winter	—136	+110	—051	—123	+106	+207	+288	+351	—000	—283	—271	—199	

Annual Inequality in the Diurnal Variation of the Vertical Force.—If we examine the average curve of the diurnal variation as observed throughout the year, and shown on diagram (A) by a full black line, we find the principal maximum value about 1 P. M., and the principal minimum value about 9½ P. M.; besides these characteristic values there is an indication of a secondary maximum about 2 A. M., and of a secondary minimum about 4 A. M. Dividing the year into a summer and winter season, the diagram exhibits the diurnal variation in summer to be a curve of but one maximum and one minimum occurring about noon and midnight respectively, whereas in winter the double feature of the curve becomes very con-

(A.) Diurnal Variation of the Vertical Force in Summer, Winter, and for the whole Year.

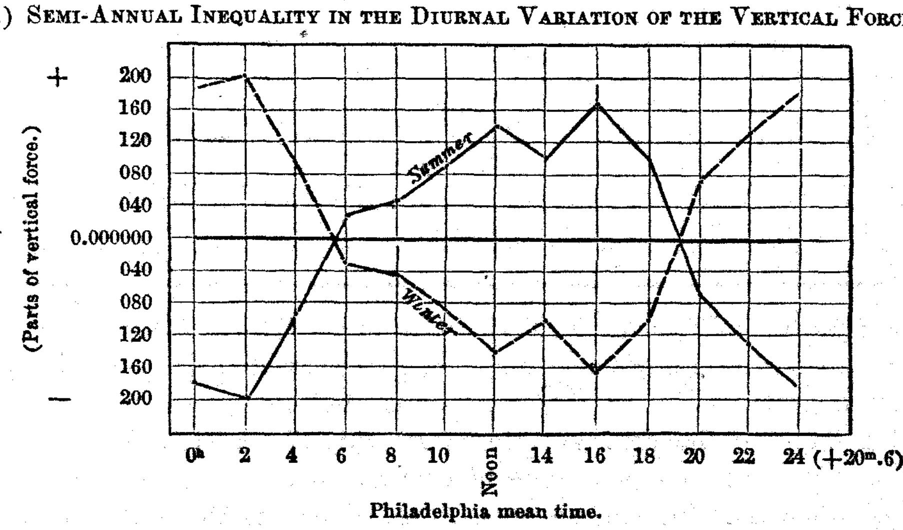


spicuous, the secondary maximum and minimum occurring about 2 and 6 A. M. respectively. The phenomenon, in the two seasons, changes therefore from a single to a double crested curve.

The semi-annual change of the diurnal variation is better shown in diagram (B), which contains the difference from the annual curve in summer and winter, viz. :—

	0h	2	4	6	8	10	Noon.	14	16	18	20	22h	(+20m.6)
Summer	—182	—201	—90	+28	+46	+93	+142	+105	+168	+101	—71	—136	
Winter	+181	+202	+91	—28	—47	—95	—142	—103	—165	—100	+71	+135	

(B.) Semi-Annual Inequality in the Diurnal Variation of the Vertical Force.



At $5\frac{1}{2}$ A. M. and 7 P. M. there is no change in the diurnal variation throughout the year; at the hours 2 A. M. and 4 P. M. the change is a maximum, viz: range equal 0.000403 parts and 0.000333 parts of the force, or equal 0.00517 and 0.00427 when expressed in absolute measure.

The turning epochs of the annual inequality as found from the hours 2 A. M. and 4 P. M. are derived from the following table, in which the numbers are expressed in parts of the force; the numbers in the last column were obtained by changing the sign of the afternoon difference before taking the mean.

	2 A. M. 0.000.	Differences. 0.000.	4 P. M. 0.000.	Differences. 0.000.	Mean difference. 0.000.
January	+238	+353	—092	—257	+305
February	+033	+148	+198	+033	+058
March	+086	+201	+020	—145	+173
April	—155	—040	+241	+076	—058
May	—345	—231	+313	+148	—190
June	—297	—182	+363	+198	—190
July	—406	—291	+386	+221	—256
August	—386	—271	+340	+175	—223
September	—304	—189	+356	+191	—190
October	+066	+181	—033	—198	+190
November	—043	+072	—010	—177	+125
December	+139	+254	—092	—257	+256
Mean	—115		+165		

The figures in the last column are represented by the equation

$$\Delta_a = + 0.000260 \, sin \, (\theta + 86°) + 0.000031 \, sin \, (2\theta + 180°)$$

the angle θ counting from January 1st at the rate of 30° a month. According to this expression the transition of the inequality from a positive to a negative value, and vice versa, takes place in the first quarter of April and October, or about 17 days after the equinoxes. The retardation of the phenomenon in the declination, horizontal and vertical force is, therefore, 10, 22, and 17 days respectively, or 16 days on the average.

Analysis of the Solar-Diurnal Variation of the Vertical Force.—For greater facility of the investigation, and for purposes of comparison, the numbers of Table 1. have been expressed analytically. The angle θ counts from midnight at the rate of 15° an hour.

For January, $\Delta_v = 714^d.2 + 4.8 \, sin \, (\theta + 134° \, 09') + 5.5 \, sin \, (2\theta + 224° \, 22') + 0.8 \, sin \, (3\theta + 61°)$

For February, $\Delta_v = 712^d.0 + 7.5 \, sin \, (\theta + 91° \, 47') + 5.1 \, sin \, (2\theta + 226° \, 22') + 1.6 \, sin \, (3\theta + 273°)$

For March, $\Delta_v = 703^d.6 + 5.5 \, sin \, (\theta + 98° \, 24') + 3.6 \, sin \, (2\theta + 220° \, 22') + 0.7 \, sin \, (3\theta + 95°)$

For April, $\Delta_v = 701^d.3 + 10.5 \, sin \, (\theta + 89° \, 12') + 2.2 \, sin \, (2\theta + 175° \, 59') + 1.3 \, sin \, (3\theta + 232°)$

For May, $\Delta_v = 693^d.5 + 13.1 \, sin \, (\theta + 85° \, 17') + 1.9 \, sin \, (2\theta + 144° \, 31') + 1.8 \, sin \, (3\theta + 278°)$

For June, $\Delta_v = 689^d.0 + 15.8 \, sin \, (\theta + 87° \, 22') + 3.1 \, sin \, (2\theta + 193° \, 56') + 0.4 \, sin \, (3\theta + 210°)$

For July, $\Delta_v = 704^d.7 + 17.4 \sin(\theta + 86° 30') + 2.6 \sin(2\theta + 174° 16') + 0.7 \sin(3\theta + 300°)$

For August, $\Delta_v = 703^d.3 + 17.1 \sin(\theta + 81° 10') + 3.7 \sin(2\theta + 215° 50') + 0.5 \sin(3\theta + 75°)$

For September, $\Delta_v = 708^d.8 + 14.3 \sin(\theta + 73° 57') + 2.9 \sin(2\theta + 210° 24') + 0.3 \sin(3\theta + 165°)$

For October, $\Delta_v = 691^d.0 + 6.1 \sin(\theta + 119° 48') + 3.1 \sin(2\theta + 236° 28') + 1.1 \sin(3\theta + 210°)$

For November, $\Delta_v = 689^d.7 + 4.4 \sin(\theta + 83° 33') + 3.0 \sin(2\theta + 254° 00') + 0.0$

For December, $\Delta_v = 701^d.2 + 4.5 \sin(\theta + 133° 49') + 4.3 \sin(2\theta + 231° 57') + 1.0 \sin(3\theta + 63°)$

We have also for summer half year (April to September inclusive), for winter half year (October to March inclusive), and for the whole year the following expressions for the diurnal variation:—

For summer, $\Delta_v = 700^d.1 + 14.6 \sin(\theta + 83° 40') + 2.5 \sin(2\theta + 191° 01') + 0.5 \sin(3\theta + 255°)$

For winter, $\Delta_v = 702^d.0 + 5.1 \sin(\theta + 108° 54') + 4.0 \sin(2\theta + 229° 58') + 0.0$

For year, $\Delta_v = 701^d.0 + 9.7 \sin(\theta + 90° 17') + 3.0 \sin(2\theta + 216° 22') + 0.2 \sin(3\theta + 255°)$

The following comparison may serve to show the general representation of the observations by the analytical expressions:—

COMPARISON FOR AUGUST.

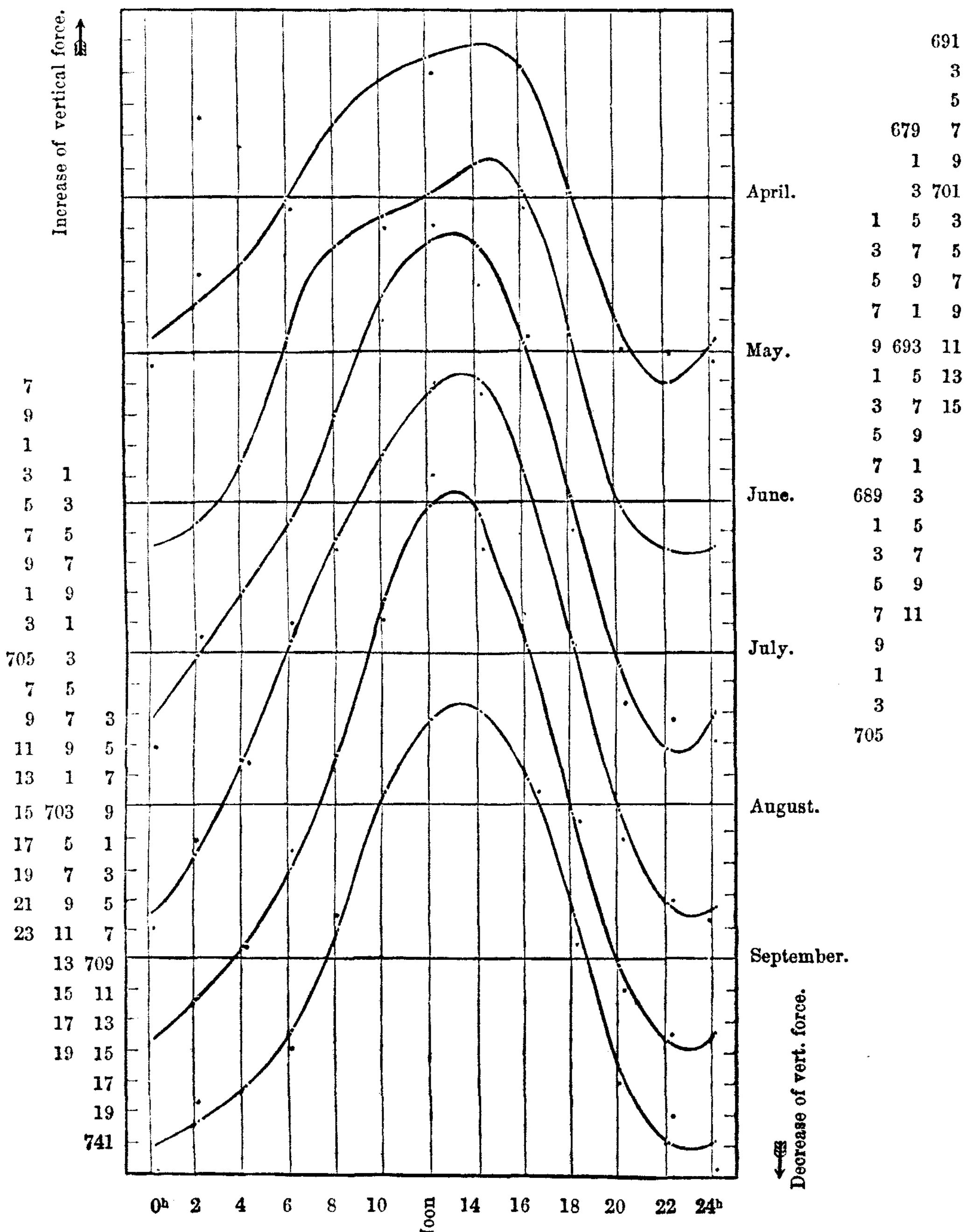
	Observed.	Computed.	Difference.
$0^h\ 20^m.6$	718	718.2	0
2 "	715	715.1	0
4 "	712	711.3	+1
6 "	706	707.0	—1
8 "	700	699.9	0
10 "	691	690.0	+1
12 "	681	683.0	—2
14 "	686	684.3	+2
16 "	693	693.5	0
18 "	704	705.0	—1
20 "	715	713.9	+1
22 "	718	718.4	0

The summer months are better represented than the winter months; in May the difference is below half a scale division; in the winter season in several instances it rises to 3, and in one case to 4 scale divisions.

Diagram C exhibits the diurnal variation, observed and computed, for the six summer months. Diagram D the same for the six winter months.

(C). SOLAR-DIURNAL VARIATION OF THE VERTICAL FORCE, APRIL TO SEPTEMBER, 1841 TO 1845.

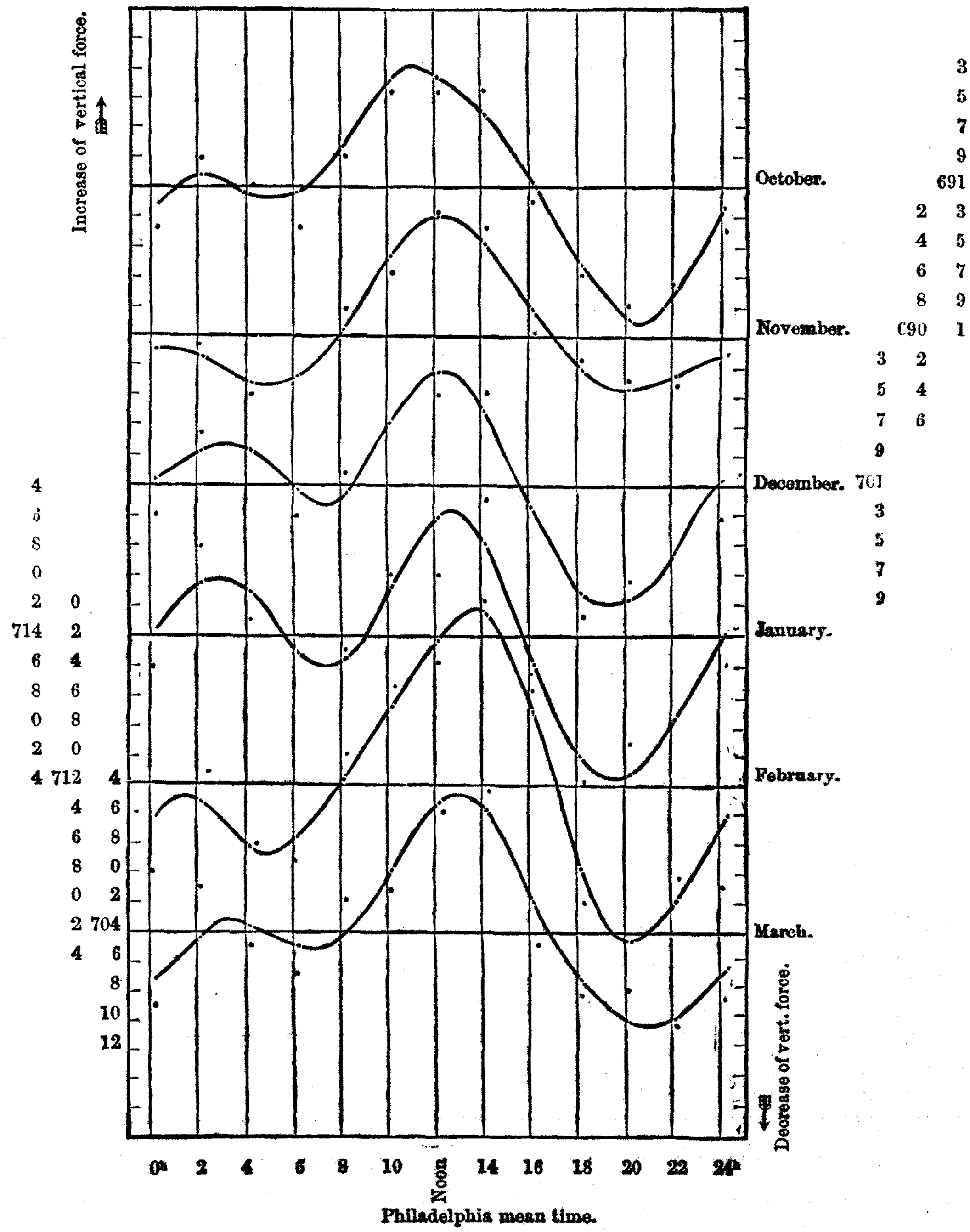
(Expressed in scale divisions.)
$1^d = 0.000033$ parts of the force.



(D). SOLAR-DIURNAL VARIATION OF THE VERTICAL FORCE, OCTOBER TO MARCH, 1841 TO 1845.

(Expressed in scale divisions.)

$1^d = 0.000033$ parts of the force.



The numerical values of the coefficients B_1 B_2 B_3 in the general equation

$$\Delta_v = A + B_1 \sin(\theta + C_1) + B_2 \sin(2\theta + C_2) + B_3 \sin(3\theta + C_3)$$

expressed in parts of the horizontal force, are given in Table V. The first three decimals (0.000) have been placed in front of the table.

TABLE V.

Month.		B_1	B_2	B_3
January		158	181	026
February		247	168	053
March		181	119	023
April		346	073	043
May		432	063	059
June	0.000	521	102	013
July		574	086	023
August		564	122	016
September		472	096	010
October		201	102	036
November		145	099	000
December		148	142	033
Summer		482	082	016
Winter		170	132	001
Year		320	099	007

The next table contains the numerical values of B_1 B_2 B_3 expressed in absolute measure, and the angles C_1 C_2 C_3 obtained by the addition of 180° to their preceding values, so as to make increasing values correspond to increasing force. The first two decimals for B are placed at the head of the columns.

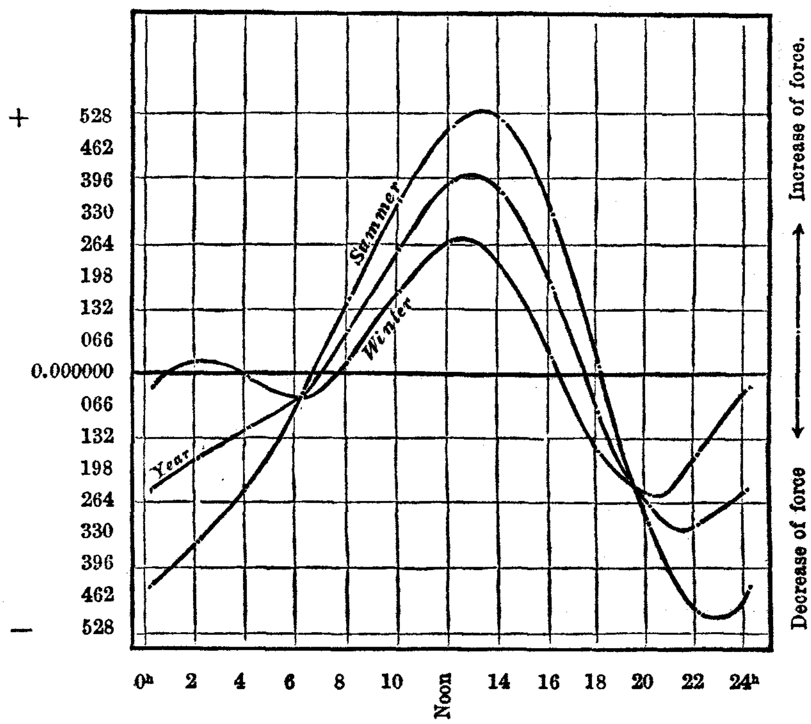
TABLE VI.

Month.	B_1 0.00	C_1	B_2 0.00	C_2	B_3 0.00	C_3
January	203	314° 09′	233	44° 22′	034	241°
February	317	271 47	216	46 22	068	93
March	233	278 24	152	40 22	030	275
April	444	269 12	093	355 59	055	52
May	554	265 17	080	324 31	076	98
June	668	267 22	131	13 56	017	30
July	736	266 30	110	354 16	030	120
August	723	261 10	157	35 50	021	255
September	606	253 57	123	30 24	013	345
October	258	299 48	131	56 28	047	30
November	186	263 33	127	74 00	000	
December	190	313 49	182	51 57	042	243
Summer	618	263 40	106	11 01	021	75
Winter	218	288 54	169	49 58	002	
Year	410	270 17	127	36 22	008	75

The next diagram (E) exhibits the general feature of the diurnal inequality for the year and its summer and winter season, as computed by means of the preceding formulæ. The greatest difference between the observed and computed values at any one hour is but $2\frac{1}{3}$ scale divisions at 2 A. M. in the winter season, and $1\frac{1}{2}$ divisions at the same hour in the annual curve. The absence of the secondary wave in the early morning hours during summer is as conspicuous as its presence in the winter season; in the annual curve there is barely a trace of it left. We also recognize again the earlier occurrence of the maximum and minimum values in winter and their later appearance in summer. If we examine the resulting curves at Toronto[1] we find there the secondary morning fluctuation equally well marked

(E). REGULAR SOLAR-DIURNAL VARIATION OF THE VERTICAL FORCE FOR WINTER, SUMMER, AND THE WHOLE YEAR.

(In parts of the force.)



Philadelphia mean time.

in summer and winter; and if we inquire into this feature for each year separately, we find great irregularities between the hours 14 (Toronto astro'l time) and 22; in 1843 the secondary maximum and minimum is plainly developed, in 1844, and especially in 1845, it cannot be traced. Diagram (F) exhibits the curves for Philadelphia for each year. In 1841 and 1842 the curves are smooth, in 1843 the wave appears well marked, in 1844 it is just perceptible. These apparent irregularities are probably due to imperfections in our instruments; on the other hand, if we take the Philadelphia series, there may be a cyclic appearance and disappearance of this wave.

[1] Vol. III of the Toronto Observations, Table LXVIII.

DIAGRAM (F).

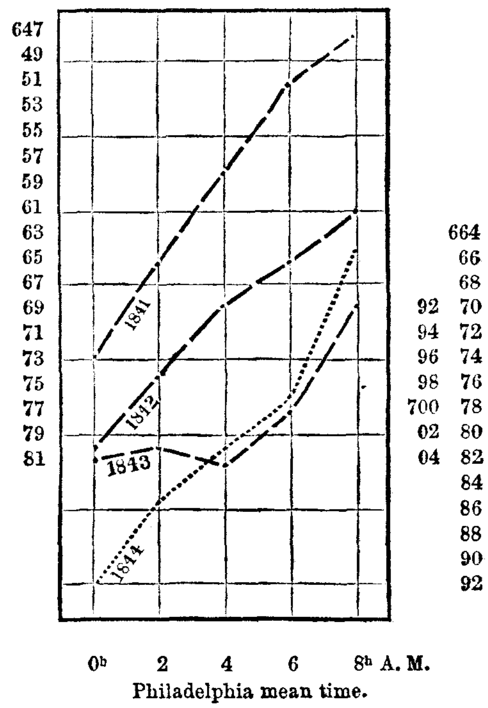


Table VII exhibits the computed times of the principal maximum and minimum of the vertical force, together with the amount of difference from its average daily value at these epochs, expressed in scale divisions; also the time and amount of the early morning secondary fluctuation traceable only in the winter months; for these last values the diagrams have been made use of.

TABLE VII.

	Maximum at	Amount in scale divisions.	Minimum at	Amount in scale divisions.	Time elapsed between max. & min.	Secondary maximum at	Amount.	Secondary Minimum at	Am't.	Secondary range.
January	$13^h\ 07^m$	$-8^d.5$	$19^h\ 27^m$	$+9^d.4$	$6^h\ 20^m$	$2\frac{1}{2}^h$	-4	7^h	$+2$	6
February	13 41	-11.5	20 21	$+10.6$	6 40	$1\frac{1}{2}$	$+1$	$4\frac{1}{2}$	$+4$	3
March	12 46	-8.9	21 04	$+6.7$	8 18	3	-1	$6\frac{1}{2}$	$+2$	3
April	13 52	-10.2	22 04	$+11.8$	8 12	-----	----	-----	----	-----
May	14 34	-13.1	22 58	$+12.6$	8 24	-----	----	-----	----	-----
June	13 09	-17.2	22 35	$+16.2$	9 26	-----	----	-----	----	-----
July	13 33	-18.2	23 10	$+17.3$	9 37	-----	----	-----	----	-----
August	13 02	-20.8	23 06	$+15.5$	10 04	-----	----	-----	----	-----
September	13 23	-16.8	23 20	$+12.6$	9 57	-----	----	-----	----	-----
October	11 16	-7.6	20 34	$+8.7$	9 18	2	-1	5	$+1$	2
November	12 35	-7.4	19 59	$+3.9$	7 24	1	$+1$	$4\frac{1}{2}$	$+3$	2
December	12 31	-7.8	19 09	$+7.8$	6 38	3	-3	7	$+1$	4
Summer	13 29	-15.8	22 55	$+14.1$	9 26	-----	----	-----	----	-----
Winter	12 43	-8.2	20 08	$+7.6$	7 25	2	-1	6	$+2$	3
Year	13 02	-11.9	21 25	$+9.7$	8 23	-----	----	-----	----	-----

The extreme variation in the time of the maximum in the course of a year is $3^h\ 18^m$, and of the minimum $4^h\ 11^m$.

At Toronto the occurrence of the maxima and minima is later than at Philadelphia; from Table LXVIII, Vol. III of the Toronto Observations, we find the maximum at 5^h, a secondary minimum at 14^h, a secondary maximum at 18^h, and the minimum at 22^h; the maximum is therefore apparently delayed at Toronto 4^h, the minimum $4\frac{3}{4}^h$, the secondary wave is likewise retarded by about 4 hours. This epochal difference I take, most likely, to be a distinctive feature due to the localities; there is also a remarkable difference in the amount of the diurnal range as will presently appear. The degrée of sensibility in the adjustment of the centre of gravity of the instrument affects most the latter difference, whereas the epochal difference may be supposed to depend, in a measure, upon the sensibility of the magnet in regard to changes of temperature and consequent changes of magnetism.

The change in the adopted value of the correction for 1° of change in the temperature (expressed in scale divisions) as used in present reduction (10.8), and as used in four volumes of record and reduction (13.5) gives us the means of a partial test of the effect on the epochs, we find from the plates in Vol. IV the time of the maximum $1\frac{1}{2}$ P. M. and of the minimum $11\frac{1}{2}$ P. M., which though somewhat nearer to the Toronto epochs, still leave a large discrepancy.

TABLE VIII.—AMPLITUDE OF THE DIURNAL VARIATION OF THE VERTICAL FORCE.

	Maximum 0.00	Minimum 0.00	Range 0.00	Maximum 0.00	Minimum 0.00	Range 0.0
January	028	031	059	359	398	0757
February	038	035	073	484	447	0931
March	029	022	051	377	283	0660
April	034	039	073	431	500	0931
May	043	042	085	555	535	1090
June	057	054	110	725	686	1411
July	060	057	117	769	733	1502
August	068	051	119	878	654	1532
September	056	041	097	712	532	1244
October	025	029	054	323	369	0692
November	024	013	037	313	166	0479
December	026	026	052	330	332	0662
Summer	052	046	098	667	597	1264
Winter	027	025	052	349	322	0671
Year	039	032	071	503	410	0913
	In parts of the force.			In absolute measure.		

The diurnal range at Toronto is very much less than at Philadelphia; in 1841–42 the range was but one-half of that observed at Philadelphia, and for later years (see Table LXVIII of Vol. III of the Toronto Observations) the ranges compare as follows: Toronto 0.00019, Philadelphia 0.00071.

In diagram G the diurnal range for each month is exhibited (expressed in parts of the force).

(G). Diurnal Range of the Vertical Force.
(In parts of the force.)

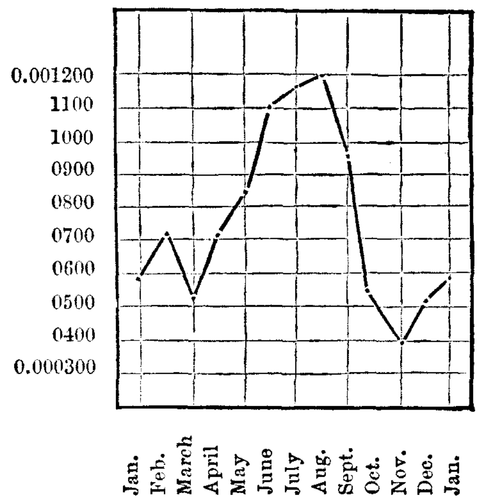


Table IX contains the times when the mean value of the vertical force is reached each day—arranged for monthly averages. In some months the average daily value is attained four times, but generally only twice. The table contains the two principal epochs (one A. M. the other P. M.); those produced by the secondary wave can easily be made out by means of the preceding diagrams.

Table IX.—Principal Epochs of Mean Vertical Force.

	A. M.	P. M.
January	$8^h\ 58^m$	$3^h\ 47^m$
February	7 42	5 16
March	8 27	4 53
April	6 15	6 13
May	5 55	6 21
June	6 36	6 04
July	6 08	6 10
August	7 30	6 04
September	7 51	6 28
October	6 34	4 19
November	8 11	4 49
December	8 27	3 39
Summer	6 43	6 13
Winter	7 52	4 29
Year	7 06	5 33

The next table contains the computed diurnal variation of the vertical force, expressed in absolute measure; compared with Table IV, it shows the differences between the observed and computed values.

TABLE X.—COMPUTED SOLAR-DIURNAL VARIATION OF THE VERTICAL FORCE, EXPRESSED IN ABSOLUTE MEASURE.

The first two places of decimals 0.00 are placed on the side; + indicates more, — less than the monthly average.

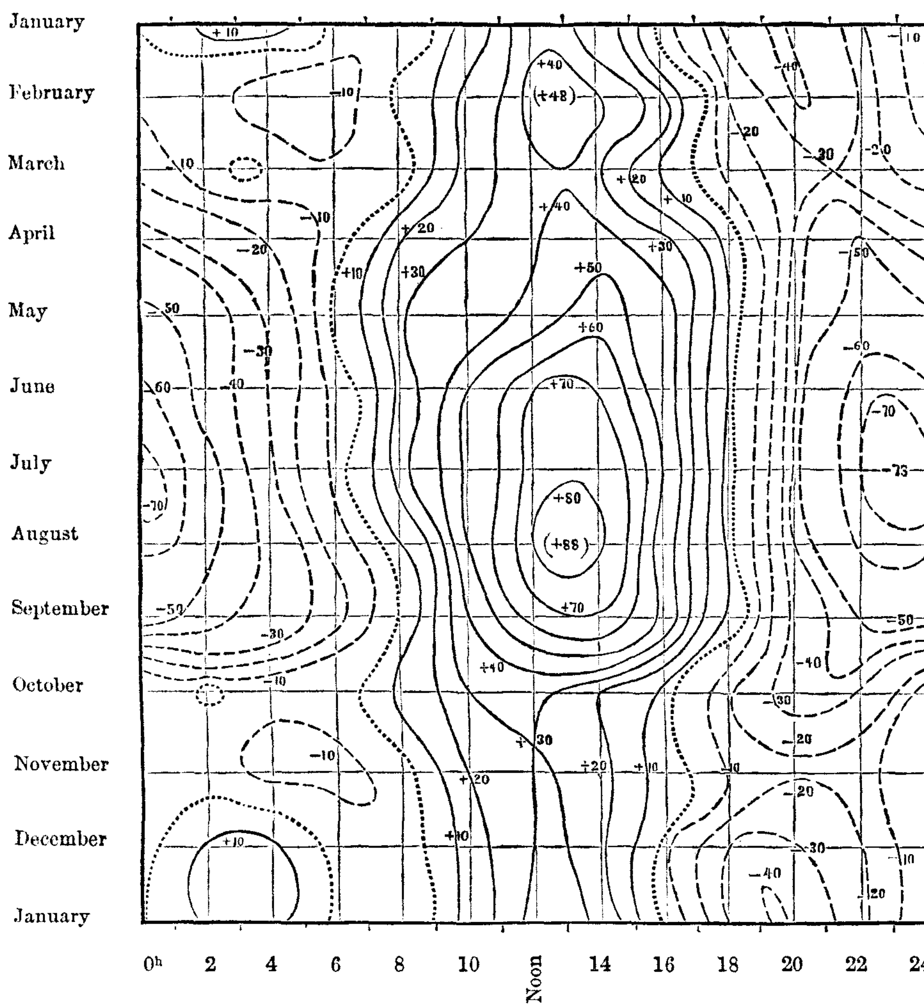
	1841–45.	0^h	2	4	6	8	10	Noon.	14	16	18	20	22^h	$+20^m.6$
	January	+025	+165	+123	—030	—046	+161	+355	+258	—080	—351	—376	—203	
	February	—072	—085	—178	—123	+063	+258	+436	+465	+203	—241	—444	—283	
	March	—135	—017	—008	—072	—013	+207	+372	+304	+059	—165	—275	—258	
	April	—385	—266	—165	+008	+216	+338	+402	+436	+317	—025	—385	—491	
	May	—516	—461	—262	+0[illegible]8	+309	+372	+448	+554	+423	—000	—402	—533	
0.00	June	—601	—419	—245	—042	+241	+533	+711	+681	+398	—068	—503	—685	
	July	—706	—550	—279	+034	+305	+537	+723	+744	+457	—051	—499	—714	
	August	—630	—499	—338	—157	+144	+563	+859	+804	+414	—072	—448	—639	
	September	—512	—431	—351	—207	+076	+431	+673	+677	+436	+047	—321	—516	
	October	—055	+030	—030	—008	+165	+309	+300	+182	—004	—237	—377	—275	
	November	—059	—089	—148	—131	+017	+207	+[illegible]13	+233	+046	—123	—161	—106	
	December	—008	+110	+097	—008	—008	+182	+330	+203	—114	—313	—304	—165	
	Year	—309	—211	—152	—055	+123	+343	+495	+457	+211	—131	—368	—402	
	Summer	—558	—440	—275	—050	+212	+465	+634	+651	+410	—025	—423	—600	
	Winter	—046	+017	—021	—051	+034	+216	+338	+271	+021	—228	—321	—216	

A graphical representation of the above tabular numbers is given in diagram H, based upon the three variables: the hour of the day, the month, and the difference of vertical force from the normal monthly value. The contour lines of the magnetic surface differ 0.001 of vertical force (in absolute measure). Full lines indicate greater value, lines of dashes less value than the normal, dotted lines represent the normal value.

(II). Differences from its Normal Value of the Vertical Force, for each Hour and Month.

(Expressed in absolute measure.)

0.00.



Philadelphia mean time.

Annual Inequality of the Vertical Force.—The minor and irregular disturbances in the adjustment of the magnetometer, as well as the effect of the progressive and secular changes, tend to make the determination of the annual inequality in the vertical force a task of some delicacy, and the results deduced from our series of observations should be considered as approximate.

Taking the monthly normals of the years 1842, 1843, and 1844, the only years which could be made complete, and correcting the monthly means for 42 scale divisions of annual increase, the following table was formed:—

Monthly Normals.												
Year.	January.	Feb'u'ry.	March.	April.	May.	June.	July.	August.	Sept'ber.	October.	Nov'ber.	Dec'ber.
1842 . . .	658	707	658	659	656	655	662	677	682	706	716	710
1843 . . .	702	710	691	707	686	678	677	691	708	710	742	746
1844 . . .	731	728	760	756	756	758	796	773	801	778	770	749
Mean . . .	697	715	703	707	699	697	712	714	730	731	743	735
Corr'd . .	+19	+16	+12	+ 9	+ 5	+ 2	— 2	— 5	— 9	—12	—16	—19
Corr'd m. . .	716	731	715	716	704	699	710	709	721	719	727	716
Mean monthly val.	— 1	—16	0	— 1	+11	+16	+ 5	+ 6	— 6	— 4	—12	— 1

The vertical force appears, therefore, to be greater in May, June, July, and August, and less in the remaining months; the range is about 32 scale divisions $= 0.00105$ parts of the force, or 0.0135 in absolute measure.

PART IX.

INVESTIGATION

OF THE

LUNAR INFLUENCE ON THE MAGNETIC VERTICAL FORCE, INCLINATION, AND TOTAL FORCE.

INVESTIGATION

OF THE

INFLUENCE OF THE MOON ON THE MAGNETIC VERTICAL FORCE.

THE method of discussion of the lunar effect on the vertical component of the magnetic force in no way differs from that employed for the horizontal component, which latter has been explained in Part VI.

The series of observations available for the lunar discussion extends from February, 1841, to June, 1845, inclusive. From February, 1841, to October, 1843, the observations are bi-hourly; from October, 1843, to the end of the series they are hourly. The record of May, 1841, is not quite complete, and in January, February, and March, 1843, but one observation a day is recorded. As increasing numbers denote a decrease of force, a positive sign of the tabulated differences between monthly normals and each individual undisturbed reading (at the normal temperature) indicates a greater force than the normal value, a negative sign indicates the reverse. 30 scale divisions being the limit beyond which difference an observation has been considered as belonging to the class of disturbances, all differences here recorded are below this limit. One scale division is 0.000033 parts of the force. The tabular numbers are expressed in scale divisions.

In tracing out the lunar effect upon the vertical force we have to contend with greater irregularities than was experienced in the case of the horizontal force. The vertical force magnetometer is more subject to changes, and the correction for temperature far exceeds that of the horizontal force.

The total number of observations and differences formed in the inquiry of the dependence of the force upon the moon's hour angle is 19513, which distribute themselves over the months and years as follows:—

TABLE I.—NUMBER OF OBSERVATIONS FOR LUNAR DISCUSSION.

Month.	1841.	1842.	1843.	1844.	1845.	Sum.
January	---	207	---	544	611	1362
February	239	198	---	549	554	1540
March	257	286	---	502	539	1584
April	256	255	250	512	581	1854
May	207	246	288	617	602	1960
June	219	275	296	529	571	1890
July	258	281	308	581	---	1428
August	283	284	314	535	---	1416
September	267	246	292	568	---	1373
October	280	298	580	607	---	1765
November	275	299	528	596	---	1698
December	239	304	541	559	---	1643
Year	2780	3179	3397	6699	3458	19513

TABLE II.—DISTRIBUTION OF NUMBERS ACCORDING TO WESTERN AND EASTERN HOUR ANGLES OF THE MOON.

Year.	Western hour angles.	Eastern hour angles.
1841	1388	1392
1842	1588	1591
1843	1694	1703
1844	3321	3378
1845	1728	1730
Sum	9719	9794

TABLE III.—DIFFERENCES FROM THE MONTHLY NORMALS, 1841.

Western hour angles of the moon.

1841.	U. cul. 0^h	1	2	3	4	5	6	7	8	9	10	11^h
January	----	---	---	---	---	---	---	---	---	---	---	---
February	+3	+1	+3	—2	+8	—5	0	+1	—1	+4	—2	—3
March	—6	+5	—3	+6	+1	+9	—6	+2	—7	—1	—3	0
April	—1	+1	0	+2	—1	+3	—1	0	—1	+7	—7	+2
May	+5	—1	+2	+1	+4	+7	+3	+4	+4	+1	+5	—5
June	0	—6	—5	+2	—8	—2	+5	—3	—4	+8	+7	0
July	—4	+8	—6	+2	—7	+5	—3	+1	—1	—7	+5	—4
August	—1	—5	+1	—6	—5	—6	0	—1	0	+1	—3	—1
September	+3	+3	+1	+1	—6	+5	—2	+2	0	—4	—3	—3
October	—8	—1	—2	0	—2	—1	—4	0	+1	0	+6	+4
November	—3	+3	—1	+3	—3	—3	—6	—5	+1	—3	+4	—1
December	—5	—4	0	—1	—2	—5	—3	—3	+5	+2	—3	—2
Year	—1.5	+0.4	—0.9	+0.7	—2.0	+0.6	—1.5	—0.2	—0.3	+0.7	+0.5	—1.2

Eastern hour angles.

1841.	L. cul. 0	1	2	3	4	5	6	7	8	9	10	11^h
January	---	---	---	---	---	---	---	---	---	---	---	---
February	—3	—2	+1	—3	+3	—2	+4	—3	+3	+2	+2	—1
March	0	+2	+3	+7	—1	+8	—7	+1	—8	0	—6	+1
April	0	—4	0	—1	0	—2	—1	—3	—1	—1	—1	+1
May	—2	—2	+1	—6	+1	—5	+1	—8	+3	—10	+3	—4
June	+1	+5	—2	+1	+5	—7	+5	+3	—1	—6	—2	—8
July	+1	—4	+6	0	—2	+5	—2	+5	—2	+2	—2	+7
August	+1	—1	+2	0	0	+1	+4	+5	+2	+4	+1	+3
September	—3	+2	—4	0	—1	+2	—1	—2	+1	0	+2	+8
October	+3	+4	+4	—1	+1	+1	+4	—1	+2	—2	—4	—4
November	+2	—3	+1	0	—1	+5	—3	+5	0	+8	—2	+2
December	+6	+6	+1	+6	+3	+2	+3	—10	0	—10	0	—7
Year	+0.5	+0.3	+1.2	+0.3	+0.7	+0.7	+0.6	—0.7	—0.1	—1.2	—0.8	—0.2

TABLE IV.—DIFFERENCES FROM THE MONTHLY NORMALS, 1842.

Western hour angles of the moon.

1842.	U. cul. 0	1	2	3	4	5	6	7	8	9	10	11^h
January	—6	0	—1	—7	—6	—5	+4	+8	—3	+9	+4	+7
February	—3	0	+5	—6	+4	—3	—9	—11	+6	+3	+7	+6
March	+5	+2	+3	0	+2	—4	—6	—6	+4	—1	—5	+5
April	+2	+2	0	0	—11	—2	—5	—3	—7	—1	+1	—5
May	—5	+3	—3	+2	—3	+4	0	0	+5	—2	+3	—7
June	+3	+5	+3	+2	—2	+1	+3	—3	+8	—7	+6	—7
July	—2	—4	—3	+2	—6	—2	0	+2	+4	+3	+7	+1
August	+2	—2	+1	—4	0	+1	—3	—2	+1	—1	+8	—3
September	+3	—5	—2	—7	—5	—3	+3	—1	—3	+6	—4	+5
October	0	0	—3	+4	+1	—5	+4	—1	+1	+4	—3	0
November	—3	+1	+1	—3	+2	+2	+3	0	+1	—4	+2	0
December	—1	—3	+1	+4	—4	+4	—1	+1	+4	—1	—3	+3
Year	—0.4	—0.1	+0.2	—1.1	—2.3	—1.0	—0.6	—1.3	+1.8	+0.7	+2.0	+0.4

Eastern hour angles.

1842.	L. cul. 0	1	2	3	4	5	6	7	8	9	10	11^h
January	+5	+4	—3	—1	—5	—3	—2	+3	—4	+4	0	—4
February	+6	+1	—2	+4	—3	+3	—1	—7	—1	+9	—5	+3
March	—4	+3	—4	—3	—4	—6	+2	—4	+2	+2	0	+4
April	—3	+4	—1	+6	+3	+10	+1	+4	+1	0	+6	+2
May	+5	+2	+6	—4	0	—2	—2	—1	—12	+8	—8	+3
June	0	—6	0	—8	+3	—5	—3	—9	+3	—5	+3	—5
July	+2	+1	+2	+1	—3	—1	+1	—2	—1	0	—3	—1
August	+5	—3	—1	0	+1	0	+2	0	+2	0	+7	—3
September	—6	—2	—1	+2	—1	—1	+2	+9	+2	+12	+4	—3
October	—9	+1	—4	—3	0	—3	+2	+2	+2	+2	0	+2
November	+3	—1	—6	—2	—2	0	+2	+2	+1	+3	—1	0
December	+5	—1	—2	—1	—1	0	0	0	—5	+1	—3	—3
Year	+0.8	+0.2	—1.3	—0.8	—1.0	—0.7	+0.3	—0.3	—0.8	+3.0	0.0	—0.4

TABLE V.—DIFFERENCES FROM THE MONTHLY NORMALS, 1843.

Western hour angles of the moon.

1843.	U. cul. 0	1	2	3	4	5	6	7	8	9	10	11^h
January	---	---	---	---	---	---	---	---	---	---	---	---
February	---	---	---	---	---	---	---	---	---	---	---	---
March	---	---	---	---	---	---	---	---	---	---	---	---
April	+10	—3	+14	—5	+8	+1	0	+1	—5	+3	—2	—3
May	+6	—3	+4	+3	+3	—3	—3	—5	—1	—4	+1	—5
June	+1	—1	—4	+2	+2	+2	+1	+2	+1	+3	+4	+4
July	+7	+5	+4	+5	+4	+8	+3	+3	+3	+1	—1	—1
August	0	—2	+2	—1	+4	0	+5	—3	+3	—4	+2	—2
September	—3	+8	0	+5	—3	+4	+4	—7	+3	—3	+1	—3
October	+2	+2	+3	+3	+2	+2	—1	—4	—2	—3	—2	—3
November	+1	+2	+2	—3	+1	—2	—1	+1	0	+1	+4	0
December	+1	+2	+3	—2	0	+2	+2	+1	—1	0	+1	—2
Year	+2.4	+1.3	+3.0	+0.4	+2.0	+1.3	+0.8	—1.1	—0.2	—0.6	+0.9	—1.7

Eastern hour angles.

1843.	L. cul. 0	1	2	3	4	5	6	7	8	9	10	11^h
January	---	---	---	---	---	---	---	---	---	---	---	---
February	---	---	---	---	---	---	---	---	---	---	---	---
March	---	---	---	---	---	---	---	---	---	---	---	---
April	—2	0	—1	—6	—5	—6	+5	+1	+4	0	—2	+1
May	—2	—6	—3	0	+2	—1	+1	+2	+4	+2	+6	+2
June	+2	0	+1	+2	—1	+2	—2	+1	—1	0	+2	—2
July	—5	—4	—5	—5	—3	—5	—6	—3	+2	+1	+3	+4
August	+2	—2	0	—1	—1	—1	+5	—2	+2	—4	+4	+2
September	—2	0	—3	—1	—6	—3	—2	+4	+3	+1	—2	+4
October	—5	—1	+1	+1	0	0	0	0	0	+2	+3	0
November	—1	0	—3	+1	+3	—1	—1	—3	—1	+2	—2	+1
December	—2	0	—2	—1	—3	—2	—3	—3	—3	—2	—3	0
Year	—1.9	—1.2	—1.6	—0.9	—1.2	—1.7	—0.6	—0.8	+0.5	0.0	+0.6	+1.1

In making up the annual means, the October, November, and December values have received double weight; they are derived from double the number of observations.

TABLE VI.—DIFFERENCES FROM THE MONTHLY NORMALS, 1844.

Western hour angles of the moon.

1844.	U. cul. 0	1	2	3	4	5	6	7	8	9	10	11h
January	+1	+5	0	0	—2	—1	—2	—1	—1	—1	—1	0
February	—1	—1	+1	+1	0	—1	+3	+3	0	+1	—1	—3
March	—4	—5	+2	—1	+3	—1	+1	—2	+1	—1	0	—3
April	+1	—1	+5	+2	0	+1	—4	+3	+3	0	—2	—3
May	0	+1	+1	+1	+1	+2	+3	+2	+2	+1	+2	0
June	+1	—2	—2	—2	—2	—1	—2	0	0	+3	0	+3
July	+2	+2	+2	+1	+1	+3	0	0	—2	0	0	0
August	+1	—2	—2	—2	—1	—2	—2	—6	—2	—4	—1	0
September	+1	+1	+1	—1	—1	—1	—2	0	—2	—2	—3	0
October	0	0	0	—2	—1	—3	—2	—3	—3	—2	0	+3
November	+2	+5	+5	+3	+2	0	+1	+3	+1	0	—2	—3
December	—1	0	+1	+3	0	—1	—1	—1	+2	+1	0	0
Year	+0.3	+0.3	+1.2	+0.3	0.0	—0.4	—0.6	—0.2	—0.1	—0.3	—0.7	—0.5

Eastern hour angles.

1844.	L. cul. 0	1	2	3	4	5	6	7	8	9	10	11h
January	—1	—4	—2	—3	—2	—2	—3	0	+1	—2	—1	+3
February	—3	—1	+2	+1	0	—3	—4	—2	+1	—1	—1	0
March	—6	+1	—4	—2	—1	+4	+2	+5	+2	—4	—4	—1
April	—1	+1	+1	0	—1	—1	—3	—1	0	—1	—1	+2
May	—1	—2	0	—2	—2	—1	—3	—2	—2	—1	0	0
June	+1	0	+1	+1	0	+1	+2	—2	—1	+2	0	—1
July	+1	0	0	+1	0	—1	+1	+2	+1	0	+1	+1
August	—2	—1	—1	0	+2	+6	+4	+4	+3	+2	+1	—1
September	—1	—2	—1	—1	0	0	+2	+3	+2	+1	+4	+4
October	+4	+1	+1	+1	0	+2	+1	+1	0	+3	+1	+3
November	—4	—4	—1	+1	+1	+1	+2	+1	0	+1	+3	+2
December	+2	+2	+1	+2	0	+1	—3	—3	—1	—1	+2	—1
Year	—1.0	—0.8	—0.2	—0.1	—0.3	+0.6	—0.2	+0.5	+0.5	—0.1	+0.4	+1.0

TABLE VII.—DIFFERENCES FROM THE MONTHLY NORMALS, 1845.

Western hour angles of the moon.

1845.	U. cul. 0	1	2	3	4	5	6	7	8	9	10	11h
January	+1	+2	+2	+2	+2	+4	+1	+1	-2	0	+3	+2
February	-2	0	-3	-2	-2	-2	0	0	+1	0	-1	+1
March	0	-1	-2	+1	+1	0	+2	+1	-2	0	+4	-1
April	-2	-4	-2	0	-1	-1	0	+2	+5	+4	+4	+2
May	-1	+1	+2	+1	+2	+3	-2	-1	0	+1	-1	0
June	+1	+1	+2	0	-2	0	-1	-1	-3	-4	-3	-2
Mean	-0.5	-0.2	-0.2	+0.3	0.0	+0.7	0.0	+0.3	-0.2	+0.2	+1.0	+0.3

Eastern hour angle.

1845.	L. cul. 0	1	2	3	4	5	6	7	8	9	10	11h
January	0	-1	-5	-3	0	-1	0	-2	-2	-4	-3	0
February	+2	+3	+3	+3	+2	0	0	-1	0	-1	-2	0
March	+4	+2	-1	+2	-2	+1	-1	+1	-2	-2	-3	-3
April	-1	+1	-1	-1	-2	-2	-1	-1	0	+1	+1	+1
May	-1	-1	-2	-3	-2	-1	-1	0	+1	+1	0	-1
June	-1	+1	0	+1	+2	+1	+2	+2	+1	+3	+2	+3
Mean	+0.5	+0.9	-1.0	-0.2	-0.3	-0.3	-0.2	-0.2	-0.3	-0.3	-0.9	0.0

Before we combine the above results of years and parts of years, it is desirable to inquire into the variability of the lunar effect in summer and winter. Considering the months from April to September (inclusive) as summer months, and those from October to March (inclusive) as winter months, and combining the differences from the monthly normals in each year according to the number of observations, we obtain the following results:—

TABLE VIII.—LUNAR-DIURNAL VARIATION IN SUMMER AND WINTER, 1841 TO 1845.

(Expressed in scale divisions.)

Western hour angles.

	U. cul. 0	1	2	3	4	5	6	7	8	9	10	11h
Summer	+1.0	-0.1	+0.6	+0.2	-1.0	+0.8	-0.4	-0.4	+0.1	-0.1	+0.6	-1.0
Winter	-0.8	+0.8	+0.9	+0.1	+0.4	-0.6	-0.5	-0.5	0.0	+0.1	+0.4	0.0

Eastern hour angles.

	L. cul. 0	1	2	3	4	5	6	7	8	9	10	11h
Summer	-0.5	-0.8	-0.3	-0.9	-0.4	-0.4	+0.4	+0.4	+0.6	+0.6	+1.1	+0.8
Winter	-0.2	+0.3	-0.9	+0.3	+0.4	+0.2	-0.5	-0.7	-0.5	0.0	-1.1	0.0

These numbers are sufficiently irregular to indicate that we cannot hope to deduce any separate results for the seasons, the number of observations (about 9800) being altogether insufficient. We can therefore in our general combination of the annual means give equal weight to the results from the six months of hourly observations in 1845, and to the results from the twelve months of bi-hourly observations in 1842; compared with these results, those of 1844 have the weight two.

TABLE IX.—RECAPITULATION OF THE ANNUAL MEANS EXHIBITING THE LUNAR-DIURNAL VARIATION FROM OVER 19,500 OBSERVATIONS BETWEEN FEBRUARY, 1841, AND JUNE, 1845, INCLUSIVE.

Western hour angles.

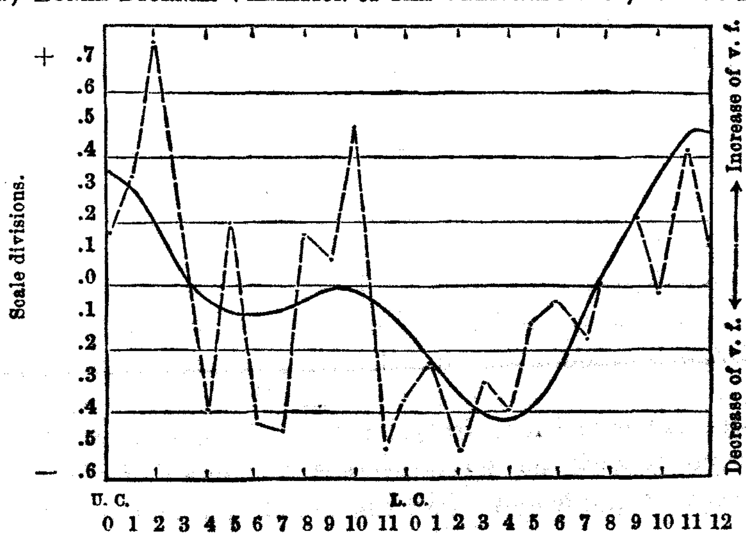
Weight.	Year.	U. cul. 0^h	1	2	3	4	5	6	7	8	9	10	11^h
1	1841	−1.5	+0.4	−0.9	+0.7	−2.0	+0.6	−1.5	−0.2	−0.3	+0.7	+0.5	−1.2
1	1842	−0.4	−0.1	+0.2	−1.1	−2.3	−1.0	−0.6	−1.3	+1.8	+0.7	+2.0	+0.4
1	1843	+2.4	+1.3	+3.0	+0.4	+2.0	+1.3	+0.8	−1.1	−0.2	−0.6	+0.9	−1.7
2	1844	+0.3	+0.3	+1.2	+0.3	0.0	−0.4	−0.6	−0.2	−0.1	−0.3	−0.7	−0.5
1	1845	−0.5	−0.2	−0.2	+0.3	0.0	+0.7	0.0	+0.3	−0.2	+0.2	+1.0	+0.3
	Mean	+0.10	+0.33	+0.75	+0.15	−0.38	+0.20	−0.42	−0.45	+0.15	+0.07	+0.50	−0.53

Eastern hour angles.

Weight.	Year.	L. cul. 0^h	1	2	3	4	5	6	7	8	9	10	11^h
1	1841	+0.5	+0.3	+1.2	+0.3	+0.7	+0.7	+0.6	−0.7	−0.1	−1.2	−0.8	−0.2
1	1842	+0.8	+0.2	−1.3	−0.8	−1.0	−0.7	+0.3	−0.3	−0.8	+3.0	0.0	−0.4
1	1843	−1.9	−1.2	−1.6	−0.9	−1.2	−1.7	−0.6	−0.8	+0.5	0.0	+0.6	+1.1
2	1844	−1.0	−0.8	−0.2	−0.1	−0.3	+0.6	−0.2	+0.5	+0.5	−0.1	+0.4	+1.0
1	1845	+0.5	+0.9	−1.0	−0.2	−0.3	−0.3	−0.2	−0.2	−0.3	−0.3	−0.9	0.0
	Mean	−0.35	−0.23	−0.52	−0.30	−0.40	−0.13	−0.05	−0.17	+0.05	+0.22	−0.05	+0.42

If we represent these values graphically, we find the general shape of the curve to be similar to that of the horizontal component, it is double crested with a principal maximum a little before the upper culmination, and a principal minimum about 3½ hours after the lower culmination of the moon; the average epoch of the vertical force tide is, therefore, about one and a half hours apparently in advance of the culminations. The secondary wave is very feeble, its greatest value

(A.) LUNAR-DIURNAL VARIATION OF THE VERTICAL FORCE, 1841 TO 1845.



happens about 9^h, western hour angle, and its least value about three hours before, giving a range of nearly one-tenth part of the principal range. The observed values for the hours 8, 9, 10 (west) however, seem to indicate that the secondary wave is really larger, but in the present case apparently reduced by the accidentally low values at the hours 11 and 12.

The following expression has been deduced to express the lunar-diurnal variation of the vertical force:—

$$V_{☾} = -0.04 + 0.27 \sin(\theta + 72°) + 0.20 \sin(2\theta + 134°)$$

θ counts from the upper culmination, westward; $V_{☾}$ is expressed in scale divisions. The smooth, full curve in the diagram is computed by the formula; the differences between the observed and computed values are sufficiently well exhibited in the diagram. The probable error of any single hourly value is ± 0.20 scale divisions.

In the following expression M signifies millionth parts of the force:—

$$V_{☾} = -\overset{M}{1.3} + \overset{M}{8.9} \sin(\theta + 72°) + \overset{M}{6.6} \sin(2\theta + 134°).$$

Maximum value of $V_{☾}$, 28^m before the upper culmination, $= + .38$ scale divisions; minimum value at $15^h\ 30^m$, -0.43 scale divisions, hence lunar-diurnal range 0.81 scale divisions $= 0.000027$ parts of the force $= 0.00034$ in absolute measure. This range is so small that the correction for temperature due to a change of but $0°.08$ would surpass it.

We have already seen that we cannot bring a sufficient number of observations to bear upon any *part* of the entire series, and are therefore not in a condition to pursue this subject of the lunar effect to any greater length.

At Toronto the curve is also double-crested with maxima three and a half hours after the moon's transits, but compared with Philadelphia the principal and secondary waves appear exchanged. The range at Toronto is 0.000012 parts of the force, nearly one-half of the Philadelphia range; we have already noticed a similar difference of range in the solar-diurnal variation, the Toronto range of which was also about one-half of that at Philadelphia. In connection with this it may be well to state that the dip at Toronto is 75° 15′, and at Philadelphia 71° 59.′

Lunar Effect upon Inclination and Total Force.—The combination of the horizontal and vertical components to inclination and total force, is effected by the formulæ:—

$$\Delta\theta = \sin\theta \cos\theta \left(\frac{\Delta Y}{Y} - \frac{\Delta X}{X}\right)$$

$$\frac{\Delta\phi}{\phi} = \cos^2\theta \frac{\Delta X}{X} + \sin^2\theta \frac{\Delta Y}{Y}$$

in which expressions $X =$ horizontal force, $Y =$ vertical force, $\phi =$ total force, and $\theta =$ inclination. The discussion of the observations for dip, in Part XII, gives the value $\theta = 71° 59'$, answering to the year 1843. Column 2 of the following table is derived from the preceding Table IX, after changing the scale divisions into their equivalents of parts of the force, one division being equal to 0.000033; column 3 is formed similarly from Table VIII, of Part VI, one division being equal to 0.0000365. Columns 4 and 5 contain the corresponding values of the lunar-diurnal variation of the inclination and total force, the former expressed in seconds, the latter in parts of the total force. The letter M, heading columns 2, 3, and 5, signifies units of the sixth place of decimals or millionth parts of the force.

TABLE X.—LUNAR-DIURNAL VARIATION OF THE INCLINATION AND TOTAL FORCE.

☾'s hour angle.	$\frac{\Delta Y}{Y}$	$\frac{\Delta X}{X}$	$\Delta\theta$	$\frac{\Delta\phi}{\phi}$
	M.	M.	″	M.
U. C.	+ 3.3	+11.0	—0.5	+ 4.0
1	+10.9	+18.2	—0.4	+11.6
2	+24.7	+32.8	—0.5	+25.4
3	+ 5.0	+40.1	—2.1	+ 8.3
4	—12.5	+ 7.3	—1.2	—10.6
5	+ 6.6	+25.5	—1.1	+ 8.4
6	—13.9	0.0	—0.9	—12.6
7	—14.8	+ 3.6	—1.1	—13.0
8	+ 5.0	—21.9	+1.6	+ 2.4
9	+ 2.3	—14.6	+1.0	+ 0.7
10	+16.5	— 7.3	+1.4	+14.2
11	—17.5	+ 3.6	—1.3	—15.4
L. C.	—11.6	+25.5	—2.2	— 8.1
1	— 7.6	+ 7.3	—0.9	— 6.2
2	—17.2	+14.6	—1.9	—14.1
3	— 9.9	—11.0	+0.1	— 9.9
4	—13.2	+ 3.6	—1.0	—11.5
5	— 4.3	—25.5	+1.3	— 6.3
6	— 1.7	—47.4	+2.8	— 6.1
7	— 5.6	—21.9	+1.0	— 7.2
8	+ 1.6	—18.2	+1.2	— 0.3
9	+ 7.3	—36.5	+2.7	+ 3.1
10	— 1.7	— 3.6	+0.2	— 1.9
11	+13.9	+ 7.3	+0.4	+13.3

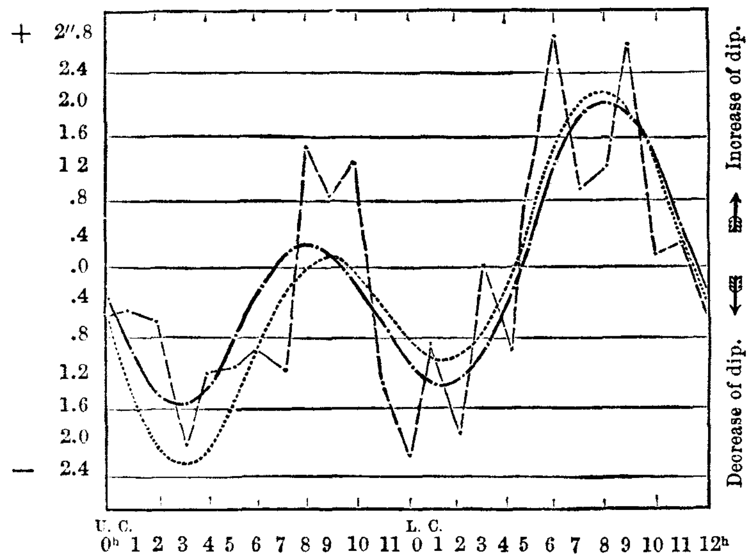
The numbers in column 2 are deduced from observations between 1841 and 1845, those in column 3 from observations between 1840 and 1845, the difference, however, is immaterial as far as it refers to the dip and total force, the lunar variations being nearly the same for a few adjacent years. The total number of observations employed in the combination is 41558.

The lunar-diurnal variation in the dip is well represented by the formula,

$$\theta_{☾} = -0''.06 + 0''.86 \sin(\theta + 156°) + 1''.30 \sin(2\theta + 206°)$$

the corresponding curve, as well as the observed values, are exhibited in the following diagram. The heavy smooth curve is the Philadelphia computed variation, the dotted curve the Toronto variation, inserted here for comparison. The correspondence between these curves is certainly remarkably close considering the minuteness of the lunar effect and the somewhat long process of deducing it.

(B.) Lunar-Diurnal Variation of the Inclination.

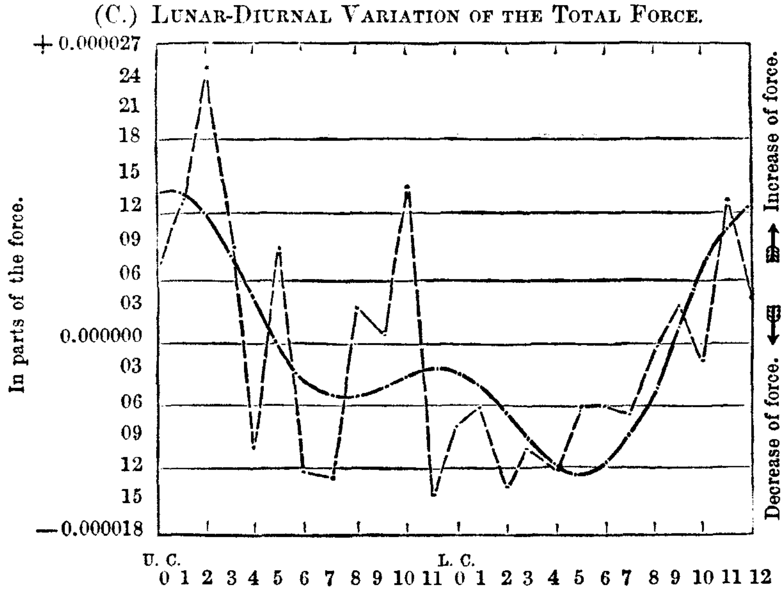


Maxima at 8^h and 20^h (principal), minima at 3^h (principal) and $13\frac{1}{2}^h$.
Total range 3″.6. Probable error of any single hourly representation $\pm 0''.7$.
The lunar diurnal variation in the total force is represented by the equation:—

$$\phi_{\text{☾}} = -\overset{M}{1.3} + \overset{M}{8.9}\,sin\,(\theta + 63°) + \overset{M}{6.3}\,sin\,(2\theta + 84°)$$

an expression not differing much from $V_{\text{☾}}$ owing to the large dip. The observed and computed values of $\frac{\Delta\phi}{\phi}$ are shown in the diagram, which nearly resembles that of the vertical force.

(C.) Lunar-Diurnal Variation of the Total Force.



Maxima at $\frac{1}{2}^h$ (principal) and 11^h; minima at $7\frac{1}{2}^h$ and 17^h (principal).
Total range 0.000026. Probable error of any single hourly representation ± 0.000006.